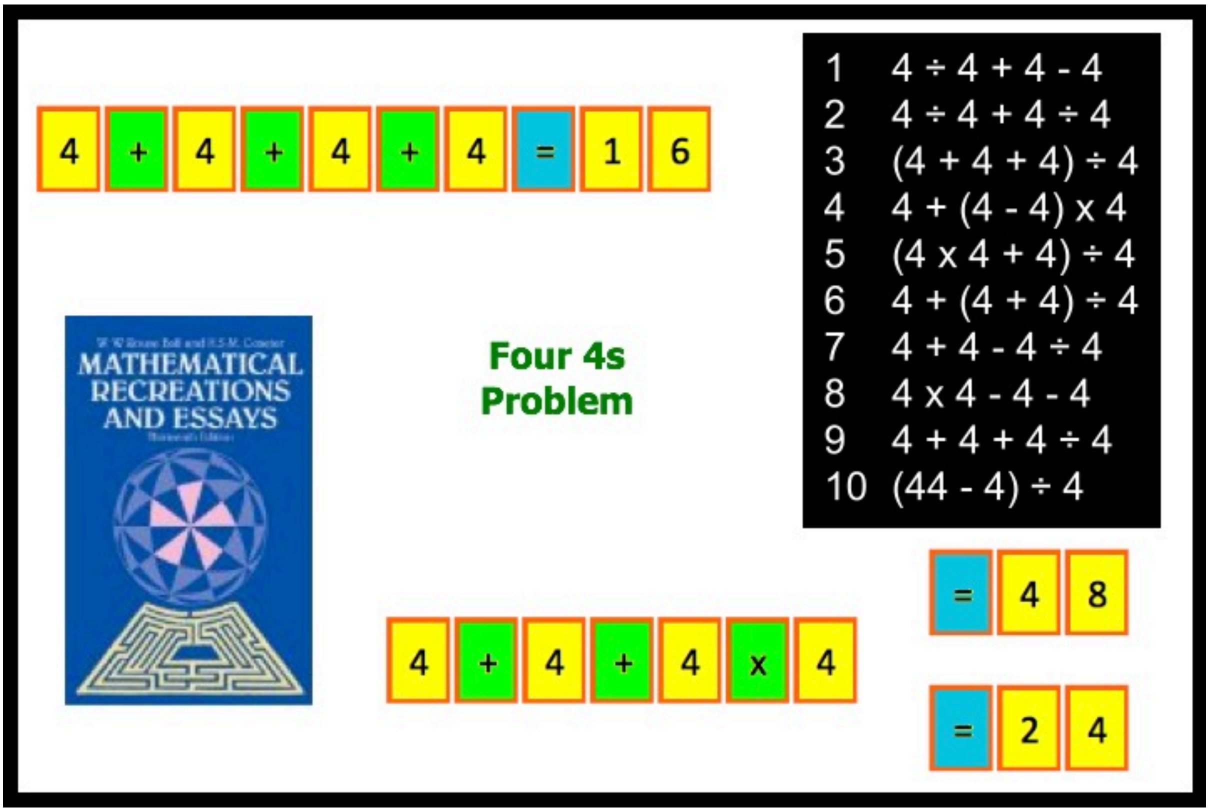
Four 4s



A Spire Maths Activity

https://spiremaths.co.uk/four4s/

This file, ActivInspire file, a spreadsheet with results, tables and charts can be found at: https://spiremaths.co.uk/four4s/

How to use exactly four 4s to make numbers up to 100 operations taken from +, -, x and ÷. It can be established that

$$1 = (4 + 4) \div (4 + 4)$$

Note that brackets are needed otherwise on a calculator:

$$4 + 4 \div 4 + 4 = 9$$

This leads us to show that without brackets ÷ takes precedence over +. Looking then at, e.g.

$$4 + 4 + 4 \times 4 = 24$$

and not 48 shows that x takes precedence over + too. Using these two examples with brackets placed in different positions can lead to many of the BIDMAS results.

In order to make numbers like 11, 13, 14 and 18 you can use the square root of 4, √4 = 2, and in order to make 19 you can use Factorial 4, 4! = 4 x 3x 2x 1 = 24.

The numbers up to 20 can be made up as follows:

1	4 ÷ 4 + 4 - 4
2	4 ÷ 4 + 4 ÷ 4
3	(4 + 4 + 4) ÷ 4
4	4 + (4 - 4) x 4
5	(4 x 4 + 4) ÷ 4
6	4 + (4 + 4) ÷ 4
7	4 + 4 - 4 ÷ 4
8	4 x 4 - 4 - 4
9	4 + 4 + 4 ÷ 4
10	(44 - 4) ÷ 4

11	44 ÷ (√4 x √4)
12	(44 + 4) ÷ 4
13	44 ÷ 4 + √4
14	4 x 4 - (4 ÷ √4)
15	4 x 4 - (4 ÷ 4)
16	4 x 4 + 4 - 4
17	4 x 4 + (4 ÷ 4)
18	4 x 4 + (4 ÷ √4)
19	4! - (4 ÷ 4 + 4)
20	4 x (4 ÷ 4 + 4)

Note how within these there are times when you get one number like 7 and then 9 is similar.

Other solutions and variations allow .4 and 4.4 but not 440. See for example:

https://en.wikipedia.org/wiki/Four_fours
http://mathforum.org/ruth/four4s.puzzle.html
http://mathforum.org/ruth/four4s.puzzle.html

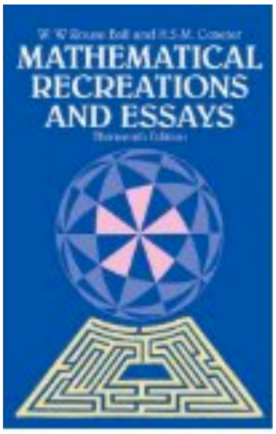
The excellent Numberphile site shows that every number can be made up to 10 with just +, -, x and ÷ but that you need concatenation (44), √ and ! to get to 20 and these plus allowing .4 gets you to 50 but for 99 you need more.

$$[4 \div 4]\% - (4 \div 4) = 99$$

since [4 ÷ 4]% means take the value in the bracket as a percentage and 1 as a percentage is 100.

The video then goes on to show that by using log you can make every number up to infinity (using log to base 4 and log to base ½).

https://www.youtube.com/watch?v=Noo4IN-vSvw



The earliest reference I have seen to this is in W W Rouse Ball's Mathematical Recreations and Essays originally of 1892 (but my version is the 1959 edition revised by H S M Coxeter). It is given in chapter 4 (page 122) as one of the Problems on a Chess-board with Counters or Pawns and is given as *First Problem with Pawns* though the problem is older. This book also contains many other investigations etc in use in contemporary mathematics classrooms.

The puzzle is found on age 16 of the book att:

https://books.google.co.uk/books?id=9lJqNJhYc9oC&pg=PP2&lpg=PP2&dq=mathematical+puzzles+rouse+ball&source=bl&ots=3Mx21wqXAM&sig=1PjIOEj12yqMGaouQr-cYSycctk&hl=en&sa=X&ved=0ahUKEwijgfHI2dnWAhWqKcAKHbbRAvAQ6AEIXTAP#v=onepage&q=mathematical%20puzzles%20rouse%20ball&f=false

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