Algebra Ideas

A Spire Maths Activity

https://spiremaths.co.uk/algebra-ideas/

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Key Ideas in Teaching Mathematics

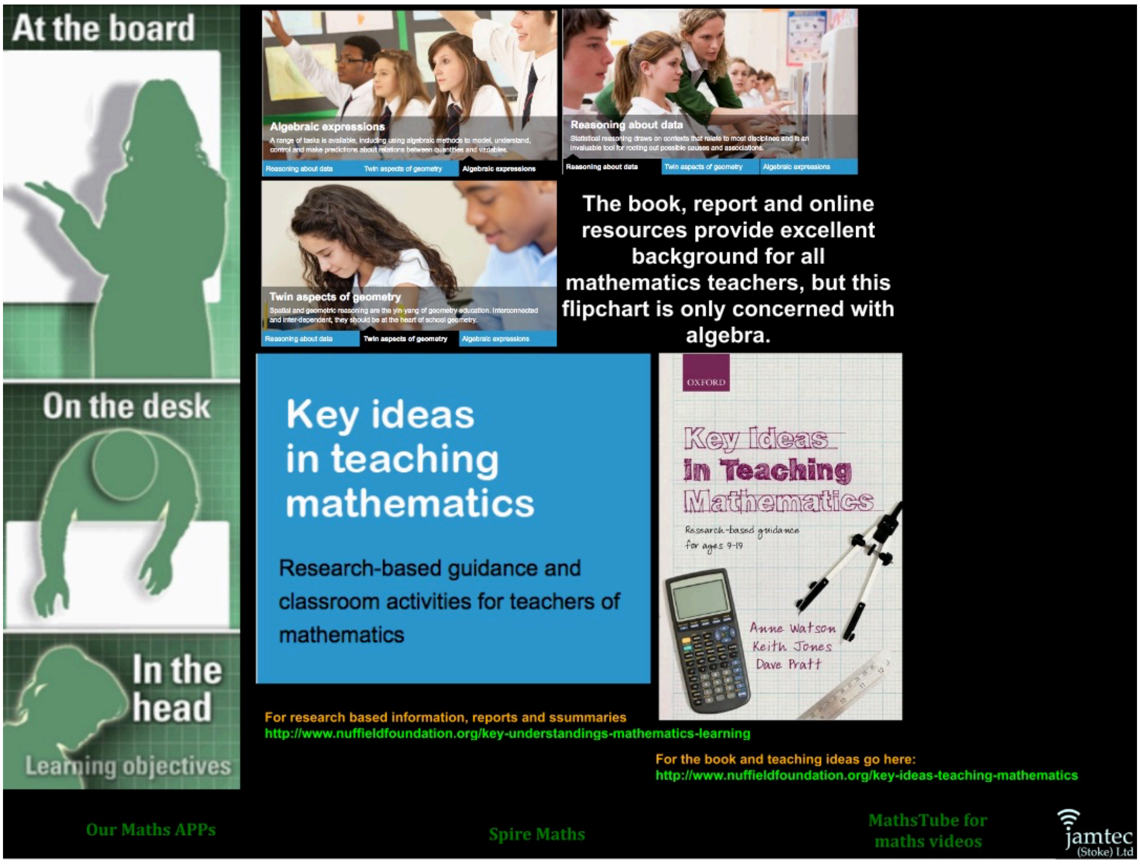
Main site for the book, summary research reports and other resources is:

http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

Research report can be found at

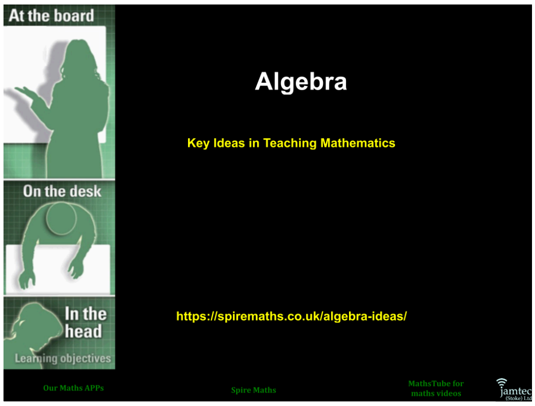
http://www.nuffieldfoundation.org/key-understandings-mathematics-learning

The book report and online resources are concerned with many topics, however this file and flipchart is only concerned with algebra ideas.



Algebra Ideas

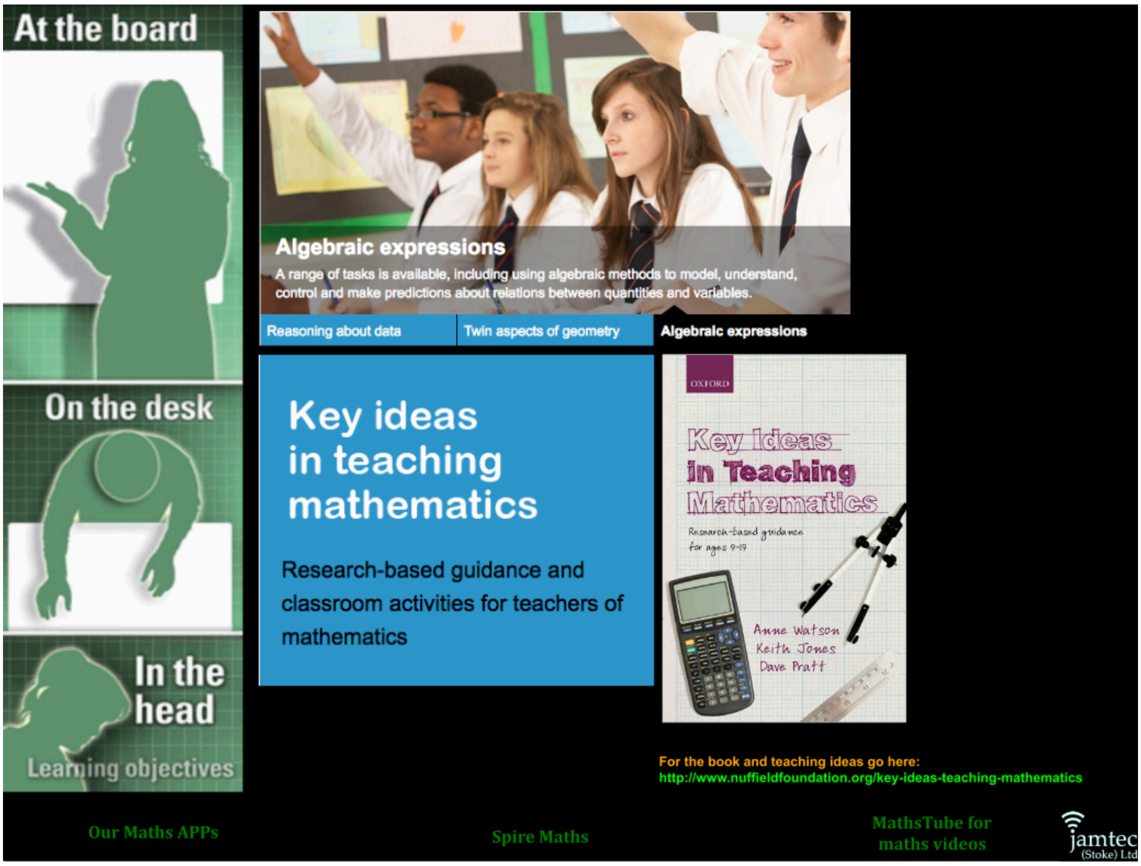
Alongside this we have provided a flipchart, copies of slides included here, plus alternative websites for some of the resources (including replacement for a broken link).



https://spiremaths.co.uk/algebra-ideas/

Other resources at:

https://spiremaths.co.uk/ilim/
https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/22489-10312-Numbers%20and%20Algerbra%203-redacted.pdf (login needed – free)
https://mathsteachers.files.wordpress.com/2014/08/algebra-makes-sense.pdf
http://www.bowland.org.uk/projects/keeping_the_pizza_hot.html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On the desk
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Research-based guidance for ages 9-19
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For the book and teaching ideas go here:
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Paper 6: Algebraic reasoning
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For the book and teaching ideas go here:
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Headlines

- Algebra is the way we express generalisations about numbers, quantities, relations and functions. For this reason, good understanding of connections between numbers, quantities and relations is related to success in using algebra. In particular, students need to understand that addition and subtraction are inverses, and so are multiplication and division.

- To understand algebraic symbolisation, students have to (a) understand the underlying operations and (b) become fluent with the notational rules. These two kinds of learning, the meaning and the symbol, seem to be most successful when students know what is being expressed and have time to become fluent at using the notation.
- Students have to learn to recognise the different nature and roles of letters as: unknowns, variables, constants and parameters, and also the meanings of equality and equivalence. These meanings are not always distinct in algebra and do not relate unambiguously to arithmetical understandings, Mapping symbols to meanings is not learnt in one-off experiences.

- Students often get confused, misapply, or misremember rules for transforming expressions and solving equations. They often try to apply arithmetical meanings to algebraic expressions inappropriately. This is associated with over-emphasis on notational manipulation, or on 'generalised arithmetic', in which they may try to get concise answers.

http://www.nuffieldfoundation.org/sites/default/files/Summary%20of%20findings.pdf

http://www.nuffieldfoundation.org/sites/default/files/P6.pdf

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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Quantities and Algebraic Expressions

Five key themes:

- notations and conventions
- structures and relations
- the equals sign
- modelling
- using algebra to reason

Recognition and construction of algebraic statements

Much manipulation of expressions and equations can now be done by software, but students still need to be able to construct and recognise algebraic statements in their various forms.

A range of tasks is available, including using algebraic methods to model, understand, control and make predictions about relations between quantities and variables. At the heart of all teaching methods is the need to recognise and move between equivalent expressions, equations and representations.

There remains a fundamental need to understand what letters and other symbols represent, and how manipulations provide different, but equivalent, expressions.

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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Notations and conventions

Letters can stand for **labels** (e.g. ABC as the vertices of a triangle), **givens** (e.g. a, b and c as lengths of sides of a triangle), **unknowns** (e.g. x in $3x+5 = 20$), **variables** (e.g. x and y in $y=mx+c$), **parameters** (e.g. m and c in $y=mx+c$) and **constants** (e.g. c in $y=mx+c$), and some letters (e.g. e, g, π) have **specific constant meanings**.

Students construct their own meanings such as:

- letters are shorthand for objects, e.g. a = apple
- letters have fixed meanings, e.g. area = l x w (so l=length and w=width)
- letters can be seen as alphabetic codes, e.g. a=1, b=2 and therefore p < q etc.
- all expressions have to be conjoined, so 3a + 5b = 8ab
- 3m could mean 3 x m, but 32 does not mean 3 x 2, and 3m might also mean '3 metres'
- To understand the need for rules of notation:compare the different answers obtained by different interpretations to see why rules are necessary.

To understand conventions: explore how different expressions work using computer algebra.

Research shows that holistic ways of relating algebra to situations are successful in helping students to learn the procedures, meanings and uses of algebra. Holistic teaching combinesarithmetic, algebra, data, graphs and functions side by side.

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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Notations and conventions 1: Evaluating Algebraic Expressions

A4 • Evaluating algebraic expressions

To enable learners to:

- distinguish between and interpret equations, inequations and identities;
- substitute into algebraic statements in order to test their validity in special cases.

This sequence of tasks enables students to compare equations, inequalities, and identities (equivalent expressions) by various means. Substitution is used to find out when expressions are equal in value. It is more common in textbooks to use substitution merely as a means to practice using symbolic conventions, but here it has a role to play in understanding the underlying relations.

The tasks are designed to challenge common misconceptions about the relations between numbers and variables and unknowns.

https://spiremaths.co.uk/ilim/

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Notations and conventions 2: Number Spirals

The intended task is on pages 37-38 and is about the intriguing and unexpected patterns that arise from spirals on number grids. These structural, not sequential patterns, and need careful explanation.

They are not predictable from number patterns alone, so they avoid a purely inductive approach to generalisation. Often these are solved by students going to and fro between number, algebra, spatial features, specific cases and generalised relations.

https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/22489-10312-Numbers%20and%20Algebra%203-redacted.pdf

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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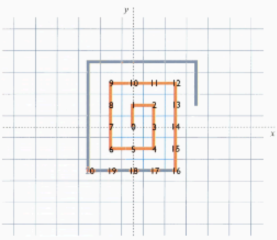
Notations and conventions 2: Number Spirals

37 *Number patterns*

D A number spiral



Copy this spiral of numbers on to a sheet of squared paper and continue it at least as far as 50.

D1 Are there any patterns in the way certain groups of numbers are distributed on the grid? For example, how are *odd* and *even* numbers spread out?

The positions of numbers on the grid can be described using x- and y-axes and counting squares horizontally and vertically from 0. Using this method 13 is at position (2, 1) and 17 is at (1,−2).

D2 Find the square numbers. Describe their position in the grid.

D3 The numbers 4, 16, 36 … are at (1, −1), (2, −2), (3, −3) …

- **a** Without filling in all the numbers, find out what the positions of 64, 100 and 144 will be.
- **b** If n is an even number where will n^2 be?
- **c** What are the positions of the square numbers 1, 9, 25, and 49?

Exploring number patterns 38

- **d** Without filling in all the numbers, say what the positions of 81, 121 and 169 will be.
- **e** If m is an odd number, where will m^2 be?

You can make a sequence of numbers by multiplying consecutive pairs of whole numbers together:

$1 \times 2 \quad 2 \times 3 \quad 3 \times 4 \quad 4 \times 5 \quad 5 \times 6 \ldots$

D4 Work out the numbers in this sequence and write down their positions in the grid.

- **a** Is there a pattern to these positions?
- **b** Is the position affected by whether the smaller number is even or odd?
- **c** Try to give a general position for $p \times q$ where p and q stand for consecutive numbers.

D5 Make a table of values to find the sequence of numbers produced by the expression $4x^2 - x$. Choose different values of x such as 1, 2, 3, 4 and complete this table.

x	1	2	3	4
x^2	1	4		
$4x^2$	4	16	…	
$-x$	−1	−2	…	
$4x^2-x$	3	14	…	

- **a** Where can you find this sequence on the grid?
- **b** What sequence do you get if you put negative values of x into the expression $4x^2 - x$? (For example if $x = -2$ you get the value 18 for the expression.) Find this sequence on the grid.

D6
- **a** Find the sequence of numbers in the grid that runs:

  71 41 19 5 3 13 31 57

- **b** Evaluate the expression $(4x^2 - 2x + 1)$ for values of x such as 1 2 3 4 … This will produce half of the sequence for you. Try to find an expression that will produce the other half.

https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/22489-10312-Numbers%20and%20Algebra%203-redacted.pdf

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Notations and conventions 3: (Generating by) Using a Formula

Exercises often ask students to reason inductively a formula for a given sequence, but the tasks on pages 42-43 turn this process on its head and offer the formula to generate the sequence.

This way round, students stand a good chance of spotting the effects of different parameters in the formula. If not, they can change parameters for themselves and see how the generated sequence changes. This is an old resource which mentions programming in BASIC in some of the tasks, but they can be adapted for use with spreadsheets.

https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/22489-10312-Numbers%20and%20Algebra%203-redacted.pdf

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Notations and conventions 3: (Generating by) Using a Formula

Finding the nth term of a sequence 42

B Using a formula

You can make a sequence by taking a formula such as

$3n + 2$

and putting different values of n into it.
Putting $n = 1, 2, 3$ … gives:

$3 \times 1 + 2 \rightarrow 5$, $3 \times 2 + 2 \rightarrow 8$, $3 \times 3 + 2 \rightarrow 11$, $3 \times 4 + 2 \rightarrow$ …

B1 Write down the first six terms of the sequence you get from the formula $3n + 2$.

B2 Write down the first six terms of the sequences generated by:

a $2n + 5$ **b** $3n - 6$ **c** $n^2 + 8$
d $3 - n$ **e** $10 - 5n$

B3 What calculator key strokes will produce the sequences in question **B2**?

B4 **a** Write down the sequence you get from the formula $4n - 7$ for values of n from 1 to 6.

b Find the 100th term of this sequence.

c How many terms would you have to go through before reaching one that is bigger than 200?

43 *Sequences*

BASIC

B5 This program in BASIC will produce the first 20 numbers of the sequence that comes from the formula $4n + 3$:

```
10  FOR N = 1 TO 20
20  PRINT N, 4*N + 3
30  NEXT N
40  END
```

a Run this program and read off the 13th and 17th terms of the sequence.

b Modify the program to produce the first 4000 terms of the sequence. Run it.

c Which is the first term in the sequence which is greater than 7000? (You may need to use the 'escape' key while the program is running.)

B6 The following program will produce the sequence of up to 500 numbers which comes from the formula $6n - 11$ but it will stop when it reaches a number greater than 1000:

```
10  FOR N=1 TO 500
20  PRINT N, 6*N-11
30  IF 6*N-11>1000 THEN 50
40  NEXT N
50  END
```

Run the program and find the first term in the sequence which is greater than 1000.

B7 **a** Write a program which will precede the first 2500 terms of the sequence that comes from the formula $1000 - 13n$.

b Read off the 1250th term and the 1750th term.

c How many terms are greater than –2000?

d Modify the program to stop once the terms in the sequence become negative.

B8 Ampshire Electricity Company charges customers according to the formula

£9.86 + 7.59p × N

where £9.86 is the fixed standing charge, 7.59p is the cost of a unit of electricity and N is the number of units used. Write a program to show the charges for supplying up to 2000 units of electricity.

https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/22489-10312-Numbers%20and%20Algebra%203-redacted.pdf

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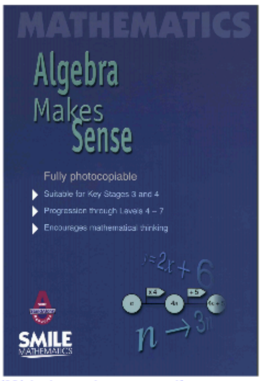
Notations and conventions 4: Language

The relevant task is on pages 16-17 and involves preparing cards for a matching exercise between words and symbols.

Expressing situations in words is a pre-cursor to using symbols, and students can then see that algebra is expressing succinctly what is sometimes long-winded and complicated in words.

By starting with words, the order of operations is easier to understand as a form of notation, rather than the notation coming first, and word problems become more accessible because students get used to mathematising them.



https://mathsteachers.files.wordpress.com/2014/08/algebra-makes-sense.pdf

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Notations and conventions 4: Language

Algebra Makes Sense — **Level 5**

Teachers notes

Matching Mappings

Type
Suitable for an individual, or for a pair or the whole class working in pairs.

Objective
To identify which functions were used to create mapping diagrams.

Framework for Teaching Mathematics
Substitute positive integers into simple linear expressions and formulae. (Y7 p. 138)
Express simple functions in words, then using symbols; represent them in mappings. (Y7 p. 160)
Substitute integers into simple formulae. (K) (Y8 p. 139)

Resources
One copy of the resource sheet (consumable) for each pair of students. Scissors and glue.

Description
Students identify which of the three functions was used to create a given mapping diagram. This initial activity could be used as an introduction to the whole class.

The main part of the activity requires students to match mapping diagrams to the correct functions by cutting out the functions and mapping diagrams. Students can either record the matching pairs by copying them into their books or alternatively they can stick the matched ones into their books. One function and one mapping diagram will not match. In order to complete the activity students are required to write down the missing function and create a mapping diagram. Mapping diagram D and F provides a useful discussion point for the plenary session highlighting subtraction and multiplication of negative numbers.

Enriching Mathematical Thinking
The functions used to create mapping diagrams D and E can be used as examples of 'many to one' mappings which have no inverse. Students can be asked to write down other functions and matching mapping diagrams for homework. Students can use their examples as a starting activity for the next lesson, e.g. by showing a mapping diagram and asking the rest of the class to find the matching function.

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Algebra Makes Sense — **Resource Sheet**

Matching Mappings

Which of the three functions below was used to create this mapping diagram?

Function: **double**
Function: **multiply by 3**
Function: **add 3**

Mapping diagram	
3	6
7	10
1	4
63	66
2½	5½

Match each of the functions at the bottom of the page to the correct mapping diagram below.

- You will find that one function does not match a mapping diagram. Draw a mapping diagram to match this function.
- You will find that one mapping diagram does not match any of the functions. Write down the function that was used to create this mapping diagram.

Mapping diagram A		Mapping diagram B		Mapping diagram C	
5	15	5	13	5	30
2	12	7	17	2	12
9	19	2	7	7	42
63	73	4	11	10	60
-10	0	9	21	2½	15

Mapping diagram D		Mapping diagram E		Mapping diagram F	
4	16	12	2	3	5
7	49	8	3	7	1
10	100	13	3	5½	2½
-1	1	5	0	9	-1
1	1	15	0	4	4

Function	Function	Function
subtract from 8	the remainder when divided by 5	add 10
subtract 2 then double	square	multiply by 6

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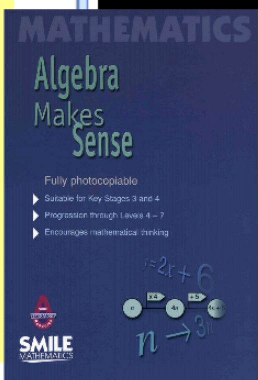

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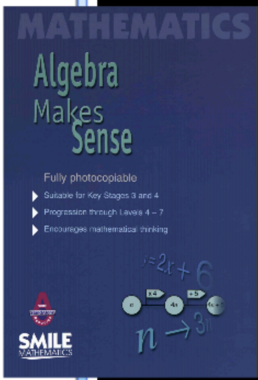
Notations and conventions 5: Equivalence

The relevant task is on pages 36-37.

Traditionally, most of the algebra curriculum used to be about learning to manipulate expressions by collecting like terms, factorising, multiplying brackets and so on. This achieves alternative forms of expressing the same relations between unknowns and variables.

For algebra to be meaningful, students can see that these manipulations (which can now be done by software) produce equivalent expressions to describe situations that the students will understand. Embedded in this kind of work, but usually left implicit, is the use of = to mean 'is equivalent to', i.e. whatever the value of the variables. This is in contrast to equations where non-equivalent expressions are temporarily equal for particular values of the variables.



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Notations and conventions 5: Equivalence

Algebra Makes Sense — **Level 7**

Teachers notes

Equivalent Expressions

Type

Suitable for a pair or the whole class working in pairs.

Objective

To use substitution and algebraic manipulation to identify equivalent expressions.

Framework for Teaching Mathematics

Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solutions). (K) (Y9 p.123)

Use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject. (Y9 p. 139)

Resources

One copy of the resource sheet (consumable) for each pair of students. Scissors and glue.

Description

There are 30 algebraic expressions. Students are required to group the algebraic expressions into 10 equivalent groups. As the activity may well take students more than a lesson to complete, the expressions have been divided into two sets. The second set can be used as an excellent homework activity. Students can decide whether to record their work in their books or alternatively stick the equivalent expressions in their books, recording their substitution of values for each expression.

A good introduction to this activity is to work through the initial example and then ask the students to write other algebraic expressions which are equivalent to $3(2a^2 + 1)$. Students can then check their expression by substituting a suitable value for a.

Enriching Mathematical Thinking

Ask students to write down three expressions which are not equivalent but which give the same result when the same value is substituted into each. For example, $a + 2$, $2a + 2$ and $3a + 2$ will all give the same value when $a = 0$. Then challenge students to find three expressions which are not equivalent but which give the same results when two values are substituted into each. For example, $2a^2 + 2$, $2a + 2$ and $2a^3 + 2$ will all give the same value when $a = 0$ and $a = 1$.

36 © RBKC Smile Mathematics

Algebra Makes Sense — **Resource Sheet**

Equivalent Expressions

These three expressions are equivalent.

$\boxed{6a^2 + 3}$ $\boxed{3(2a^2 + 1)}$ $\boxed{4a^2 + 3 + 3a + 2a^2 - 3a}$

Check that they are equivalent by substituting values for a.

For example let $a = 7$.

$6 \times 7^2 + 3$	$3(2 \times 7^2 + 1)$	$4 \times 7^2 + 3 + 3 \times 7 + 2 \times 7^2 - 3 \times 7$
$6 \times 49 + 3$	$3(2 \times 49 + 1)$	$4 \times 49 + 3 + 3 \times 7 + 2 \times 49 - 21$
$= 294 + 3$	$= 3(98 + 1)$	$= 196 + 3 + 21 + 98 - 21$
$= 297$	$= 297$	$= 297$

For each group of 15 expressions below:

- Cut out the expressions and match them in groups of three equivalent expressions.
- Check your groups of three equivalent expressions by substituting values for m and x.

$2m - m^2$	$\frac{1}{2}(10m + 10)$	$3m^2 - 2$
$m(m + 1) + m$	$3m^2 + 2m - 2 - 2m$	$5(m + 1)$
$\frac{1}{2}(6m^2 - 4)$	$-m(m - 2)$	$(m + 1)^2 - 1$
$2m(2m - 1)$	$5m + 5$	$m^2 + 2m$
$4m^2 - 2m$	$m^2 + 2m - 2m^2$	$2(2m^2 - m)$

$x(\frac{x}{2} + 3)$	$0.5x^2 + 3x$	$3x - 21$
$6x + 3x^2$	$3(x - 8) + 3$	$\frac{x}{2} + 2$
$3(2x + x^2)$	$2x + 6$	$3(x - 7)$
$2(x + 3)$	$\frac{1}{2}(x + 2)(x + 4) - 4$	$2(x + 2) + 2$
$x - 0.5x + 2$	$\frac{1}{2}(x + 4)$	$3x(2 + x)$

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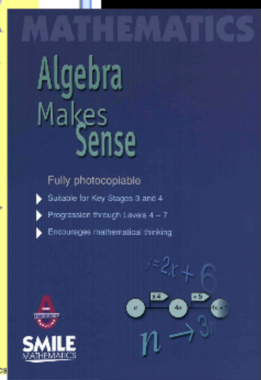

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Structures and Relations

Students know many numerical relations from their early work with arithmetic. School algebra is meaningful when it is interpreted to show the relations between numbers that students already know.

- The additive relation, e.g. 2 + 3 = 5, is expressed as a + b = c, so we can find x if 2 + x = 5 and also graph the relation between x and y if x + y = 5
- The multiplicative relation, e.g. 2 x 3 = 6, is expressed as c=ba, c/b = a and c/a = b
- Addition and subtraction have an inverse relationship, one ‘undoes’ the other.
- Addition and multiplication are both commutative
- In 37 + 49 – 37 students can spot that they do not have to do calculations. a + b – a = b expresses the same relation.
- This equation: 3(p+q) = 3p + 3q, expresses a fact that students often use in mental arithmetic

Structures and Relations

Students need to meet and understand a range of mathematical statements: formulae, equations, identities, properties, functions, and know what they mean and how to use them.

Students need to know how to ‘read’ algebra with meaning.

Students have to be fluent in transforming expressions into simpler or more usable forms, and how to use substitution of one expression for another to reduce complexity.

Research shows that holistic ways of relating algebra to situations are successful in helping students to learn the procedures, meanings and uses of algebra. Holistic teaching combines arithmetic, algebra, data, graphs and functions side by side.

Structures and Relations 1: Perimeter Expressions

Perimeter Expressions

This problem gives students plenty of opportunity to practise manipulating algebraic expressions within a purposeful context. Along the way, the challenges will provoke some insights that will be worth sharing. Issues relating to the dimensions in formulae for areas and lengths might emerge.

This activity is on the NRICH website. Perimeter is a rich context for exploring the value of algebra as a reasoning tool.

Use perimeter as a context to express sums of known and unknown quantities. The unknown values are positioned within a linear expression, and students can work backwards (solving a linear equation) to find possible missing lengths.

Use of the familiar situation of length to provide an image of the meaning of the letters provides a bridge between the missing number problems used in primary mathematics and solving equations algebraically.

http://www.nationalstemcentre.org.uk/elibrary/maths/resource/3163/number-and-algebra-3

For the book and teaching ideas go here:
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The Equals Sign

The equals sign can be used to indicate:

- formulae such as A = ½ bh;
- equations such as x+y=5;
- identities such as 3a+3b=3(a+b);
- properties such as ‘vertically opposite angles are equal’ and
- functions such as f(x) = 2x+3.

In school algebra the equals sign is usually used to indicate equality of value, such as 3+5=9-1, or angle ABC = angle ACB.

Typical misunderstandings about how to use the equals sign include:

- Letters are objects to move around, e.g. ‘move x to the other side’
- If there are letters in an expression you always have to find their value, so how can you find x in ‘5x+10=5(x+2)’?
- The equals sign means ‘calculate’

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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The next 3 slides repeat slides from above.

The Equals Sign 1: Evaluating Algebraic Expressions

A4 • Evaluating algebraic expressions

To enable learners to:

- distinguish between and interpret equations, inequations and identities;
- substitute into algebraic statements in order to test their validity in special cases.

https://spiremaths.co.uk/ilim/

This sequence of tasks enables students to compare equations, inequalities, and identities (equivalent expressions) by various means. Substitution is used to find out when expressions are equal in value. It is more common in textbooks to use substitution merely as a means to practice using symbolic conventions, but here it has a role to play in understanding the underlying relations.

The tasks are designed to challenge common misconceptions about the relations between numbers and variables and unknowns.

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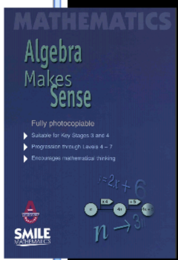
The Equals Sign 2: Equivalence

The relevant task is on pages 36-37.

Traditionally, most of the algebra curriculum used to be about learning to manipulate expressions by collecting like terms, factorising, multiplying brackets and so on. This achieves alternative forms of expressing the same relations between unknowns and variables.

For algebra to be meaningful, students can see that these manipulations (which can now be done by software) produce equivalent expressions to describe situations that the students will understand. Embedded in this kind of work, but usually left implicit, is the use of = to mean 'is equivalent to', i.e. whatever the value of the variables. This is in contrast to equations where non-equivalent expressions are temporarily equal for particular values of the variables.



https://mathsteachers.files.wordpress.com/2014/08/algebra-makes-sense.pdf

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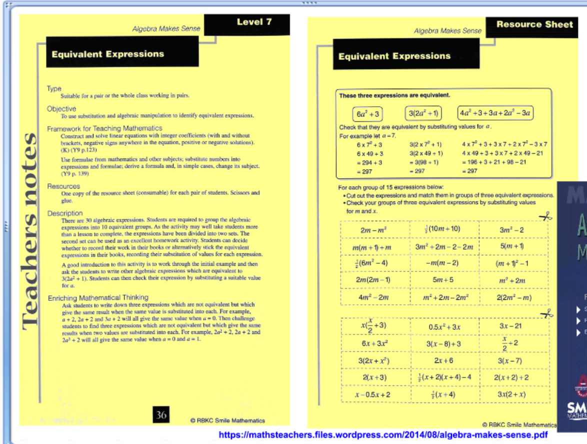
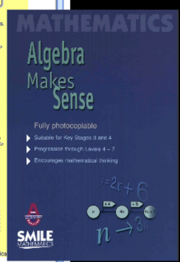
Notations and conventions 4: Equivalence





https://mathsteachers.files.wordpress.com/2014/08/algebra-makes-sense.pdf

For the book and teaching ideas go here:
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Modelling

The usefulness of algebra for students who are not going to become pure mathematicians lies in:

- understanding and using symbolic mathematical models to predict, explain and prove in mathematical and other situations;
- controlling, using, understanding and adapting spreadsheet, graphing, programming and database software

Students can learn to use algebra successfully if they have multiple experiences, over time, of modelling situations in and out of mathematics; exploring and explaining what happens using various representations, and relating these back to the situation. Research shows that having personal control of digital technology is important as it gives students a way to vary expressions, variables and parameters and see the effects.

Conversely, research also shows that holistic ways of relating algebra to situations are successful in helping students to learn the procedures, meanings and uses of algebra. Holistic teaching combines arithmetic, algebra, data, graphs and functions side by side.

http://www.bowland.org.uk/projects/keeping_the_pizza_hot.html

For the book and teaching ideas go here:
http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics

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Modelling: Keeping the Pizza Hot

The Case Study looks at the problem of how a pizza shop can maximize its market for home delivered pizzas by keeping the pizza warm (and so edible) for longer.

- They use mathematical modeling to find how to ensure that the pizza arrives hot!
- They examine cooling curves for pizzas with different packaging and to explore ways to keep the pizza warm and the implications of doing so.
- Pupils then use the model to address the market problem of the pizza shop by answering the questions: how long does it take a pizza to cool, how far can the delivery travel in that time and what difference does the packaging make?

Keeping the pizza hot

- Overview
- Assessment
- Mathematical content
- Organisation and pedagogy
- Resources provided
- Resource requirements

http://www.bowland.org.uk/projects/keeping_the_pizza_hot.html

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Using Algebra to Reason

Algebra lets us apply additive and multiplicative relations to solve problems where we don't know everything. For example:

- If I have four more sweets than you have, then there is a method of equalising the amount so long as you know the relation of 'difference' – and we don't have to know exactly how many sweets we have in total
- If I know I run twice as fast as you on average, then I know I need half the time to run the distance - without knowing the total time or distance.
- If I know that $a + b = c$, then: $b + a = c$ $\quad c = a + b$ $\quad c = b + a$ $\quad c - b = a$ $\quad c - a = b$ $\quad a = c - b$ $\quad b = c - a$

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Using Algebra to Reason

Algebra lets us apply additive and multiplicative relations to solve problems where we don't know everything. For example:

- Associativity: $a + (b + c) = (a + b) + c$
- Distributivity: $a(b+c) = ab + ac$
- If I know that $ab=c$, then: $ba=c$, $c = ab$ *and* $c = a$, $c/b=a$ and $c/a=b$
- $(a + d) - (b + d) = a - b$
- If $a > b$ and $b > c$, then $a > c$
- If $a = 2b$ and $b = 2c$ then $a = 4c$

For the book and teaching ideas go here:
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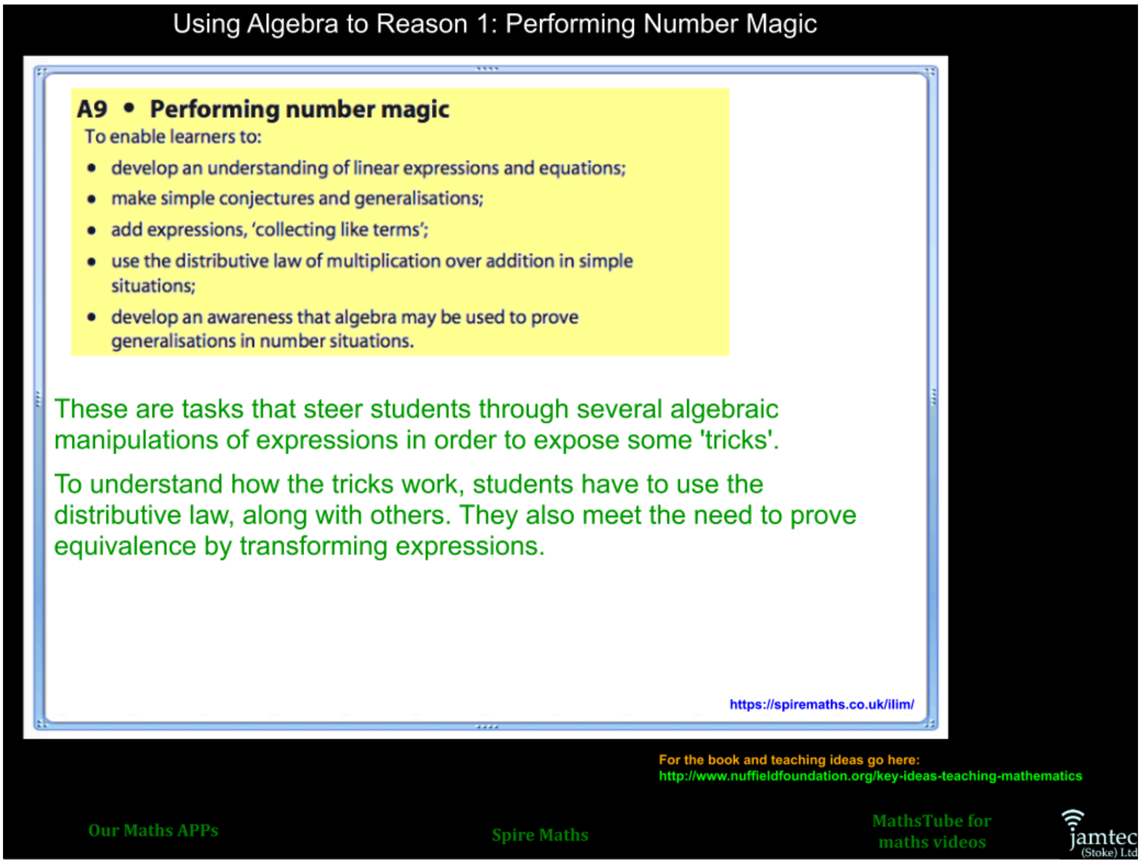

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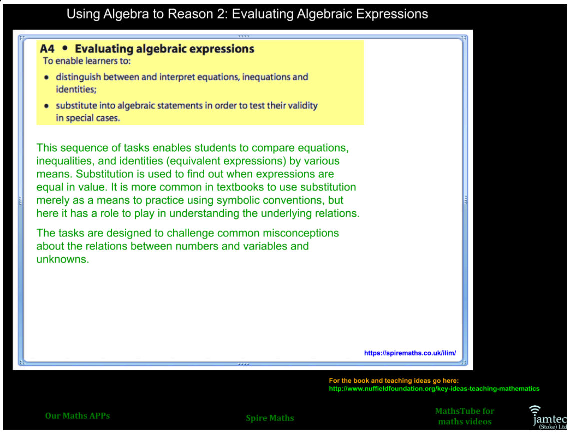
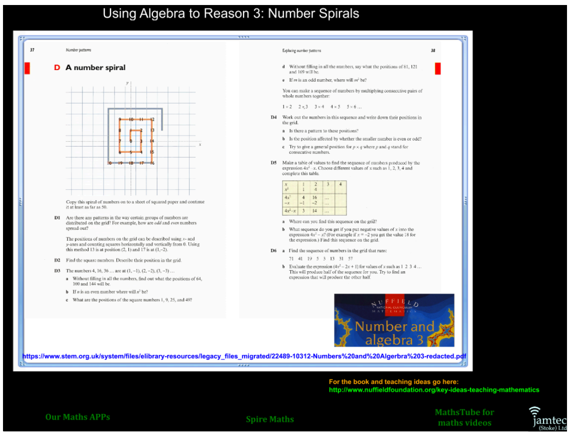




Our iPad and iPhone resources

Search for Jamtec on the AppStore. We also have other non-mathematics apps. Prices correct at 30 January 2017.


Age-ulator Free: Randomised £0.99

Directed Numbers £0.99: Equivalents £0.99: Multiplication Pairs £0.99

Maths Charts for Jenny Eather Free:
Maths Charts for Jenny Eather (Deluxe version) £4.99


Grids4Maths £0.99: GeoDraw £0.99 (iPad only)

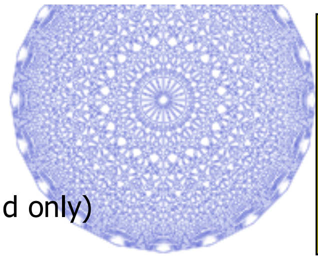
Education APPs from Apple

Half price for volume purchase of some Education APPs

Maths APPs for iPads and iPhones

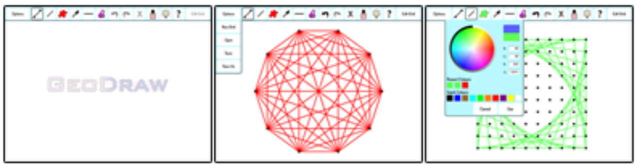
GEODRAW

Available on iPad iOS 5.0 or later!



(iPad only)

Grids
Circular
Isometric: horizontal
Isometric: vertical
Polar
Square

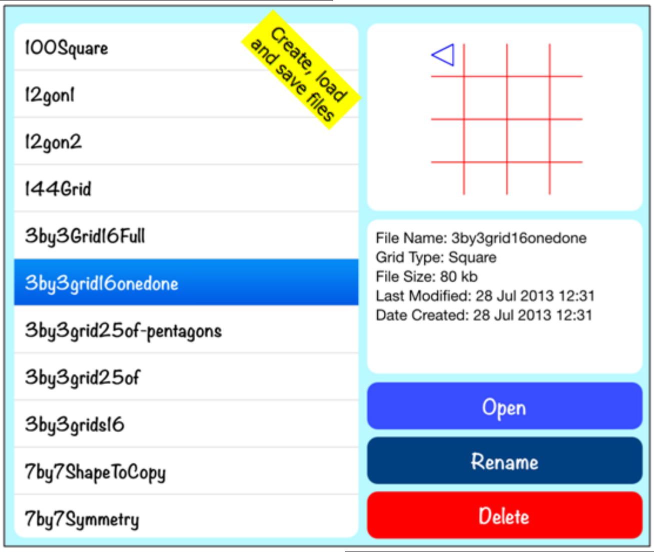


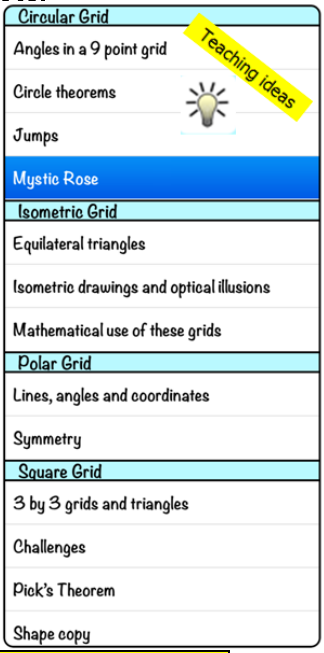
£0.99

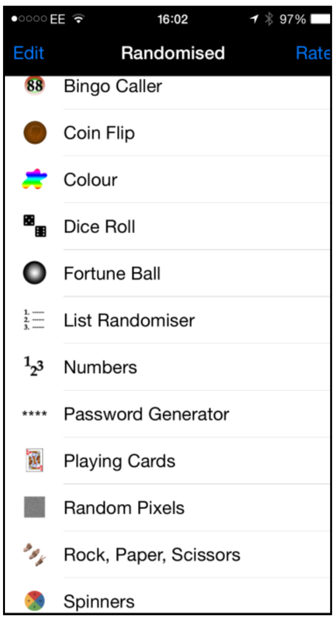
Change
Number of grid points
Grid point size
Line thickness
Line colour

GeoDraw offers users a choice of 5 grids for use in mathematics and D&T lessons. Send/export images with/without grid using: Bluetooth, Email, Facebook, Twitter and into Pages or Keynote.

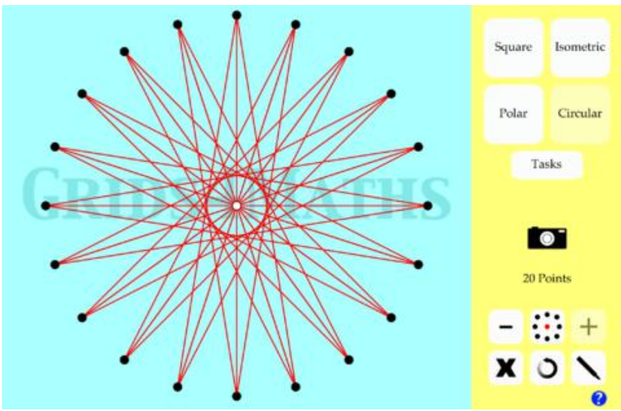
Eligible for VPP discount (see next page).



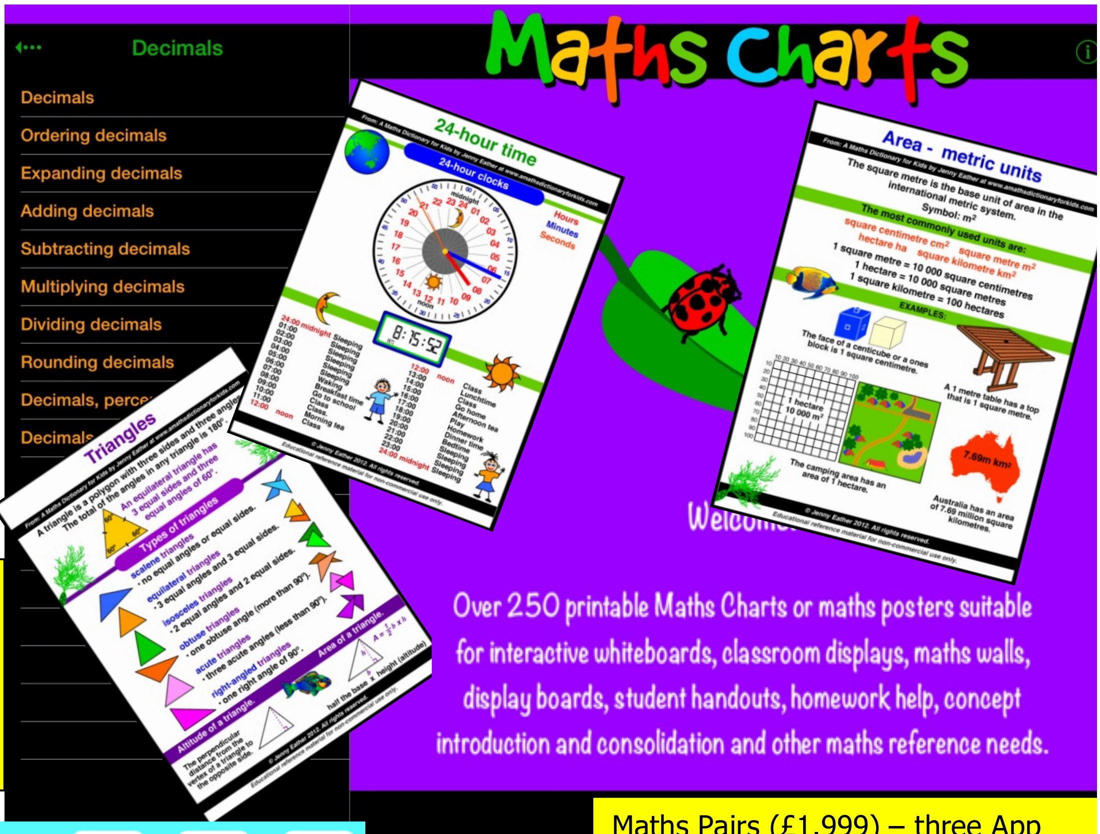




Randomised (99p): for probability lessons.
Age-ulator (free): for large number work and problem solving.
Grids4Maths (99p): much simpler version of GeoDraw for iPhones.

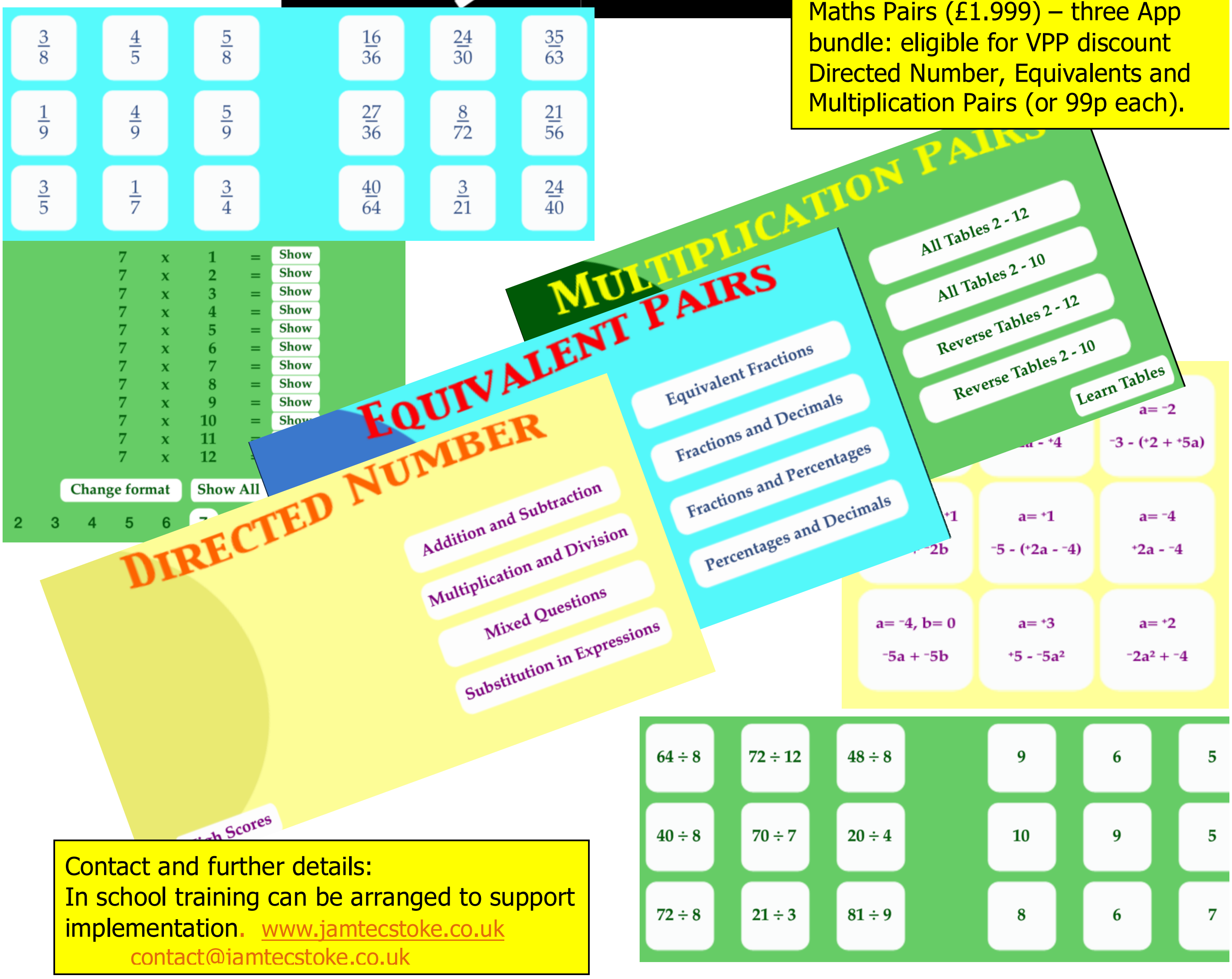


We've teamed up with Jenny Eather to bring her Maths Charts web resources to the iPad/iPhone.
Try Maths Charts by Jenny Eather for free, then buy full Deluxe version for £4.99 (half this if you sign up for VPP with Apple and buy 20 or more copies).



Volume Purchase Programme (VPP) lets you buy Apple apps at discount rate of half price for 20 or more of the same app.

Maths Pairs (£1.999) – three App bundle: eligible for VPP discount Directed Number, Equivalents and Multiplication Pairs (or 99p each).



Contact and further details:
In school training can be arranged to support implementation. www.jamtecstoke.co.uk
contact@jamtecstoke.co.uk

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