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Stimulating, Practical, Interesting, Relevant, Enjoyable Maths For All

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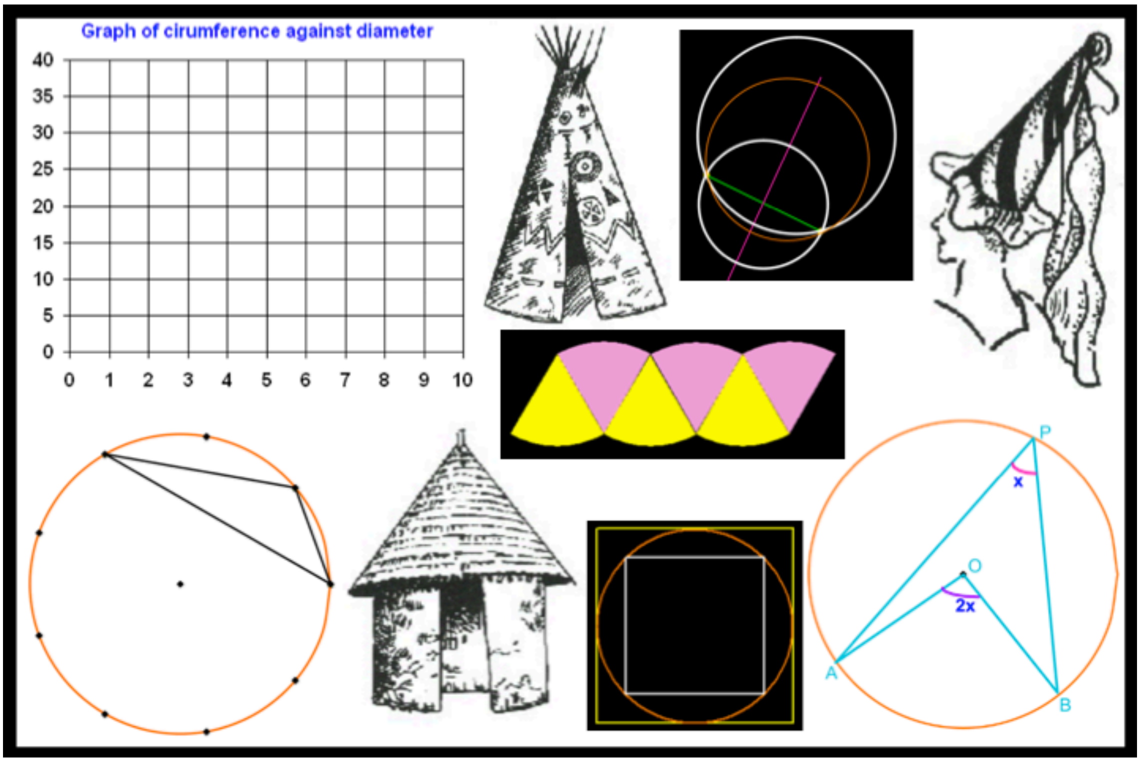
Circles



A Spire Maths Activity

https://spiremaths.co.uk/circle/

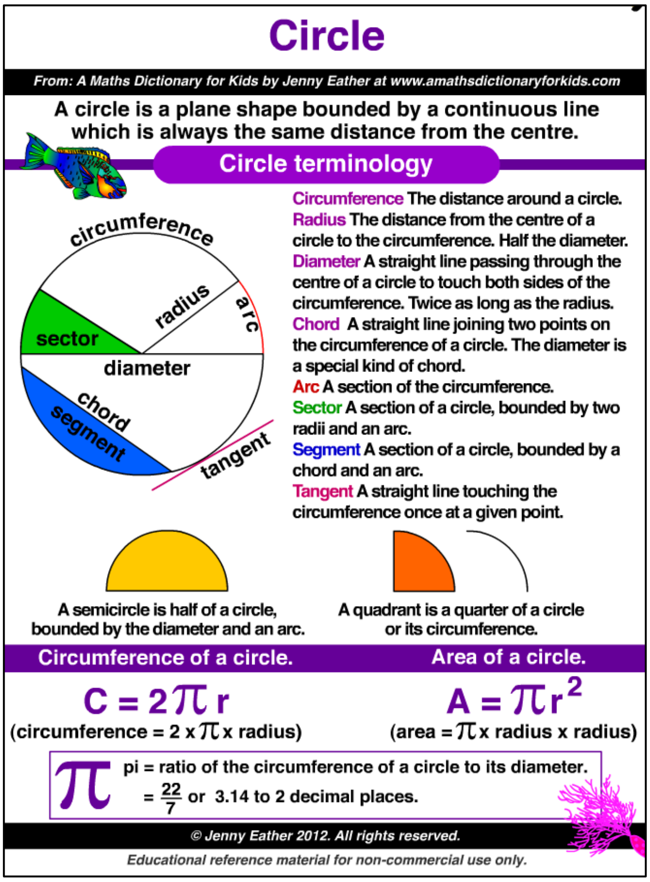
Table of Contents

What is a circle?

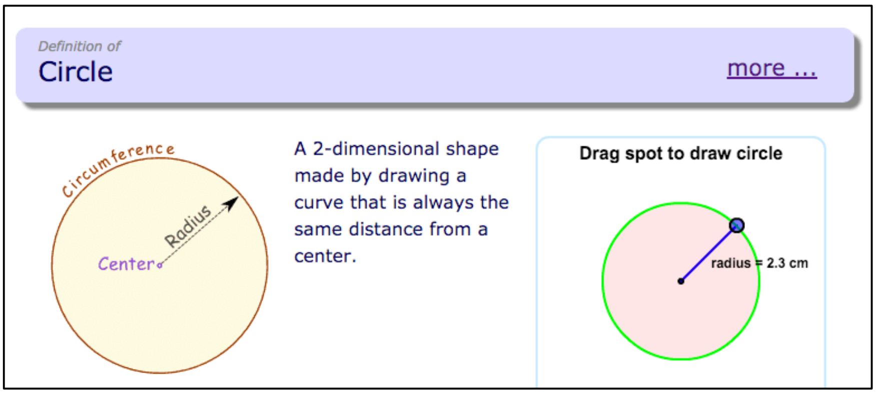
Definitions, parts and formulae

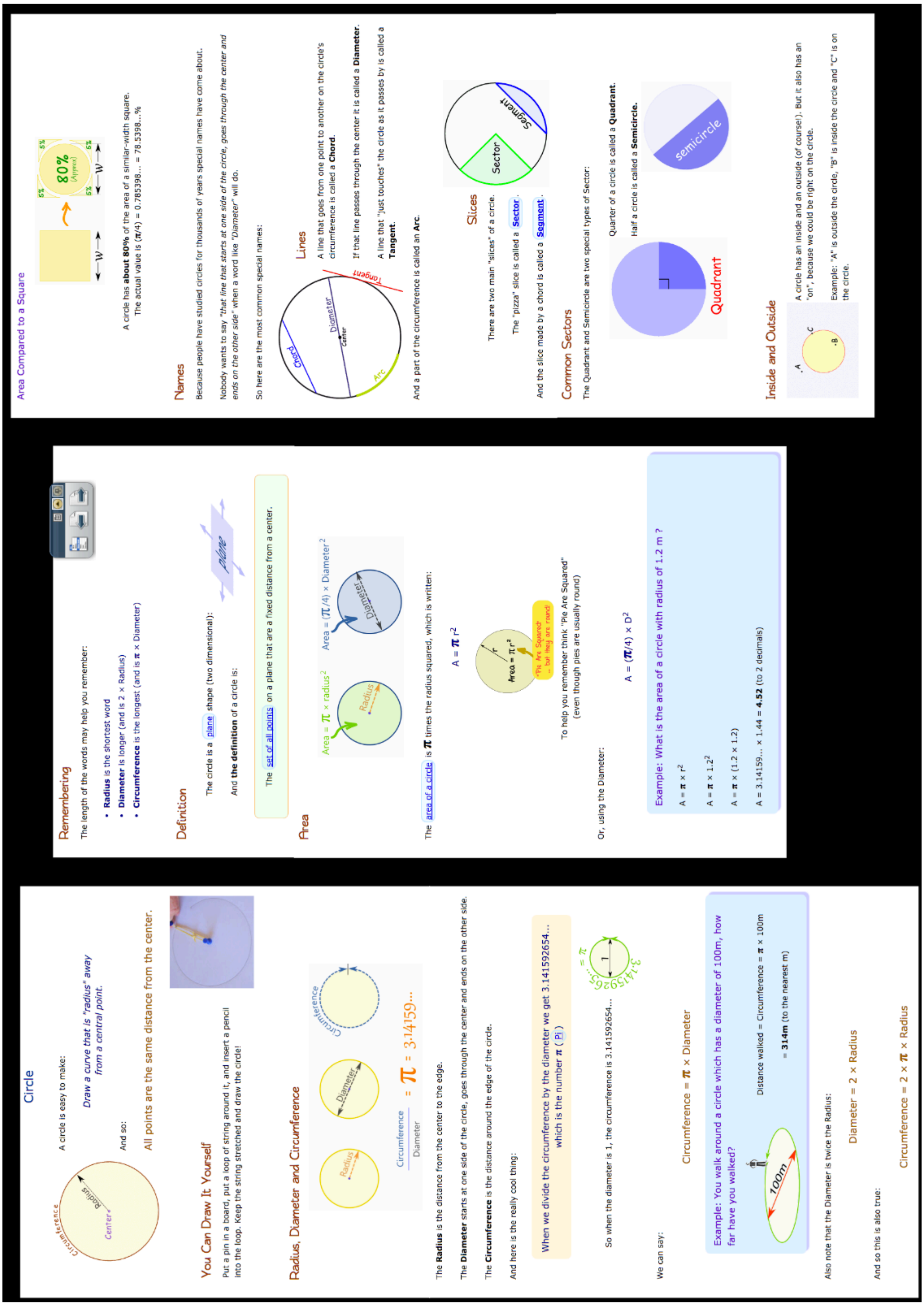
A circle definition: the locus of a point on a plane that is a fixed distance from a given point.
An alternative circle definition: the set of all points on a plane that are a fixed distance from a given point.

http://www.amathsdictionaryforkids.com/dictionary.html
http://www.amathsdictionaryforkids.com/mathsCharts.html



http://www.mathsisfun.com/definitions/circle.html
See index for information about blocking adverts from webpages, since maths is fun, like too many other websites, has lots of adverts.





Circle

A circle is easy to make:

Draw a curve that is "radius" away from a central point.

And so:

All points are the same distance from the center.

You Can Draw It Yourself

Put a pin in a board, put a loop of string around it, and insert a pencil into the loop. Keep the string stretched and draw the circle!

Radius, Diameter and Circumference

$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159...$

The **Radius** is the distance from the center to the edge.

The **Diameter** starts at one side of the circle, goes through the center and ends on the other side.

The **Circumference** is the distance around the edge of the circle.

And here is the really cool thing:

When we divide the circumference by the diameter we get 3.141592654... which is the number π (Pi)

So when the diameter is 1, the circumference is 3.141592654...

We can say:

Circumference = π × Diameter

Example: You walk around a circle which has a diameter of 100m, how far have you walked?

Distance walked = Circumference = π × 100m

= **314m** (to the nearest m)

Also note that the Diameter is twice the Radius:

Diameter = 2 × Radius

And so this is also true:

Circumference = 2 × π × Radius

Remembering

The length of the words may help you remember:

- **Radius** is the shortest word
- **Diameter** is longer (and is 2 × Radius)
- **Circumference** is the longest (and is π × Diameter)

Definition

The circle is a plane shape (two dimensional):

And **the definition** of a circle is:

The set of all points on a plane that are a fixed distance from a center.

Area

Area = π × radius^2

Area = (π/4) × Diameter^2

The area of a circle is π times the radius squared, which is written:

$A = \pi r^2$

"Pie Are Squared" – but they are round!

To help you remember think "Pie Are Squared" (even though pies are usually round)

Or, using the Diameter:

$A = (\pi/4) \times D^2$

Example: What is the area of a circle with radius of 1.2 m ?

$A = \pi \times r^2$

$A = \pi \times 1.2^2$

$A = \pi \times (1.2 \times 1.2)$

$A = 3.14159... \times 1.44 =$ **4.52** (to 2 decimals)

Area Compared to a Square

A circle has **about 80%** of the area of a similar-width square.
The actual value is (π/4) = 0.785398... = 78.5398...%

Names

Because people have studied circles for thousands of years special names have come about.

Nobody wants to say *"that line that starts at one side of the circle, goes through the center and ends on the other side"* when a word like *"Diameter"* will do.

So here are the most common special names:

Lines

A line that goes from one point to another on the circle's circumference is called a **Chord**.

If that line passes through the center it is called a **Diameter**.

A line that "just touches" the circle as it passes by is called a **Tangent**.

And a part of the circumference is called an **Arc**.

Slices

There are two main "slices" of a circle.

The "pizza" slice is called a **Sector**.

And the slice made by a chord is called a **Segment**.

Common Sectors

The Quadrant and Semicircle are two special types of Sector:

Quarter of a circle is called a **Quadrant**.

Half a circle is called a **Semicircle**.

Inside and Outside

A circle has an inside and an outside (of course!). But it also has an "on", because we could be right on the circle.

Example: "A" is outside the circle, "B" is inside the circle and "C" is on the circle.

http://www.mathopenref.com/tocs/circlestoc.html

Math Open Reference

Home Contact About Subject Index Word Index Feedback

About these ADVERTISEMENTS

Circles and Ellipses

- Circle - definition
- Parts of a circle - a pictorial index
- Radius
- Diameter
- Circumference
- Pi
- Area of a circle
- Annulus
- Area of an Annulus
- Chord
- Tangent
- Secant
  - Intersecting Secant Lengths Theorem
  - Intersecting Secant Angles Theorem
- Intersecting Chords Theorem
- Concentric circles
- Incircles
- Circumcircles

Parts of a Circle

- Semicircle
- Sector of a Circle
  - Area of a Sector
- Segment of a circle
  - Area of a Segment given central angle
  - Area of a Segment given segment height

Angles in a Circle

- Inscribed Angle
- Central Angle
- Central Angle theorem
- Thales' Theorem
- Angle inscribed in a semicircle

Arcs

- Arc definition
- Major and Minor Arcs
- Arc length
- Angle measure of an arc
- Adjacent arcs
- Intercepted Arc
- Radius of an Arc or Segment
- Sagitta - Height of an Arc or Segment

Finding the Center of a Circle

- Finding the center with compass and ruler
- Finding the center with any right-angled object

Equations of a Circle

- See 'Equations of a circle' Section

Ellipses

- See Ellipse Section

http://www.mathopenref.com/tocs/constructionstoc.html
http://www.mathopenref.com/worksheetlist.html

Circles, Tangents

- Constructing the center of a circle or arc
- Finding the center of a circle or arc with any right-angled object
- Tangents to a circle through an external point
- Tangent to a circle through a point on the circle
- Tangents to two circles (external)
- Tangents to two circles (internal)
- Circle through three points

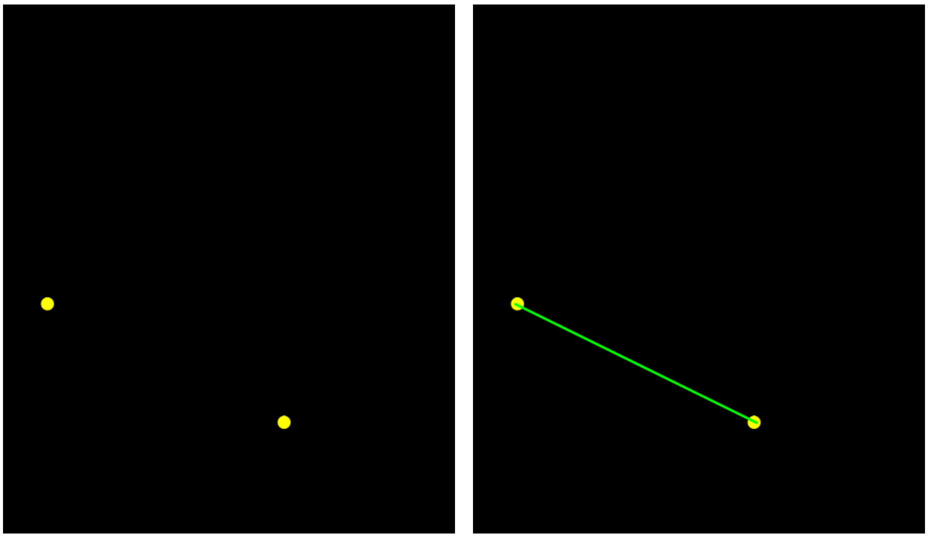
Constructions

- Introduction to Euclidean Construction - tools and rules
- Printable constructions worksheets

Circles and Tangents

Finding center of a circle
Circle through 3 points
Tangent through a point
Tangent through a point on the circle
Tangent to two circles (external)
Tangent to two circles (internal)

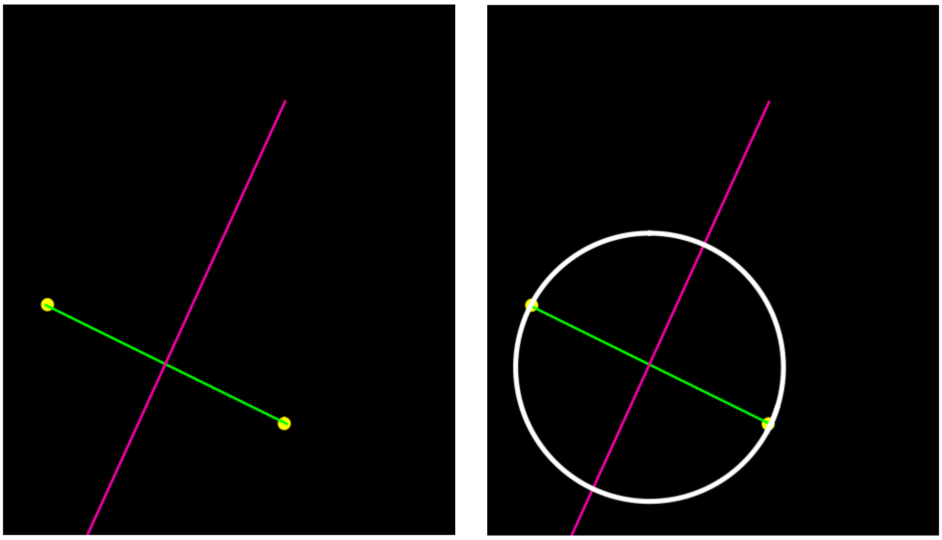
Circles through 2 points



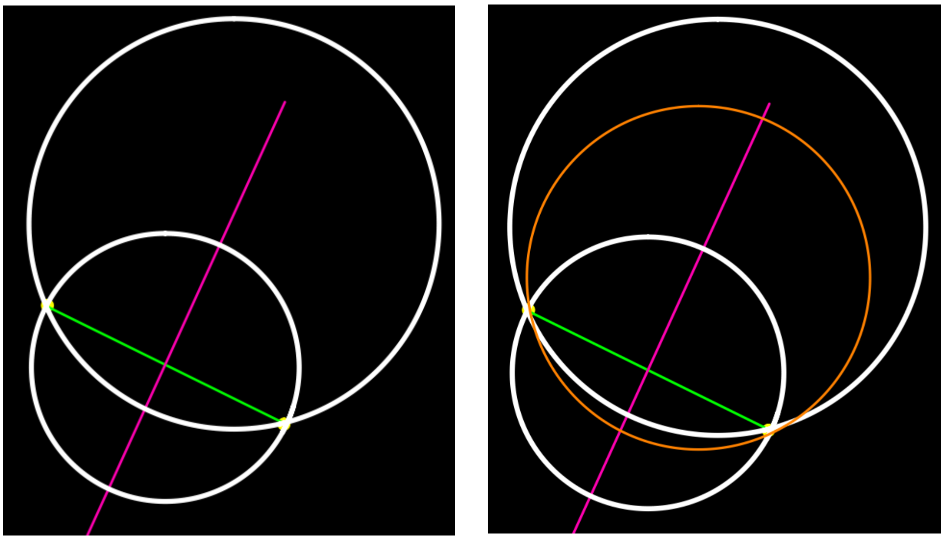
Ask pupils to see how many circles they can draw through two points.

These diagrams show how many circles pass through 2 points.

A green line joins the two points, shown in yellow.



The pink/purple line is the perpendicular bisector of this line.



Two white and one orange circle are shown passing through these two points.

An infinite number of circles pass through these two given points.

The centres of all these circles are found on the perpendicular bisector of the two points.

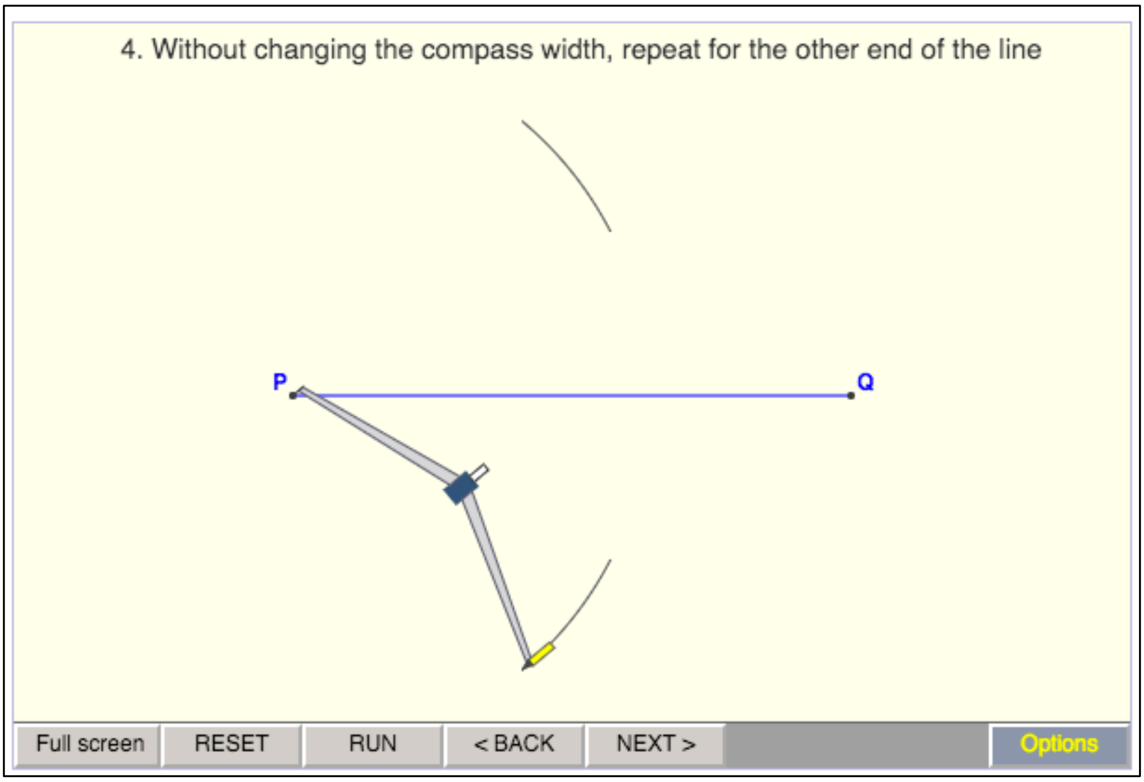
Pupils need time to understand why the circle is important for constructions:

- it allows you to draw lines of equal length
- all lines from the centre are the same length

Perpendicular bisector of a line segment

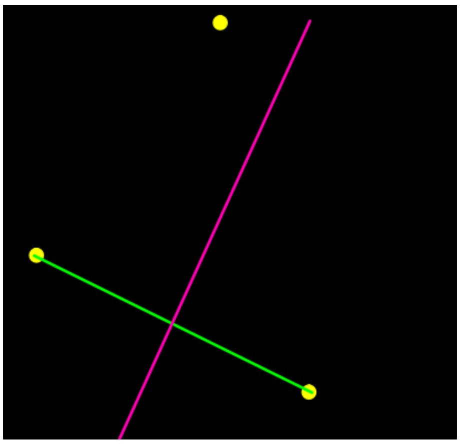
An animation, set of printable instructions and a proof can be found at the excellent Math Open Reference site:

http://www.mathopenref.com/constbisectline.html
Diagram shows part of the animation).



This page also has a list of other construction pages available on the Math Open Reference site.

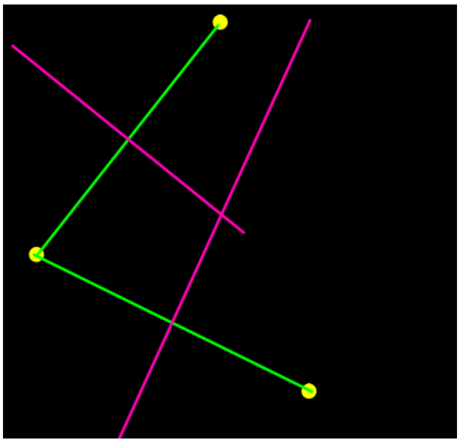
Circles through 3 points

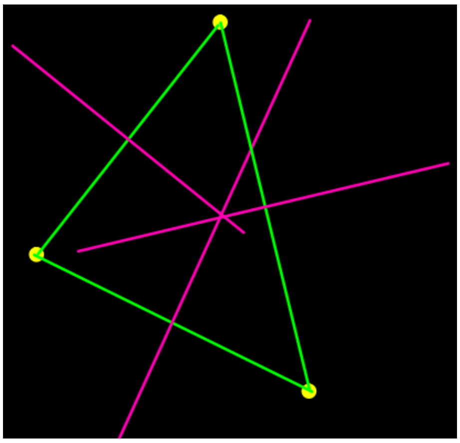


The idea is much as before.

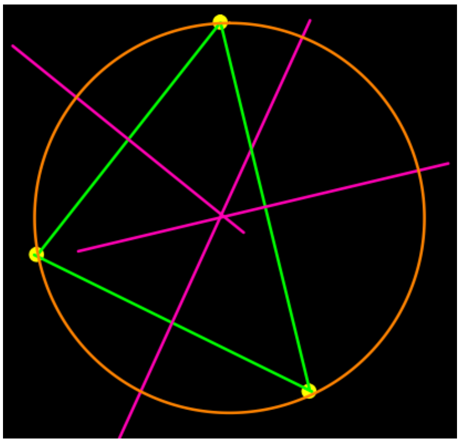
Only one circle will pass through the vertices of a triangle.
Because its centre must be on the perpendicular bisector of each of the sides of the triangle it means that the three perpendicular bisectors of a triangle must meet at a point. There is a proof of this at:

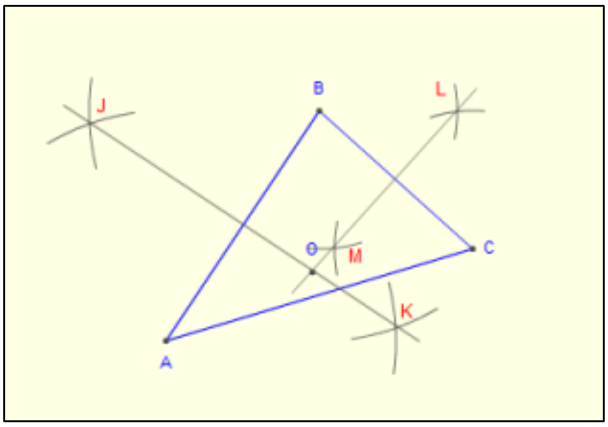
http://www.mathopenref.com/constcircumcenter.html





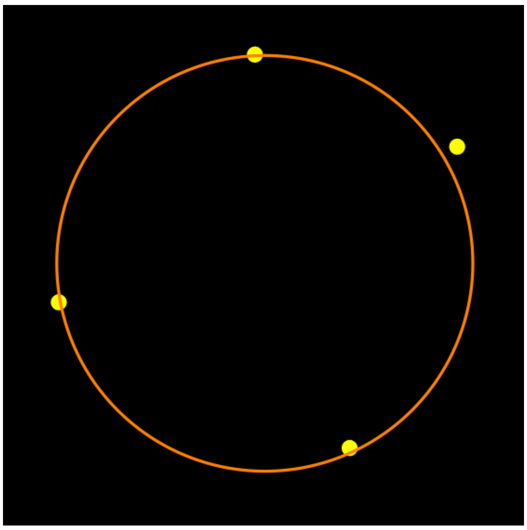
The circle is known as the Circumcircle of the triangle, and the centre of it is referred to as the Circumcentre of the triangle. Two versions follow: on the right green lines are the sides of the triangle; the pink/purple lines are the perpendicular bisectors.



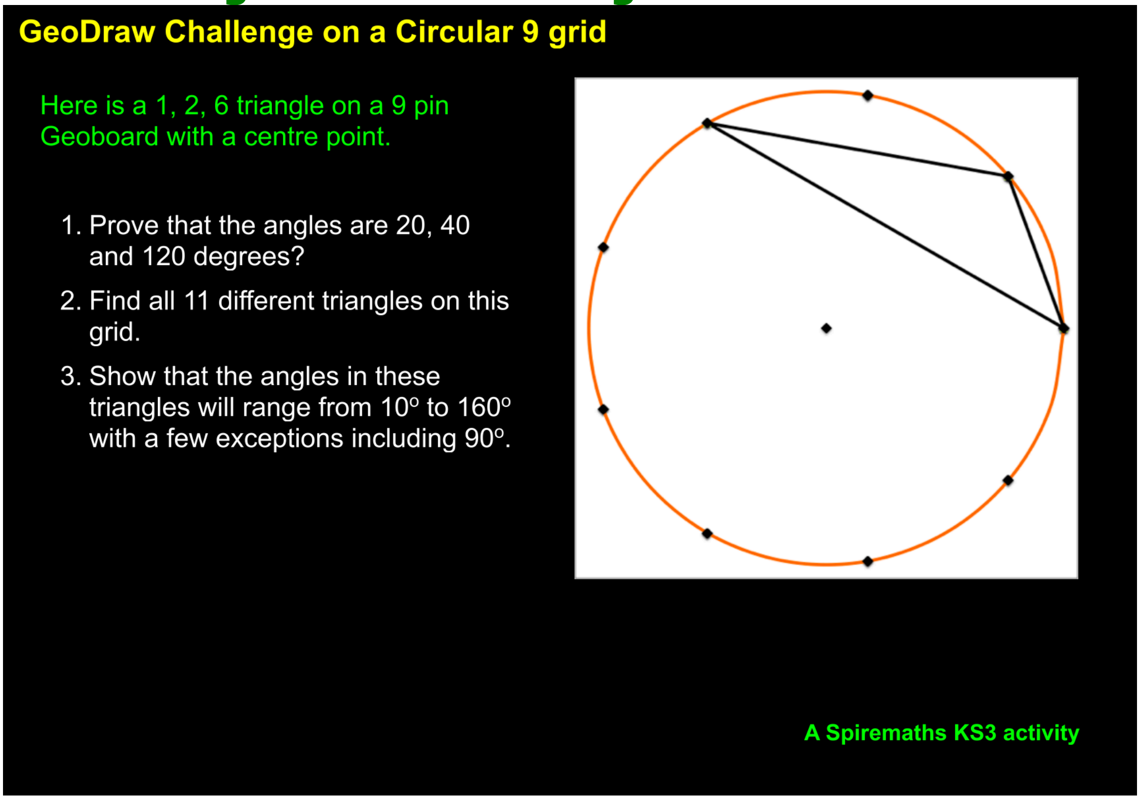


Circles through 4 points

In general, a circle will not pass through 4 random points on a plane. Since only one circle will pass through any 3 given non-collinear points on a plane (shown by the orange circle below), so placing a fourth point randomly on the same plane will not usually lie on the circle. When it does the quadrilateral made is said to be a Cyclic Quadrilateral.



GeoDraw Challenge on a Circular 9 grid

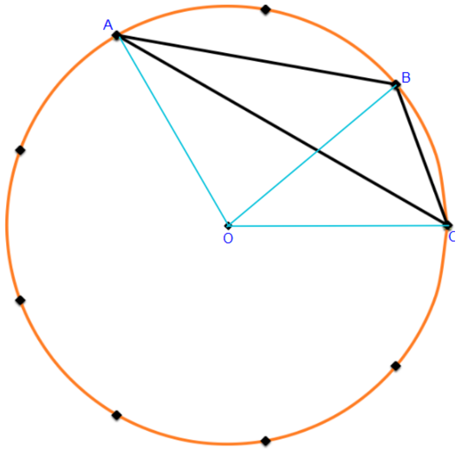


To solve this you need to '**know the facts**' about this circular grid. Basically at KS3 (years 7 to 9) this means

- knowing that the angle a the centre when two adjacent radii are drawn in - here it is 40^o since the central angle is 90^o ($360^o \div 4$)
- knowing that a triangle joining the centre point to any two points on the circumference of a circle makes an isosceles triangle (this point cannot be emphasised enough, since it will solve most angle problems on a circular grid)

At KS4 (years 10 and 11) work might also assume the circle theorems, or be used to help establish the circle theorems.

Angles of a 1, 2, 6 triangle on a circular geoboard



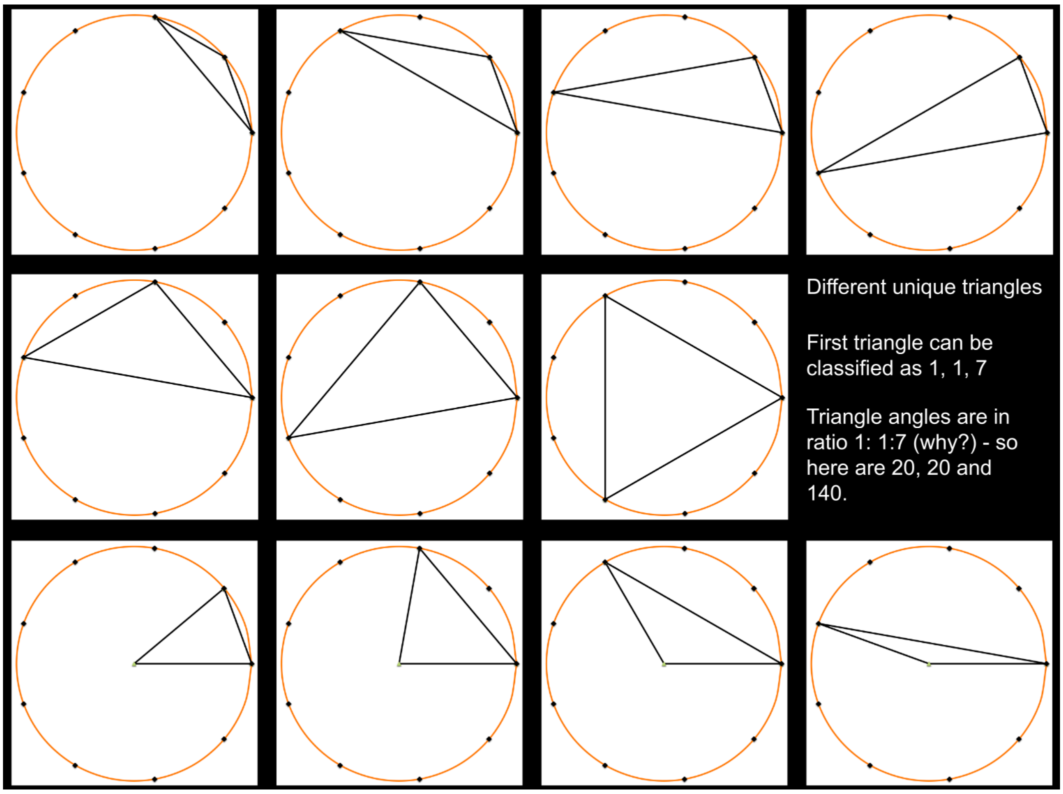
For the triangle above it helps to construct three radii OA, OB, OC (as right) and realise that this creates three isosceles triangles (OAB, OAC and OBC) where the angles can be found directly.

- Triangle OBC has angles 40, 70, 70.
- Triangle OAB has angles 80, 50, 50.
- Triangle OAC has angles 120, 30, 30.

So angles are 20, 40 and 120 (50 – 30, 70 – 30 and 70 + 50 respectively).

The 11 possible solutions are given below. By constructing these from 'East' and counting round dots etc. with the first step one dot round only 4 can be made, the fifth would repeat the fourth etc.; similarly for those with a first step of two dots round; and so on.

The triangles on a circular 9 grid (our GeoDraw iPad app is good for this)



There are 7 that are unique up to reflection and rotation that do not involve the centre point (1 is equilateral, 3 are isosceles and 3 are scalene: (3, 3, 3) is equilateral; (1, 1, 7); (1, 4, 4) and (2, 2, 5) are isosceles; and (1, 2, 6), (1, 3, 5) and (2, 3, 4) are scalene. You cannot make a right-angled triangle.

These give you angles of 20, 40, 60, 80, 100, 120 and 140.

There are 4 more that have a vertex at the centre; all are isosceles. These allow you to get angles of 40, 80, 120 and 160 at the centre and 70, 50, 30 and 10 at the circumference. Hence angles that are possible in a triangle are:

10, 20, 30, 40, 50, 60, 70, 80, 100, 120, 140 and 160

		Length Side 2			
		1	2	3	4
Length Side 1	1	7			
	2	6	5		
	3	5	4	3	
	4	4	3	2	1
	5	3	2	1	
	6	2	1		
	7	1			

The table that follows (from a spreadsheet) shows yellow shaded item which give unique triangles around a circular 9 grid, where 'length side 1' refers to number of dots you count round to get to next vertex, etc. These refer only to those that don't have a vertex at the centre.

Ratios and angles of triangles on a circular 9 grid

Looking at the triangle in the challenge it can be considered as a 1:2:6 triangle since starting at East you count round 1 dot, then 2 and then 6 in turn to get to the next vertex. These total 9, so with 180 degrees to share in the triangle, each share is 20 degrees. Hence angles are 20, 40 and 120 degree respectively, where the angle is opposite to the appropriate part of the ratio.

Circle Theorems

Good resources for teaching circle theorems

Geogebra interactive sites

Green Maths – Geogebra examples in an order that builds on the theorems in order, but examples don't flip when e.g. on the other arc:
https://www.geogebra.org/b/kQysXXvn#material/F9Kh99tc

Michael Borcherds – one of the main people behind Geogebra: 20 pages to support this work, starting by emphasising the importance of isosceles triangles.
https://www.geogebra.org/m/PFf7ehXE

Ideas sites

Jo Morgan's excellent Resourceaholic blog – details where to find loads of ideas
http://www.resourceaholic.com/2014/11/circletheorems.html

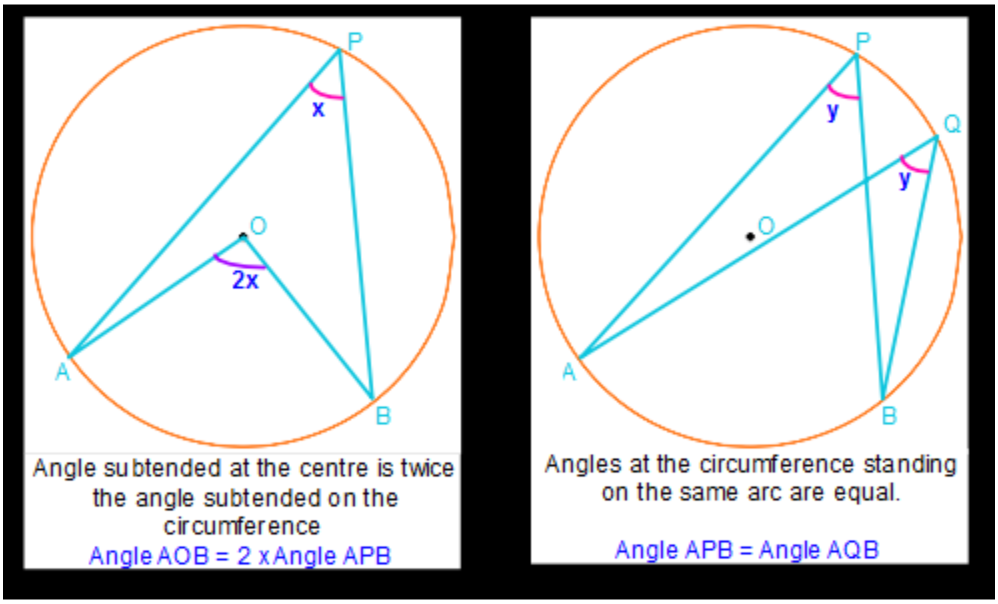
Miss Brookes – again another list of good sites with ideas.
https://www.missbrookesmaths.com/single-post/2015/04/25/Circle-Theorems

Problem solving

nrich lists some problems that can introduce circle theorems.
http://nrich.maths.org/6007

The theorems

There are 8 in total: the first four primarily about angles the other four about chords or tangents.

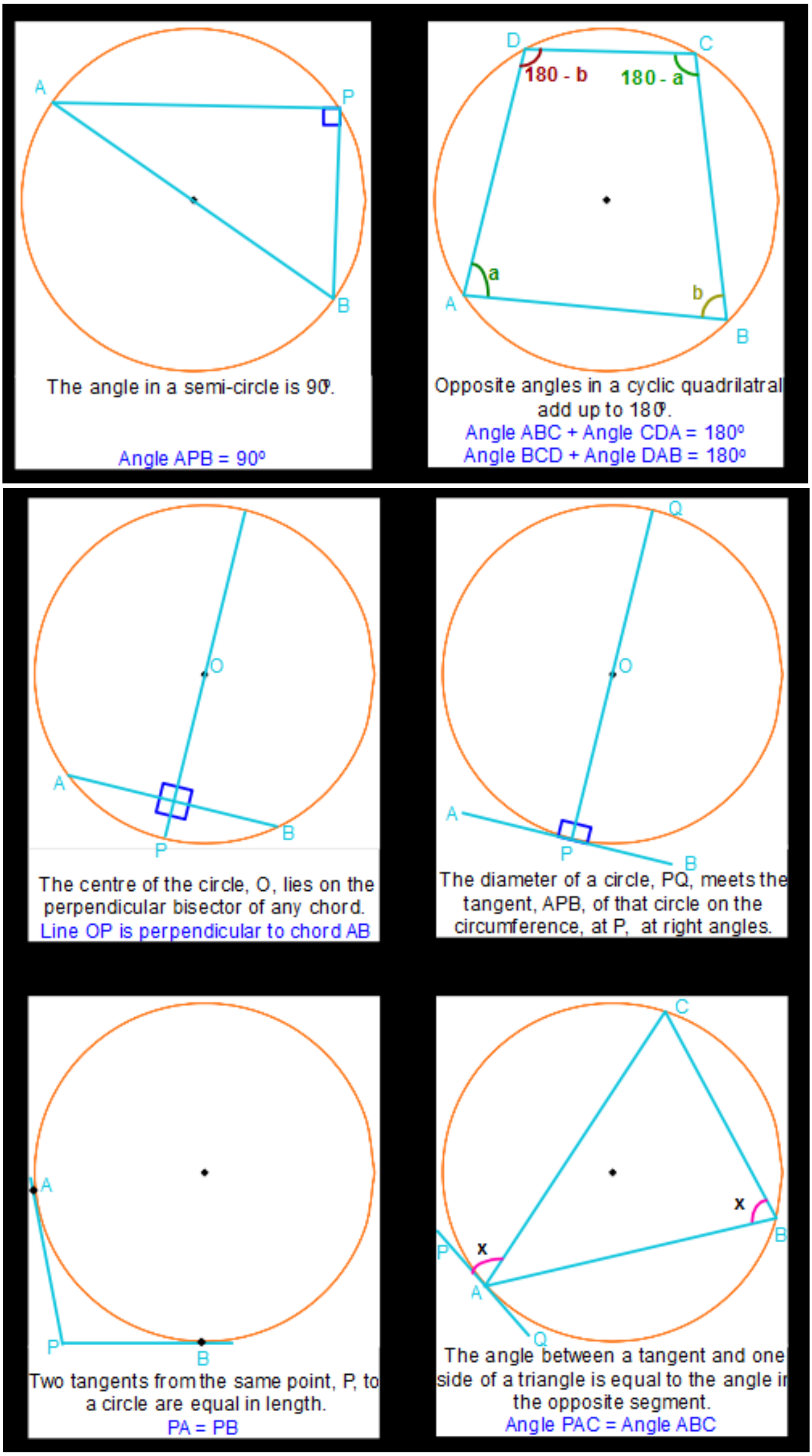


1. The angle subtended at the centre by two points on a circle is twice the angle subtended on the circumference.

2. Angles subtended at the circumference by two points on a circle are equal.

3. The angle in a semi-circle is 180°.

4. Opposite angles in a cyclic quadrilateral are equal.



5. The centre of the circle lies on the perpendicular bisector of any chord of the circle.

6. The diameter and tangent that meet at a point on the circle are perpendicular.

7. Two tangents from a point to a circle have the same length.

8. The alternate segment theorem. The angle between a tangent and one side of a triangle is equal to the angle in the opposite segment.

The proofs

The angle subtended at the centre by two points on a circle is twice the angle subtended on the circumference

Proved by constructing extra radii and then looking at all the angles that result from the isosceles triangles found.

Angles subtended at the circumference by two points on a circle are equal

Uses the first since both the circumference angles have the same central angle.

The angle in a semi-circle is 180o

A special case where the angle at the centre is 180 degrees, so the angle at the circumference is a right angle.

Opposite angles in a cyclic quadrilateral are equal

As a consequence of the first it looks at both the major and minor angles at the centre which add up to 360 degrees, so the corresponding angles at the circumference must add up to 180 degrees. The cyclic quadrilateral is the special name given to a quadrilateral whose vertices fall on a circle.

All squares and rectangles are cyclic quadrilaterals; unless it is one of these a parallelogram and a rhombus is not a cyclic quadrilateral.

The centre of the circle lies on the perpendicular bisector of any chord of the circle

It is proved by adding radii to the ends of the chord and using congruence (RHS = Right-angle, Hypotenuse and Side on triangles created.

The tangent and radius that meet at a point on the circle are perpendicular

This follows by considering if it were not the case and joining the centre to another point on the tangent where it does make a right angle, which leads to a contradiction.

Two tangents from a point to a circle have the same length

Proved by joining that common point to the centre of the circle and using number 6 again with RHS.

The alternate segment theorem. The angle between a tangent and one side of a triangle is equal to the angle in the opposite segment

See diagram above. Proved by constructing triangle AOC and using results 6 and 1 (O is the centre): angle OAC = 90 – x (result 6), so OCA is the same (isosceles triangles with radius) so angle AOC = 2x (angles of triangle =180). So angle ABC is half this (result 1).

Use of the Circle Theorems and finding angles in circles

Virtually all problems with angles involve recognising where a radius creates an isosceles triangle and/or the circle theorems.

General Circle Work

Problems arise with measurement associated with the circle, because it does not fill space and the numbers are not convenient in terms of calculations related to perimeter, area and volume (when circle are associated with 3-D shapes).

In fact there has been trouble with circle from early on when the Greeks attempted, but failed, to square the circle.

Also there are misconceptions and misunderstandings associated with pi, though many people know that it is linked to the circle.

In many cases pupils are confused about the circumference and area of a circle. Problems can occur later on when some people need to make use of relevant formulae: since to cut metal up to make pipes or cans will involve changing lengths into circumferences.

Problems Associated with Circle Work

1. Confusion of squaring with doubling, that is, 2r and r^2.
2. The mystery of pi, that it is actually a number.
3. Misunderstanding commutativity and that

$$\begin{aligned} \pi \times 2r &= 2r \times \pi \\ &= \pi \times 2r \\ &= \pi \times 2 \times r \\ &= 2 \times \pi \times r \\ &= 2\pi r \end{aligned}$$

4. Non-appreciation that circumference is a linear dimension and that area is a product of two linear dimensions.

Key Things to Remember about Circles

Every radius for a circle is the same length, so triangles involving two radii are isosceles.

Suggested Early Activities

1. Plenty of experience of circles all around, in nature and in designs
2. Practical work to find areas and perimeters (using grids and string), use all sorts of objects, not all round, and build up the idea of perimeters surrounding areas
3. Cutting up shapes and putting them together in different ways
4. Make use of compasses and establish the relationship between the radius and the circle
5. Cut up and fold circles and explore the geometry of the circle

Suggested Later Activities

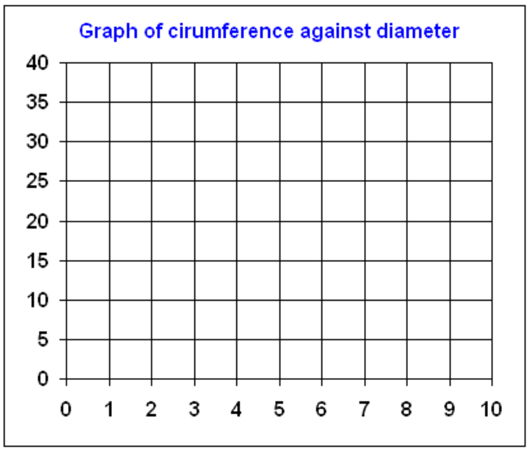
6. Investigate on a calculator or spreadsheet to establish π as a number (need not be linked to circles)
7. Consider whether it is best to have π approximated by 3
8. Use shapes to approximate to the area of a circle
9. Think about enclosing and inscribed squares and hexagons,
10. Historical significance
11. Consider dimensions of equations and the need for equations to balance

Examples of Activities

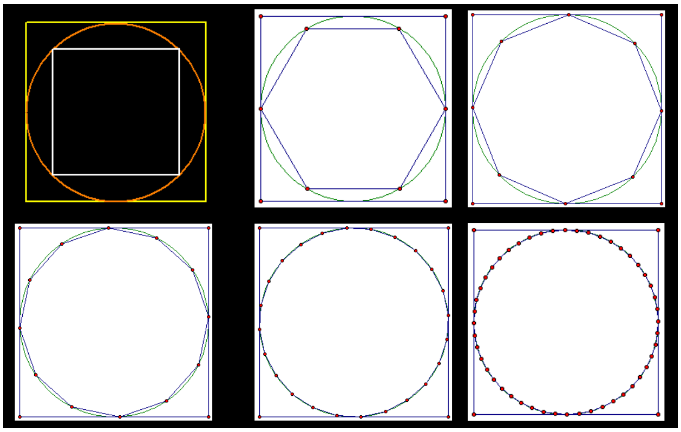
Practical work: measure to establish relationship between circumference and diameter

Name of object	Diameter in cm	Circumference in cm	Circumference divided by diameter

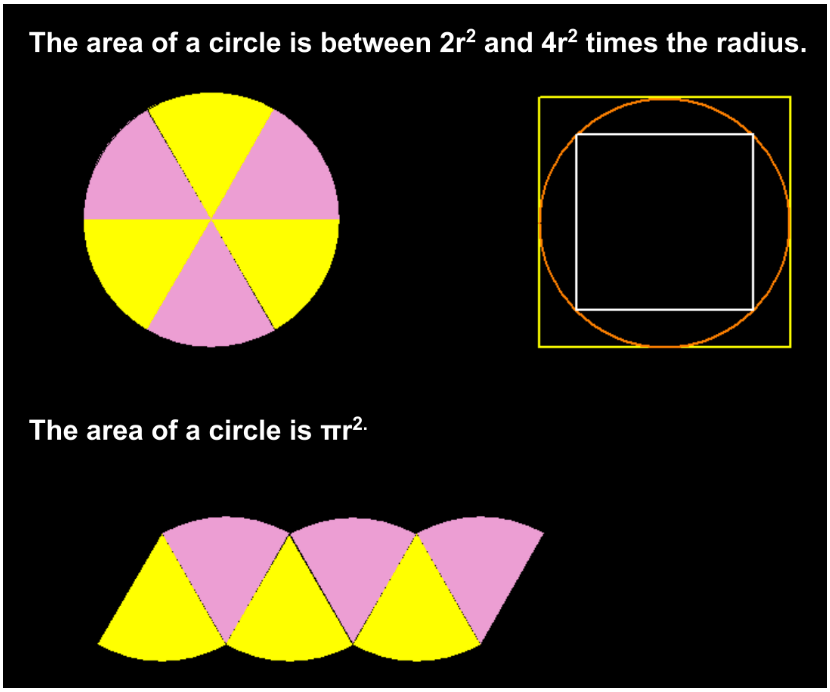
Plot C against d on e.g. a spreadsheet to see there is a possible linear relationship. You need lots of different objects and discussion should focus on which will give more accurate measurements. A4 graph paper is good for measuring around the circumference of objects.



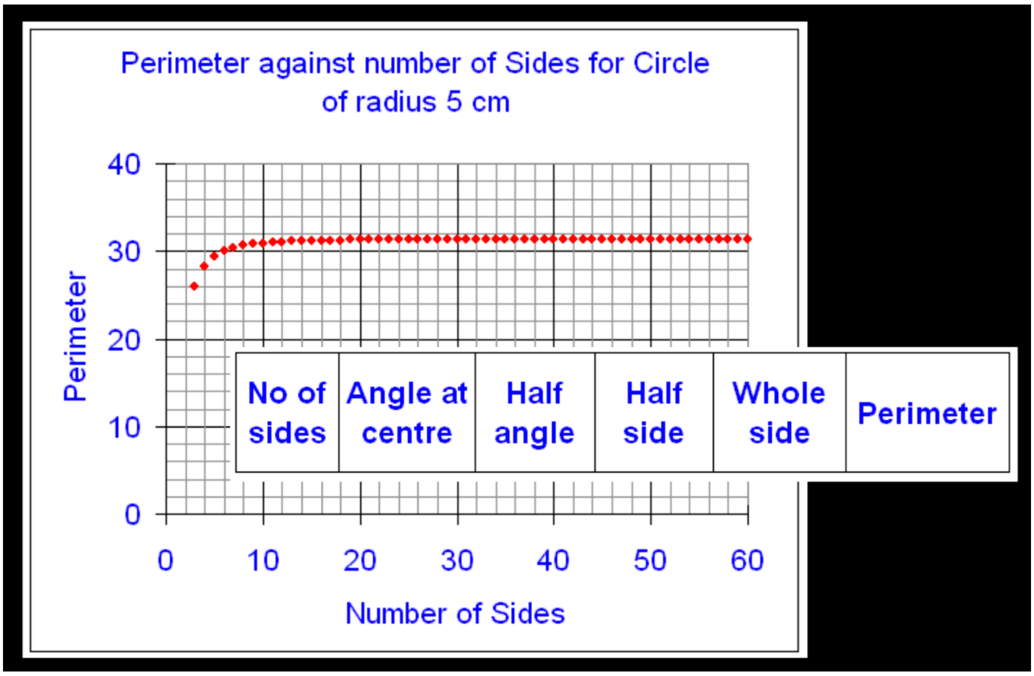
Measure Perimeters



Measure areas by cutting sectors from different size circles



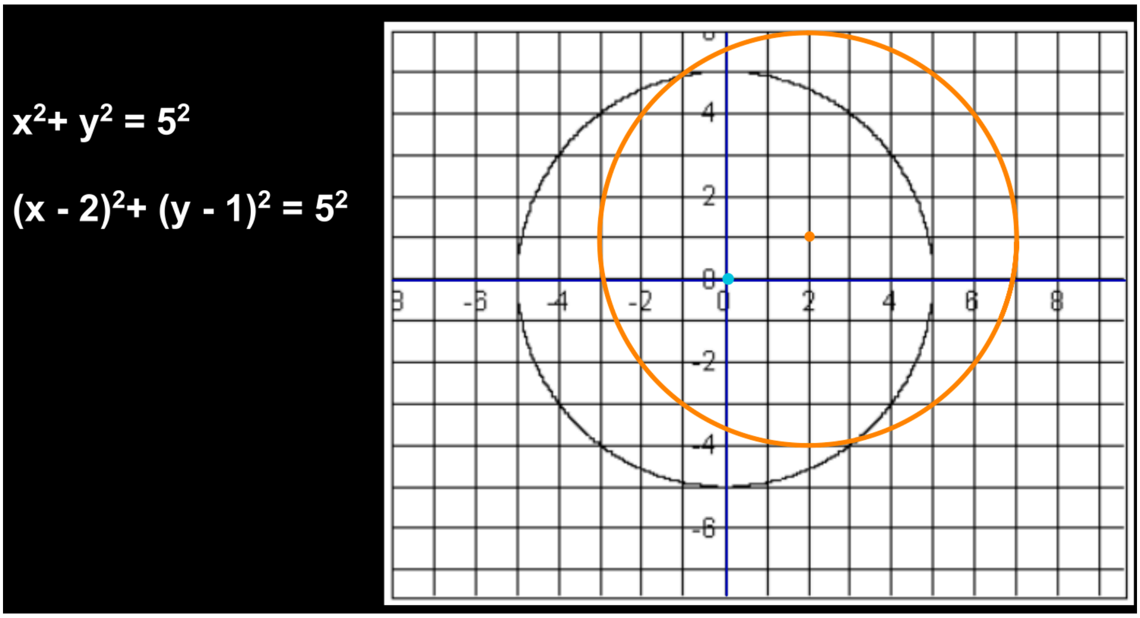
Perimeter – Sides Graph



The flipchart contains a spreadsheet and there is also one at;

https://spiremaths.co.uk/circle/

The Circle at A level



Cones and Circles

Right cones (where vertex is over centre of the base) are made by cutting a sector from a circle, since every point on the cone base perimeter is equidistant from the cone vertex.

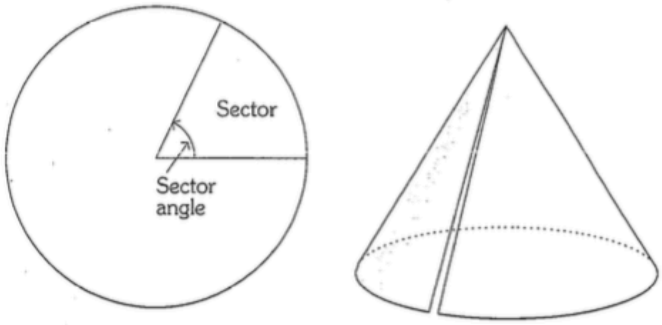
A set of practical activities follow:

10 Creating Cones (10a)

In this unit you are going to discover how a paper circle can be used to make a variety of cones. You will then use your knowledge to make a cone of a particular size.

Making cones

A cone can be made by removing a sector from a circle and then joining together the straight edges. Paper clips and sellotape will help.



1. Make several paper circles of the same size.
2. Remove sectors of different sizes from each circle to make a variety of cones.
3. Copy the table below and complete a row for each of your cones.

Radius of starting circle =		
Sector angle cut out	Cone height	Cone base radius

4. Compare your results with other people.
5. Could you calculate the dimensions of a cone if you knew the radius of the starting circle and the sector angle?

The diagram above and the following one are reproduced with full permission.

10 Creating Cones (10b)

The cone in society

Here are cones that can be found in different times and places. Read the information about each of them and then make a scale model (or representation) of one of them.

A Kikuyu dwelling

A hennin, a 13–14th century hat

A tipi of the North American Plains Indian

The Kikuyu dwelling

The Kikuyu people of Kenya used to build their round dwellings, which were made from wattle and daub, with a base diameter between 4 m and 5 m. The cylindrical part used to be about 2.5 m tall: it was frequently painted white. The cone shaped roof, made of thatch, would overhang the hut by about a metre to provide a shady verandah.

The tipi of the North American Plains Indians

The North American Plains Indians used to build tipis (also spelt tepee) that were between 4 m and 6 m tall. A tipi was made of 20 to 30 pine poles which were covered with buffalo hides. It was easy to put up and take down. The Crow used to build tall tipis and the Cheyenne low, squat tipis.

The hennin, a 13–14th century hat

The hennin, or steeple headdress, was a cone with a veil attached at the top. It was fashionable in France, though it never became popular in Britain.

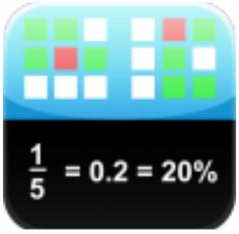
Extensions

1. Exchange cones with a friend. Calculate the radius and sector angle of the starting circle.
2. Write a computer or graphic calculator program, or a spreadsheet file to help with this work.

Our iPad and iPhone resources

Search for Jamtec on the AppStore. We also have other non-mathematics apps. Prices correct at 5 March 2017.


Age-ulator Free: Randomised £0.99



Directed Numbers £0.99: Equivalents £0.99: Multiplication Pairs £0.99

Maths Charts for Jenny Eather Free:
Maths Charts for Jenny Eather (Deluxe version) £4.99

GeoDraw £0.99 (iPad only)

Education APPs from Apple

Half price for volume purchase of some Education APPs
All non-free APPs above are eligible for this discount.

AdBlocker software for browsers

Many mathematics sites unfortunately have adverts: we really like the AdBlocker software which we download from Softonic at: http://bit.ly/adblockmw

SPIRE MATHS

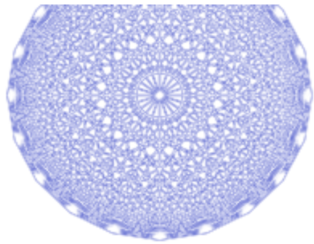
jamtec (Stoke) Ltd

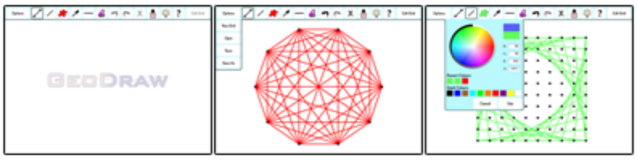
Stimulating, Practical, Interesting, Relevant, Enjoyable Maths For All

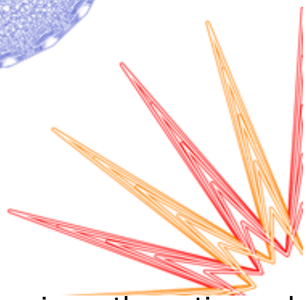
Maths APPs for iPads and iPhones

GEODRAW

Available on iPad iOS 5.0 or later! (iPad only)



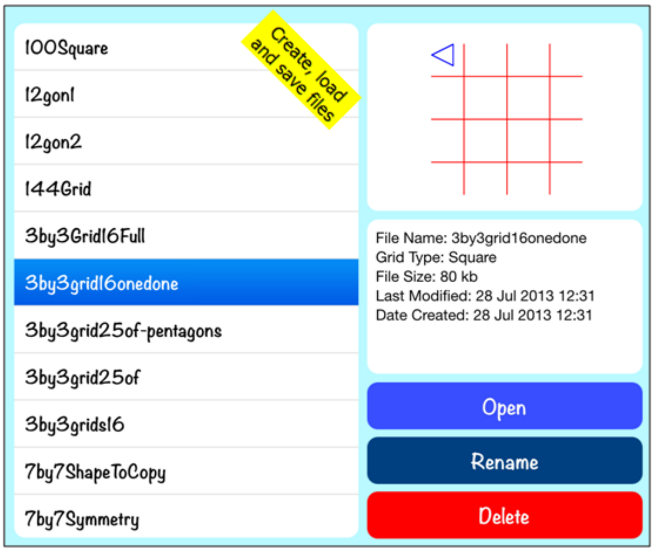


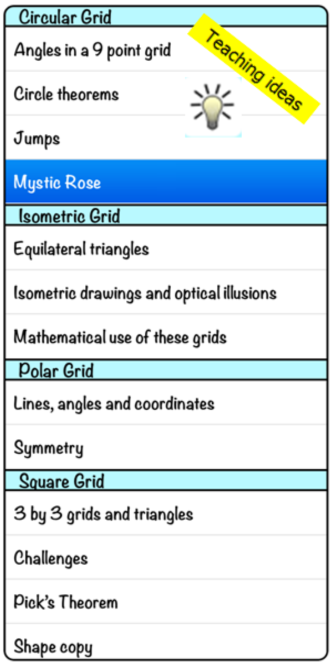


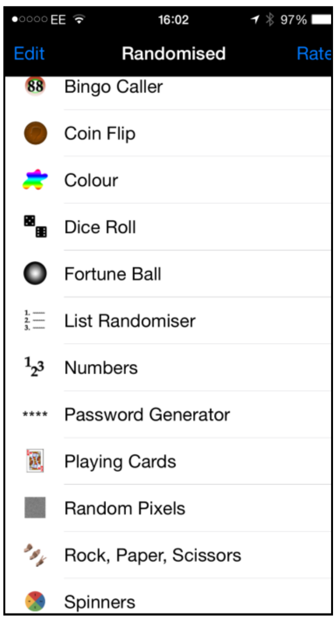
Change
Number of grid points
Grid point size
Line thickness
Line colour

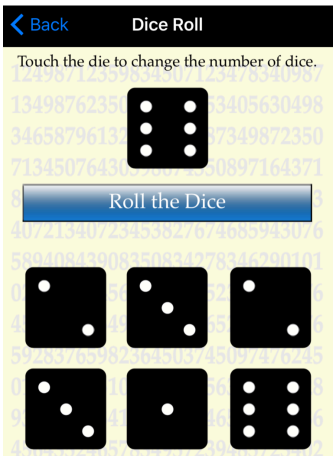
- GeoDraw offers users a choice of 5 grids for use in mathematics and D&T lessons. Send/export images with/without grid using: Bluetooth, Email, Facebook, Twitter and into Pages or Keynote.

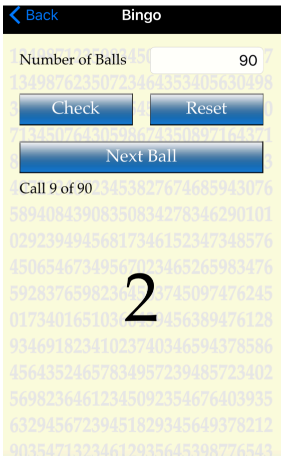
Eligible for VPP discount (see next page).



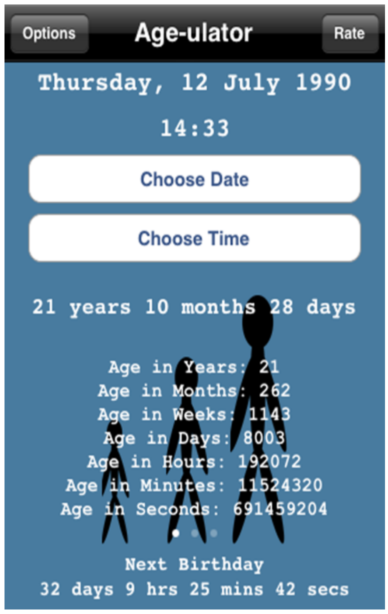








Randomised (99p): for probability lessons.
Age-ulator (free): for large number work and problem solving.



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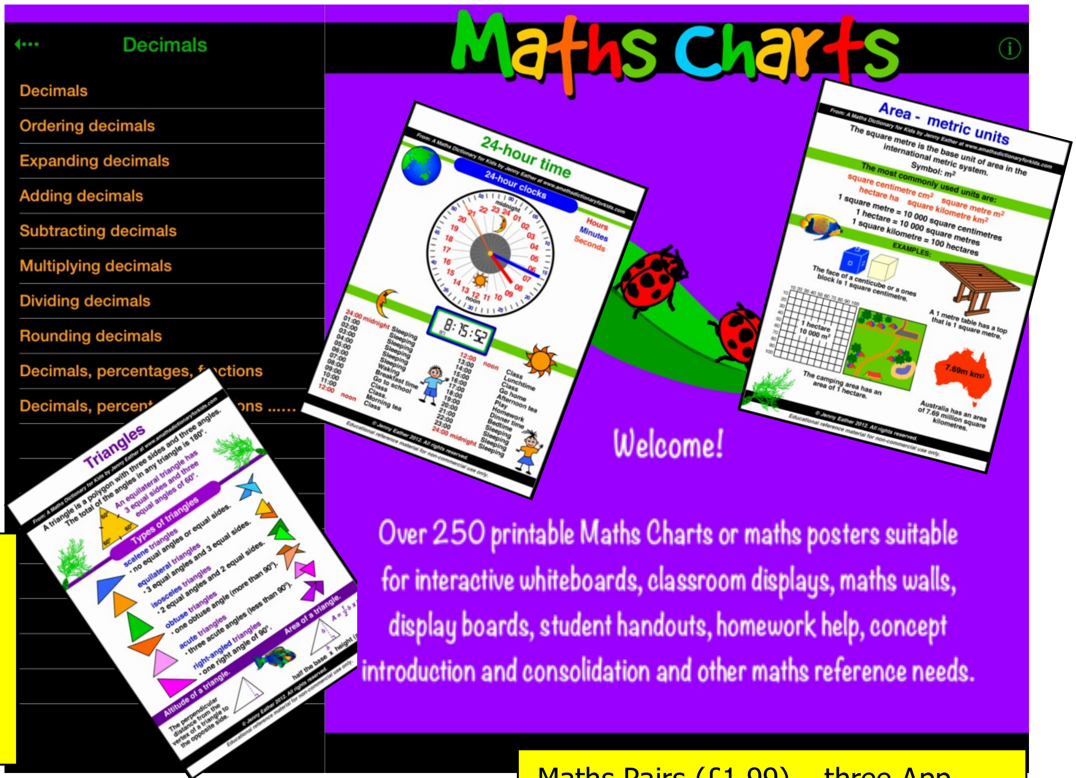
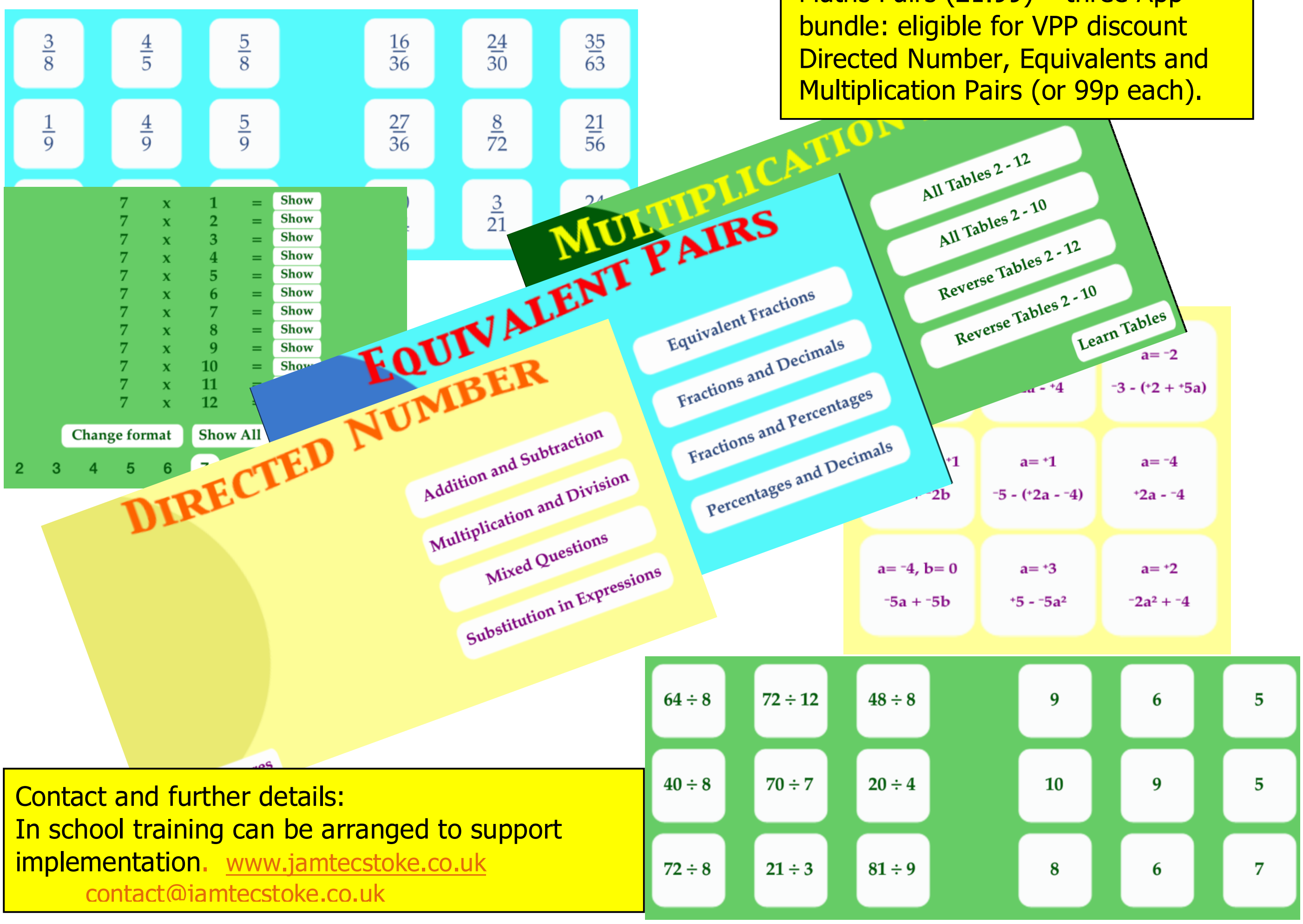
We've teamed up with Jenny Eather to bring her Maths Charts web resources to the iPad/iPhone.
Try Maths Charts by Jenny Eather for free, then buy full Deluxe version for £4.99 (half this if you sign up for VPP with Apple and buy 20 or more copies).



Volume Purchase Programme (VPP) lets you buy Apple apps at discount rate of half price for 20 or more of the same app.

Maths Pairs (£1.99) – three App bundle: eligible for VPP discount Directed Number, Equivalents and Multiplication Pairs (or 99p each).



Contact and further details:
In school training can be arranged to support implementation. www.jamtecstoke.co.uk
contact@jamtecstoke.co.uk