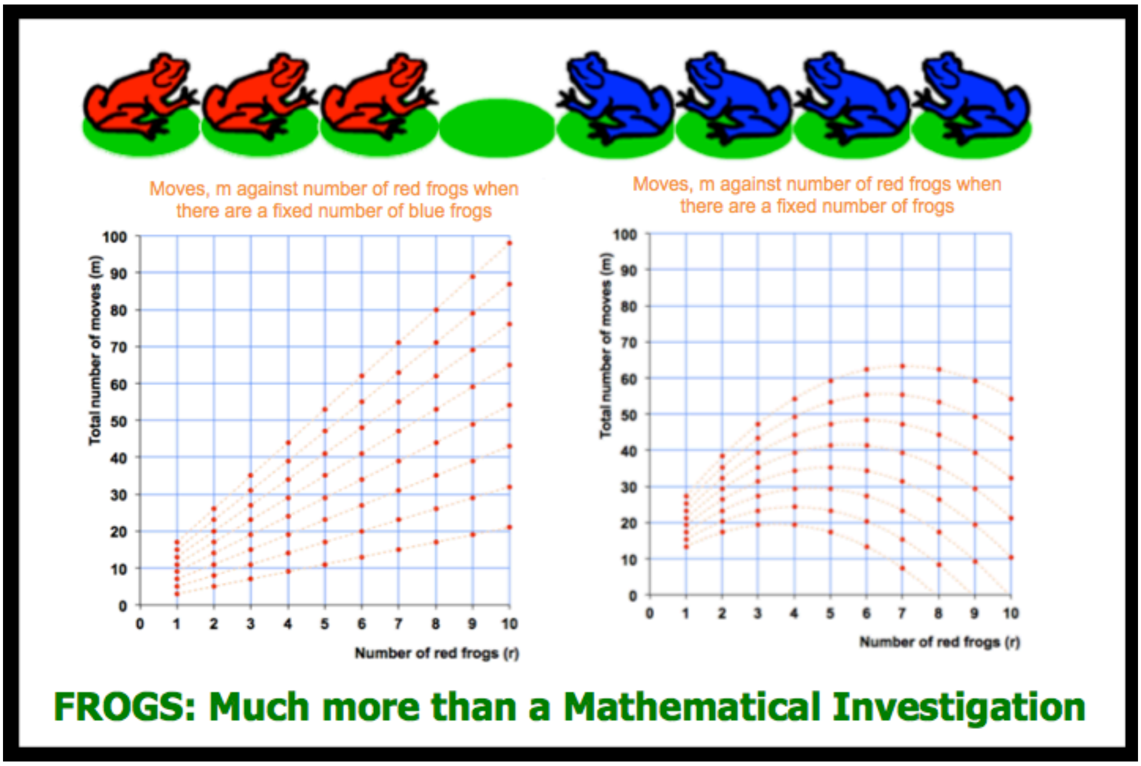
Frogs: Much More than a Mathematical Investigation



A Spire Maths Activity

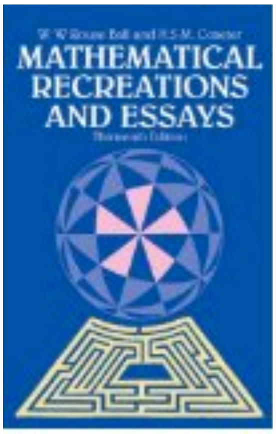
https://spiremaths.co.uk/frogs/

Table of Contents

Resources

This file, ActivInspire file, a spreadsheet with results, tables and charts can be found at:

https://spiremaths.co.uk/frogs/



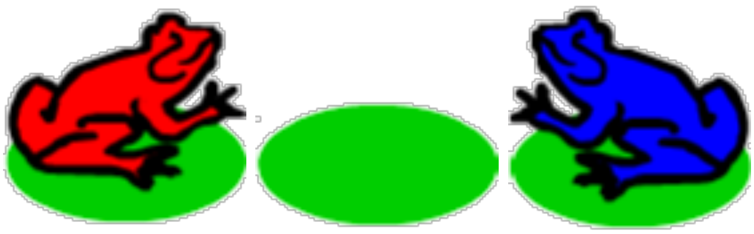
The earliest reference I have seen to this is in W W Rouse Ball's Mathematical Recreations and Essays originally of 1892 (but my version is the 1959 edition revised by H S M Coxeter). It is given in chapter 4 (page 122) as one of the Problems on a Chess-board with Counters or Pawns and is given as *First Problem with Pawns* though the problem is older. This book also contains many other investigations etc in use in contemporary mathematics classrooms.

Pages from Rouse Ball and Edouard Lucas books also found on the website above.

1. Pages shown are from the ActivInspire resource for this work.
2. The interactive Excel spreadsheet has multiple 'pages' that allow similar activities to those on the IWB file and follow the ideas below.
3. There are two flash files related to this from the Spire Maths Interactive Collection: a main lesson section and a plenary section. (These will not work on iPads or iPhones.)
4. There are teacher notes also for these two flash files.

Frog Images

These can be used to make your own IWB resource. Place the blanks first, then the red/blue frogs – the blanks are needed as place holder.



Mathematics in School Article (Free, but you have to register with jstor)

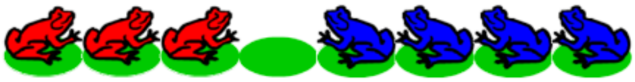
Patterns Which Are — Need Reasoning
Ruth Eagle
Mathematics in School
Vol. 24, No. 5 (Nov., 1995), pp. 44-46
Published by: The Mathematical Association
Stable URL: http://www.jstor.org/stable/30215225
Article gives a proof for the formula.

Frogs: video solution to the 3 by 4 problem

https://youtu.be/BLdCSVWX4lo

The Standard Problem

Three red and four blue frogs are shown on eight lily pads. Each can only move in the direction it is facing and can slide to an adjacent empty pad or hop over one frog of the other colour. The object is to swap the sides of the frogs. Click link at top for Spire Maths interactive for this problem.



Red	Blue	Hops	Slides	Moves
3	4			

Drag each frog in the appropriate direction and it will snap into place. Collect more or use fewer of each colour. Count the total slides and hops to solve any given start point. Find a formula for total moves.

Predict the Number of Hops, Slides and Moves

Hide the Hops, Slides and Moves columns.

Predict the number of Hops, Slides and Moves for the different combinations shown.
Find m, the total number of moves in terms of r, the number of red frogs and b, the number of blue frogs.

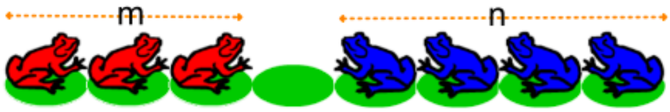
Red	Blue	Hops	Slides	Moves
3	4	12	7	19
2	3	6	5	11
3	10	30	13	43
3	8	24	11	35
5	5	25	10	35
1	25	25	26	51
4	9	36	13	49
3	12	36	2	38
2	20	40	3	43
1	19	19	4	23
n	n	n^2	$2n$	$n^2 + 2n$
r	b	rb	$r + b$	$rb + r + b$

Formula is: $m = rb + (r + b)$

Drag down each of the large pink rectangles above. Tap the pink beside the formula to see the formula.

Steps to a Proof

Assume m frogs on one side and n frogs the other side.



How many lily pads does each frog move?

Each red frog moves by n + 1 lily pads.

This gives m (n + 1) lily pads in total.

In the same way the total moves by the blue frogs is

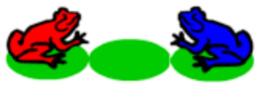
$$m\ (n + 1) \text{ lily pads}$$

So total moves, in lily pads, is the sum of these two

$= m\ (n + 1) + n\ (m + 1)$

$= mn + m + nm + n$ But mn = nm

$= 2mn + m + n$



How many hops are there?

It can be thought of that for each red/blue one of them has to hop the other.

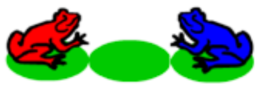
This is the same as thinking of it as each red hopping each blue. It does not matter which one hops, but one of them has to.

So there are mn hops in total.

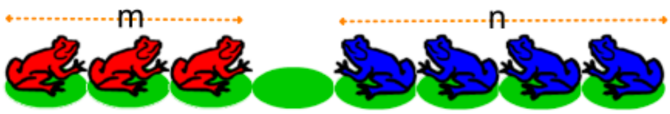
But each hop moves a frog by 2 lily pads.

So total lily pad movement because of these hops

= 2mn



In summary



Total movement in lily pads is

= m (n + 1) + n (m + 1)

= mn + m + nm + n

= 2mn + m + n

Total movement in lily pads from the hops

= 2mn

The difference must come from the slides and each of these is a movement of one lily pad.

Total slides = m + n

Reversing the Problem: Find the Number of Frogs

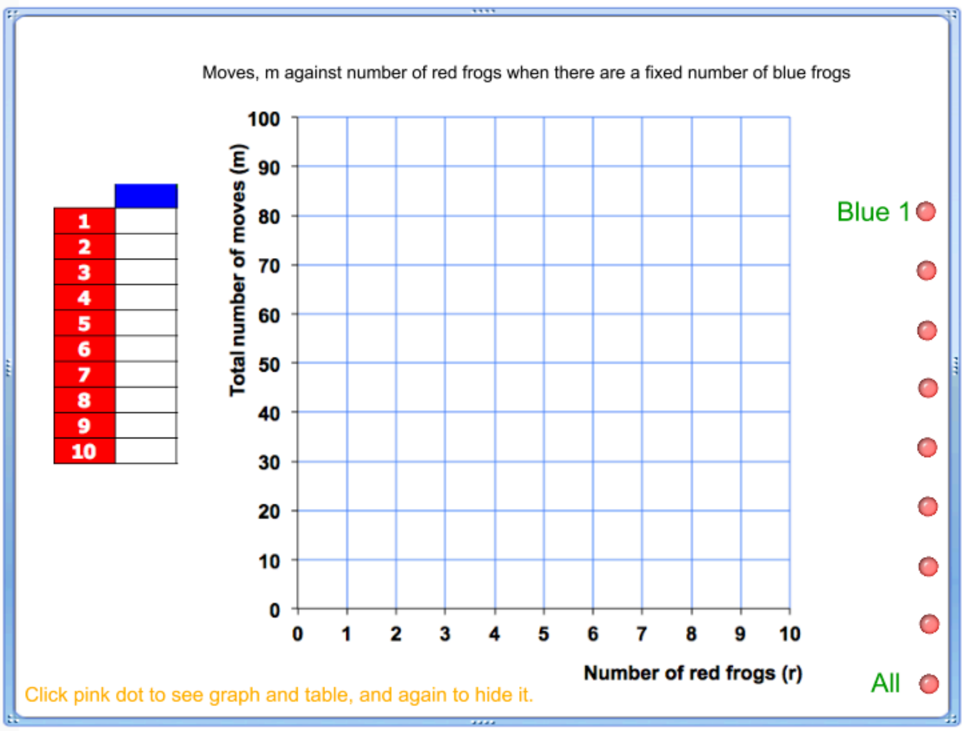
Hide the Red and Blue columns. Although you cannot have negative moves the ‘number’ puzzle solution can be found. Note this is about factorising.

Red	Blue	Hops	Slides	Moves
3	4	12	7	19
4	5	20	9	29
11	3	33	14	47
2	9	18	11	29
7	7	49	14	63
6	8	48	14	62
7	9	63	16	79
11	12	132	23	155
-3	-7	21	-10	?
6	-5	-30	1	?
8	0.5	4	8.5	?
20	0.2	4	20.2	?

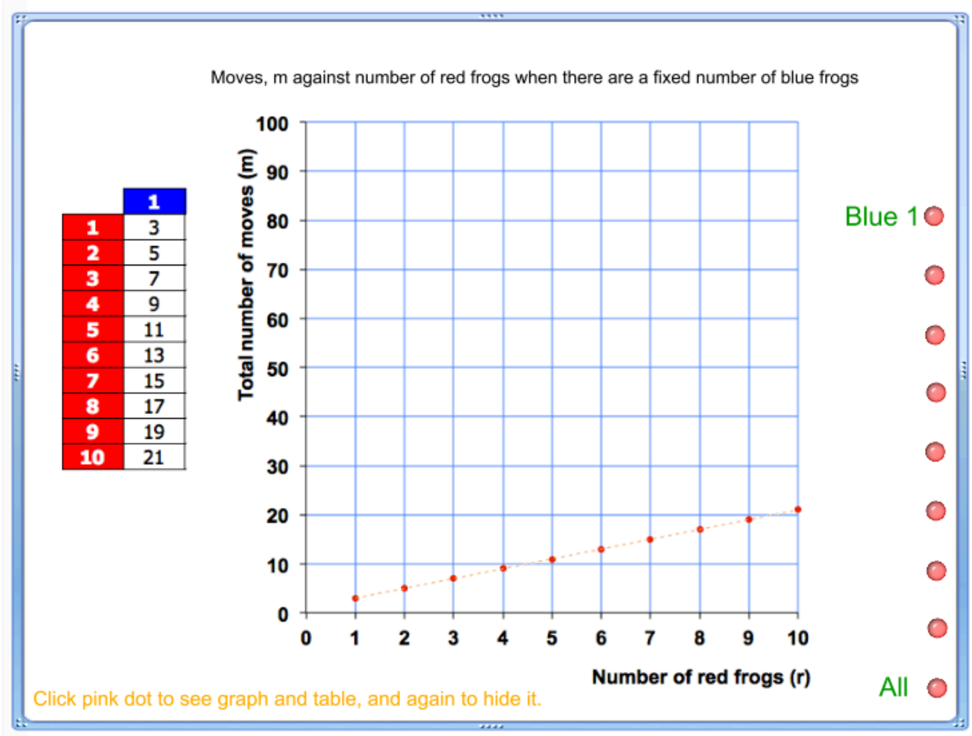
Drag down each of the large pink rectangles above.

Fixing the Number of Blue Frogs

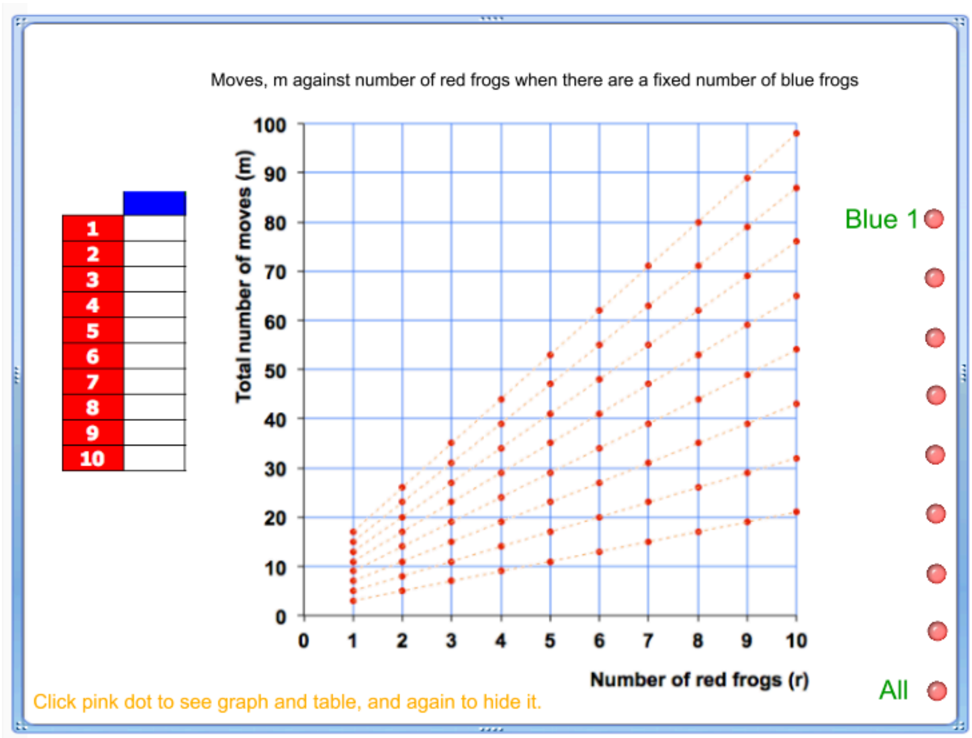
This is really about graphs! Dotted lines are shown connecting points (since graph only has meaning at these points, but line helps with understanding the rule).
What happens when you have e.g. 4 blue frogs and vary the number of red frogs?



Here is the graph and the table of values when there is 1 blue frog (and number of red frogs vary).

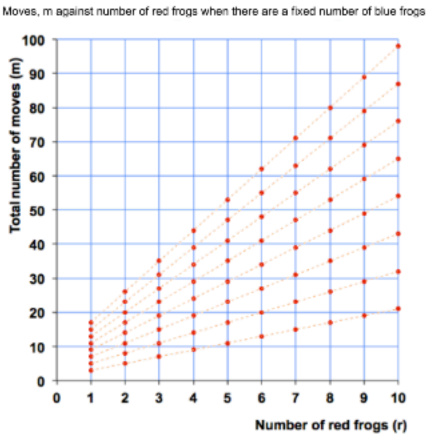


This shows all graphs for when there is 1 blue frog to when there are 8 blue frogs (and the reds vary).



Tables of values, graphs and equations related to situation above.

	1	2	3	4	5	6	7	8
1	3	5	7	9	11	13	15	17
2	5	8	11	14	17	20	23	26
3	7	11	15	19	23	27	31	35
4	9	14	19	24	29	34	39	44
5	11	17	23	29	35	41	47	53
6	13	20	27	34	41	48	55	62
7	15	23	31	39	47	55	63	71
8	17	26	35	44	53	62	71	80
9	19	29	39	49	59	69	79	89
10	21	32	43	54	65	76	87	98



Formula for b = 1:
$m = 2r + 1$

Formula for b = 2:
$m = 3r + 2$

Formula for b = 3:
$m = 4r + 3$

Formula for b = 4:
$m = 5r + 4$

Formula for b = 5:
$m = 6r + 5$

Formula for b = 6:
$m = 7r + 6$

Formula for b = 7:
$m = 8r + 7$

Formula for b = 8:
$m = 9r + 8$

Formula for b = n:
$m = (n + 1)r + n$

Drag pink rectangles to side to show different column(s) of figures. Click blue to see a formula. Click again to hide it.

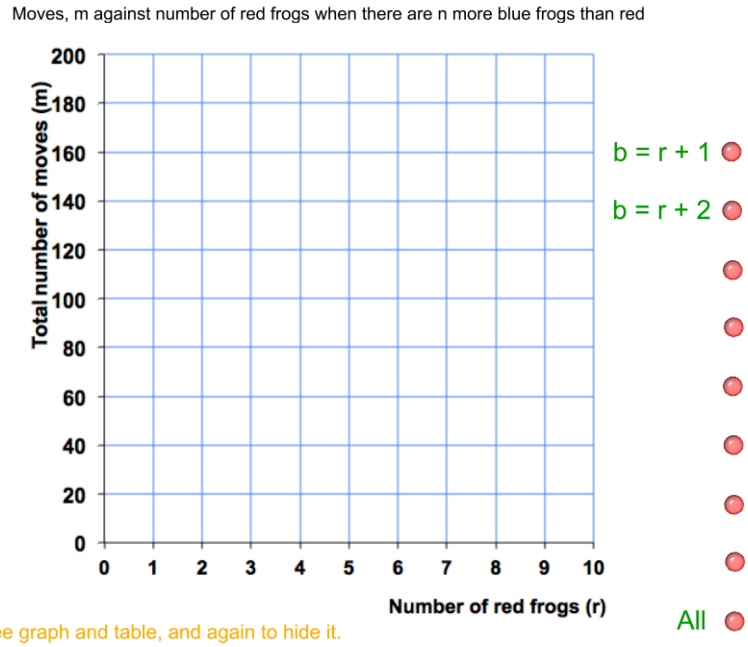
Your Own Rules for the Number of Frogs

Two more examples are given.

Fixing the Number of Blue Frogs to be n more than the Red

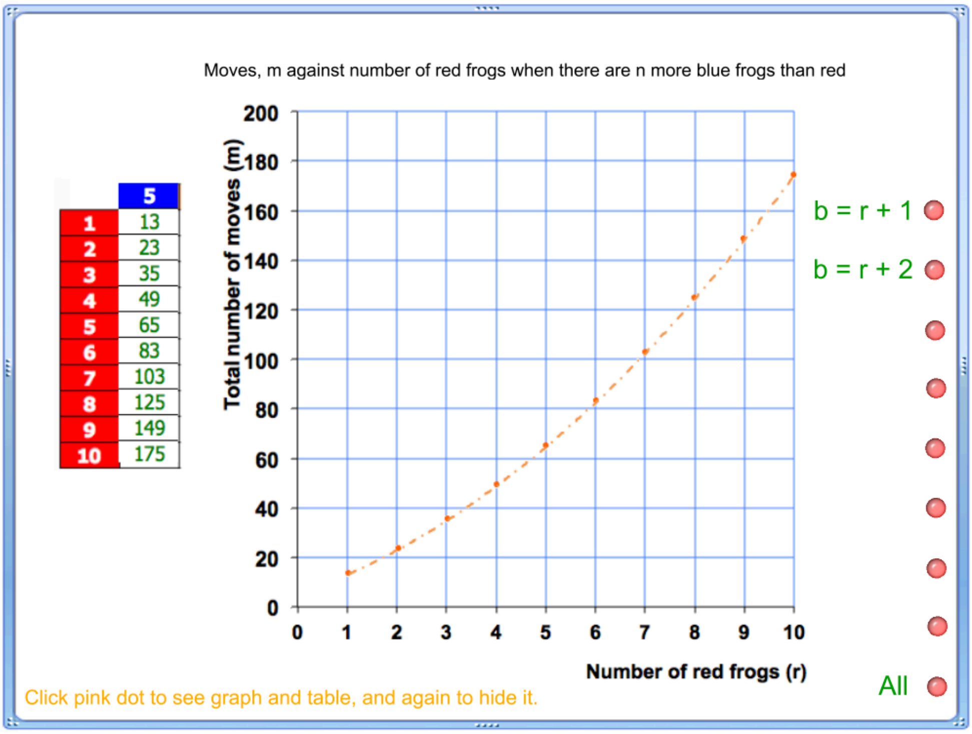
Always have one more blue than red frogs and vary the number of red frogs.

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

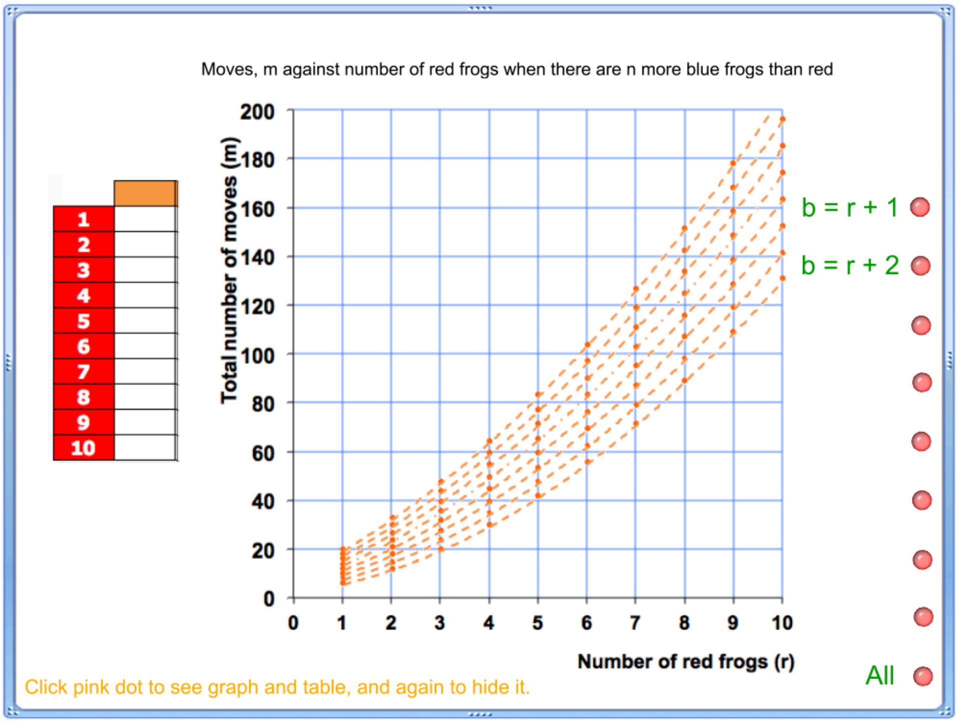


Click pink dot to see graph and table, and again to hide it.

Here the table of values and the graph show the case when there are 5 more blue frogs than red frogs.

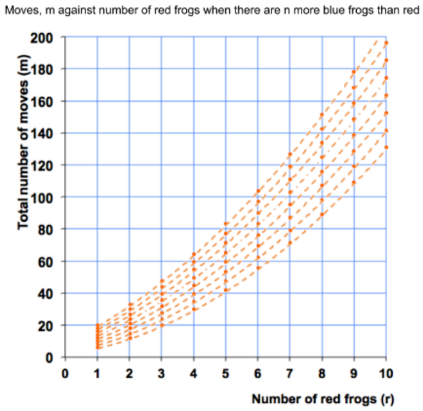


Graphs for when number of blue is between 1 and 8 more than the number of red frogs (which vary).



Tables of values, graphs and equations related to situation above.

	1	2	3	4	5	6	7	8
1	5	7	9	11	13	15	17	19
2	11	14	17	20	23	26	29	32
3	19	23	27	31	35	39	43	47
4	29	34	39	44	49	54	59	64
5	41	47	53	59	65	71	77	83
6	55	62	69	76	83	90	97	104
7	71	79	87	95	103	111	119	127
8	89	98	107	116	125	134	143	152
9	109	119	129	139	149	159	169	179
10	131	142	153	164	175	186	197	208



Formula b = r +1:
$m = r^2 + 3r + 1$

Formula b = r + 2:
$m = r^2 + 4r + 2$

Formula b = r + 3:
$m = r^2 + 5r + 3$

Formula b = r + 4:
$m = r^2 + 6r + 4$

Formula b = r + 5:
$m = r^2 + 7r + 5$

Formula b = r + 6:
$m = r^2 + 8r + 6$

Formula b = r + 7:
$m = r^2 + 9r + 7$

Formula b = r + 8:
$m = r^2 + 10r + 8$

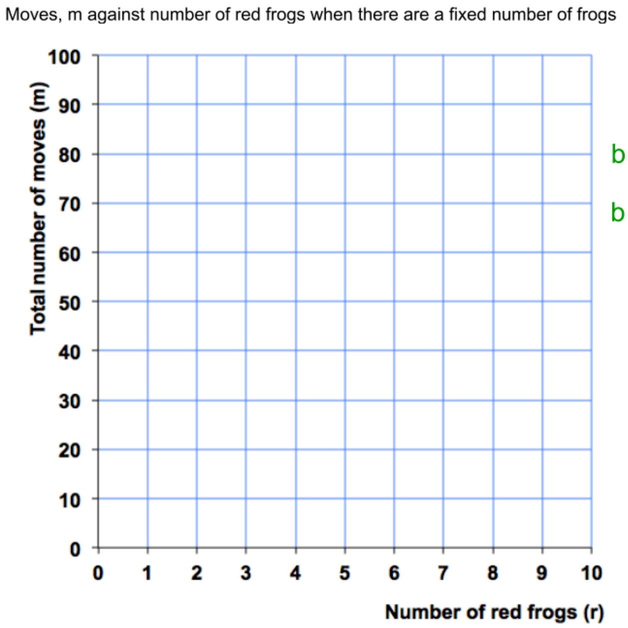
Formula b = r + n:
$m = r^2 + (n + 2)r + n$

Drag pink rectangles to side to show different column(s) of figures. Click blue to see a formula. Click again to hide it.

Keeping the Number of Frogs Constant

Here we start with exactly 7 frogs and vary the number of each colour.

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



b + r = 7

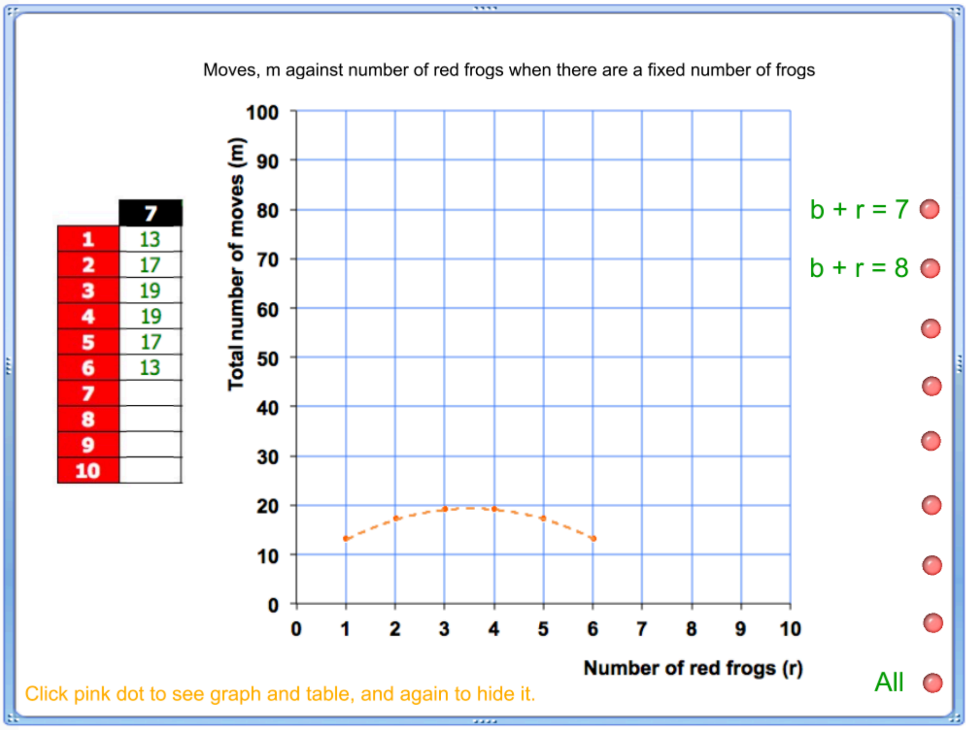
b + r = 8

All

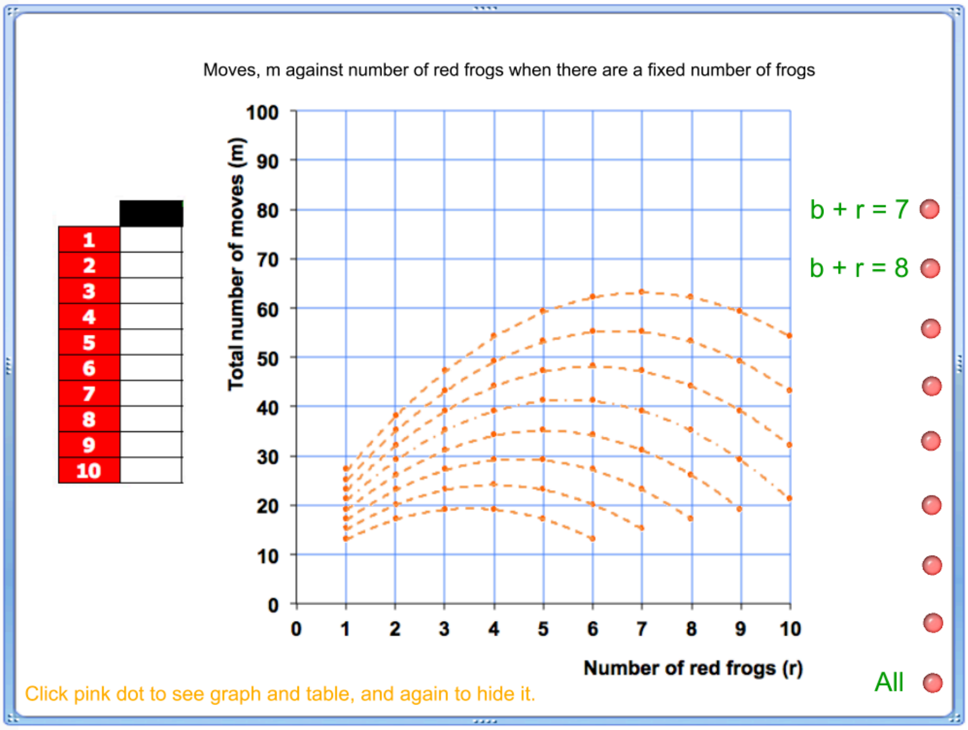
Click pink dot to see graph and table, and again to hide it.

This shows the table of values and the graph for when there are exactly 7 frogs, with the number of red frogs varying. This time it does not a straight line.

Note that although the situation from the numbers could be extended to e.g. 7 of one colour and none on the other this is not a sensible frogs example.



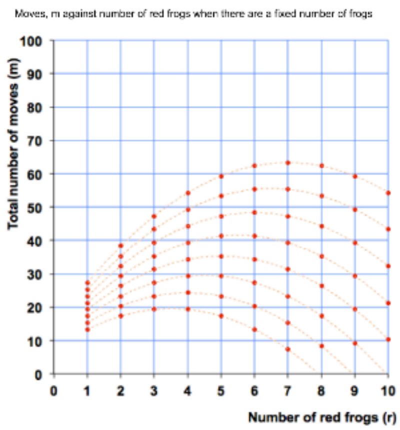
All the graphs for total number of frogs between 7 and 14 (the top quadratic).



Tables of values, graphs and equations related to situation above.

Total number of Frogs is e.g. n

	7	8	9	10	11	12	13	14
1	13	15	17	19	21	23	25	27
2	17	20	23	26	29	32	35	38
3	19	23	27	31	35	39	43	47
4	19	24	29	34	39	44	49	54
5	17	23	29	35	41	47	53	59
6	13	20	27	34	41	48	55	62
7	7	15	23	31	39	47	55	63
8	-1	8	17	26	35	44	53	62
9	-11	-1	9	19	29	39	49	59
10	-23	-12	-1	10	21	32	43	54



Formula 7 frogs:
$m = -x^2 + 7x + 7$

Formula 8 frogs:
$m = -x^2 + 8x + 8$

Formula 9 frogs:
$m = -x^2 + 9x + 9$

Formula 10 frogs:
$m = -x^2 + 10x + 10$

Formula 11 frogs:
$m = -x^2 + 11x + 11$

Formula 12 frogs:
$m = -x^2 + 12x + 12$

Formula 13 frogs:
$m = -x^2 + 13x + 13$

Formula 14 frogs:
$m = -x^2 + 14x + 14$

Formula n frogs:
$m = -x^2 + nx + n$

Drag pink rectangles to side to show different column(s) of figures. Click blue to see a formula. Click again to hide it.

Keys to the Frogs Solution

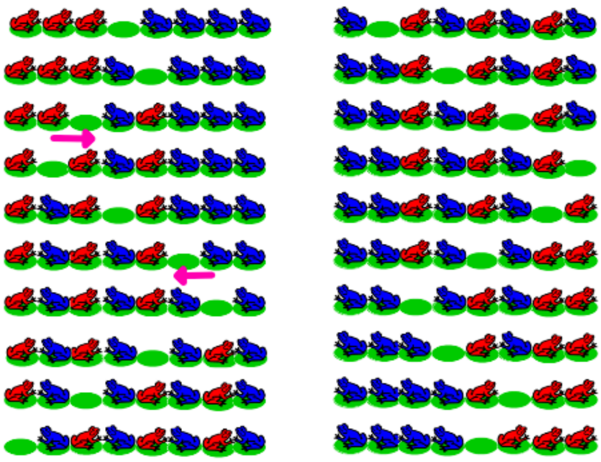
The key to being able to solve this is to always make sure you follow hops with a slide of the same colour frog. Moves 2 and 5 are crucial below.

Whenever you can follow eg a red hop with a final red slide; similarly for blue.

Start

Key move for red

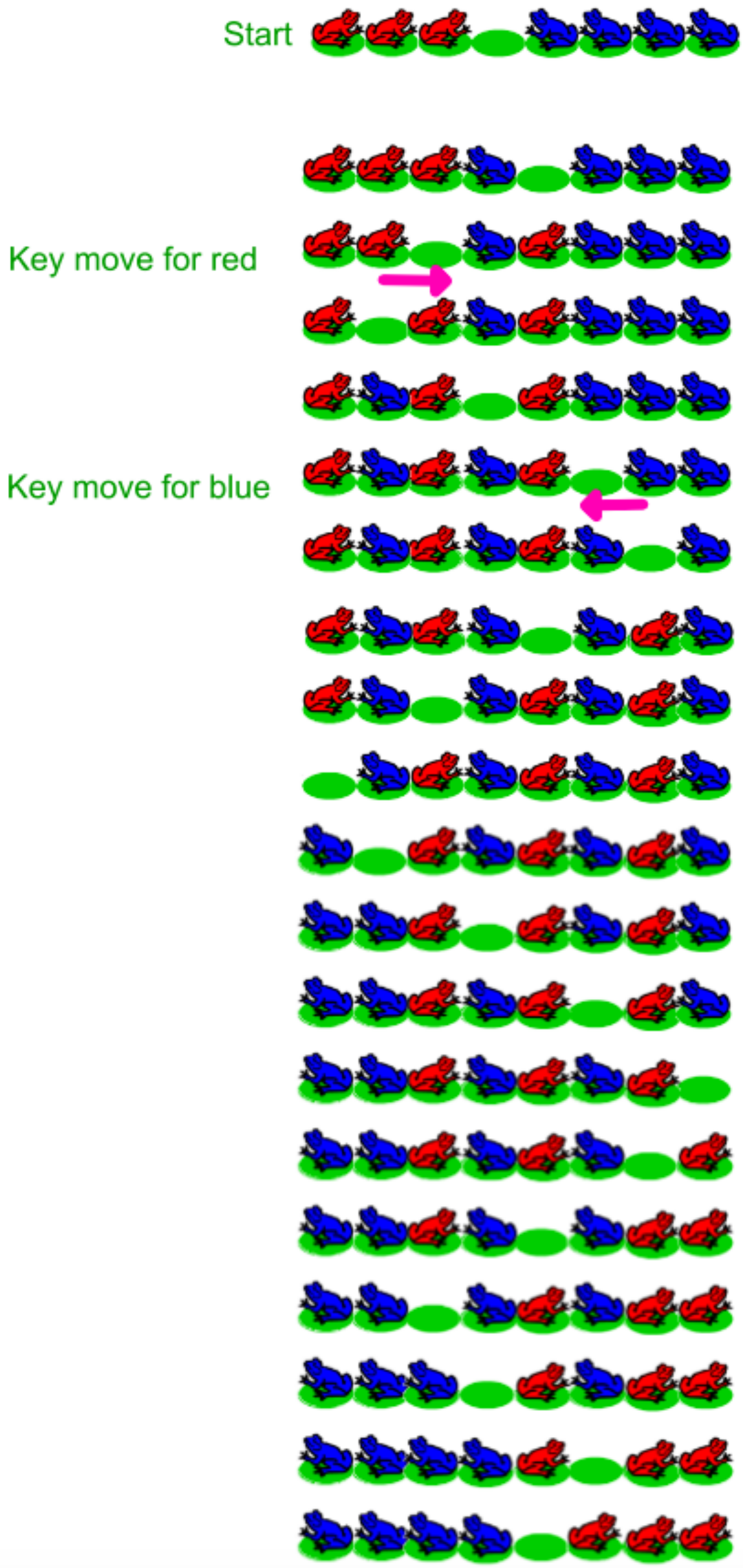
Key move for blue



For an 8 by 3 solution
https://www.youtube.com/watch?v=zwRF2j9eKQY

See:
https://www.youtube.com/watch?v=zwRF2j9eKQY

The 19 Move Solution to the 3 by 4 Frogs Problem



The Spreadsheet Pages

Frogs: Much More Than a Mathematical Investigation
A Spire Maths Activity

All resources for the activities at https://spiremaths.co.uk/frogs/

You can use this file on an interactive whiteboard or with a data projector
It contains moveable frogs, tables where answersgraphs/formulae can be shown/hidden

Resources on the website include the following
ActivInspire flipchart
pdf file with notes, solutions
Spreadsheet of graphs, tables linked to frogs work
Relevant book(s) etc.
2 flash files with ideas for main and plenary activities - will not work on iPads or iphones

Click tabs below to see the activities

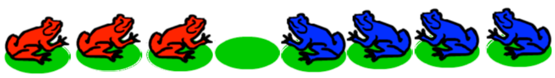
All you can do on this page is read the info above, link to the website or change to one of the other tabs for the activities.

The frogs in spreadsheet can be moved. The slider allows answers to be shown.

Frogs: The Standard Problem
Predict the number of Hops, Slides and Moves for the different combinations shown.
Find m, the total number of moves in terms of r, the number of red frogs and b, the number of blue frogs

Red	Blue	Hops	Slides	Moves
3	4			
2	3			
3	10			
3	8			
5	5			
1	25			
4	9			
3	12			
2	20			
1	19			
n	n			
r	b			

Move scroll bar down to see solutions (to bottom to see all)



You can move the frogs above to help find the solution to the problem

Frogs: The Standard Problem
Predict the number of Hops, Slides and Moves for the different combinations shown.
Find m, the total number of moves in terms of r, the number of red frogs and b, the number of blue frogs

Red	Blue	Hops	Slides	Moves
		12	7	19
		20	9	29
		33	14	47
		18	11	29
		49	14	63
		48	14	62
		63	16	79
		132	23	155
		21	-10	?
		-30	1	?
		4	8.5	?
		4	20.2	?

Move scroll bar down to see solutions (to bottom to see all)

The example shows the tables filled in for when there is 1 blue frog and the number of red frogs vary – the graph for this is shown too. Moving the top slider on the spreadsheet shows a different column of the bottom table for when there are, for example, 2 blue frogs and the number of red frogs varies. The graph then shown would match the table of values given to the left of the graph.

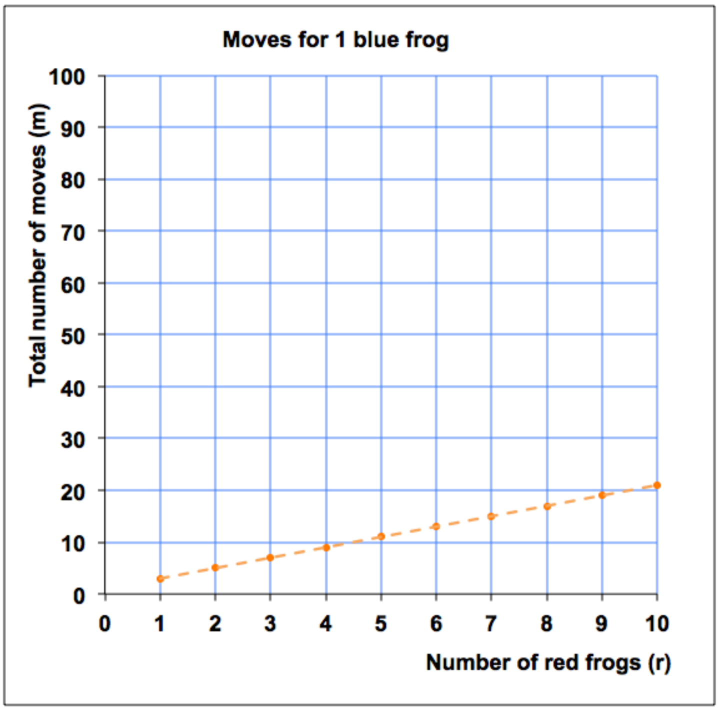
Relevant formulae are given in appropriate section above.

Fixing the Number of Blue Frogs

Moves for 1 blue frog

	1
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

Move scroll bar across to change number of fixed blue frogs



Formula hidden

Move scroll bar across to hide/show formula

Formula for number of moves for 1 blue frog

where m = total number of moves and r = number of red frogs

	1	2	3	4	5	6	7	8
1	3							
2	5							
3	7							
4	9							
5	11							
6	13							
7	15							
8	17							
9	19							
10	21							

The example shows the tables filled in for when there are 7 blue frogs and the number of red frogs vary – the graph for this is shown too. Works as page above.

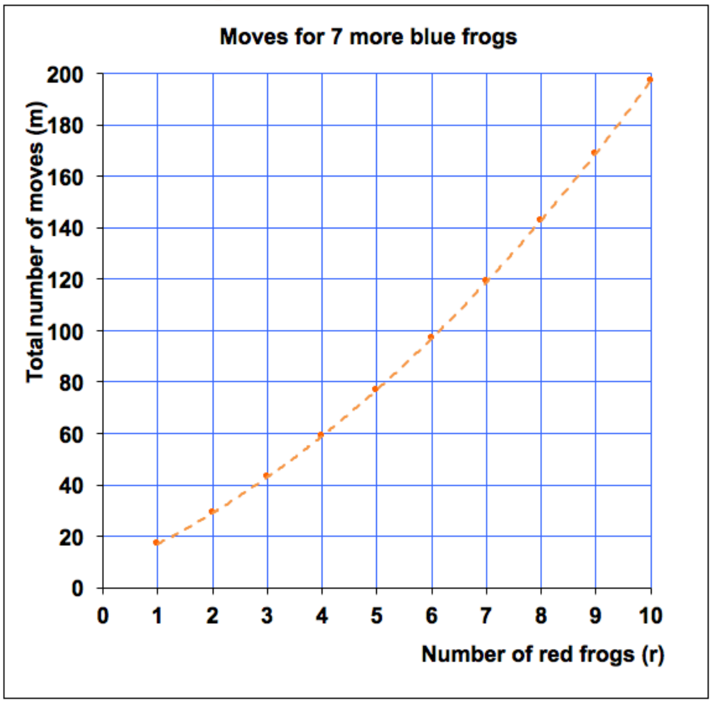
Relevant formulae are given in appropriate section above.

Number of Blue Frogs is n More Than Number of Red Frogs

Moves for 7 more blue frogs

	7
1	17
2	29
3	43
4	59
5	77
6	97
7	119
8	143
9	169
10	197

Move scroll bar across to change number of fixed blue frogs



Formula shown

Move scroll bar across to hide/show formula

Formula for number of moves for 7 more blue frogs

$m = r \times r + 9r + 7$ where m = total number of moves and r = number of red frogs

	1	2	3	4	5	6	7	8
1							17	
2							29	
3							43	
4							59	
5							77	
6							97	
7							119	
8							143	
9							169	
10							197	

The example shows the tables filled in for when there are exactly 7 frogs – the graph for this is shown too. Works as page above.

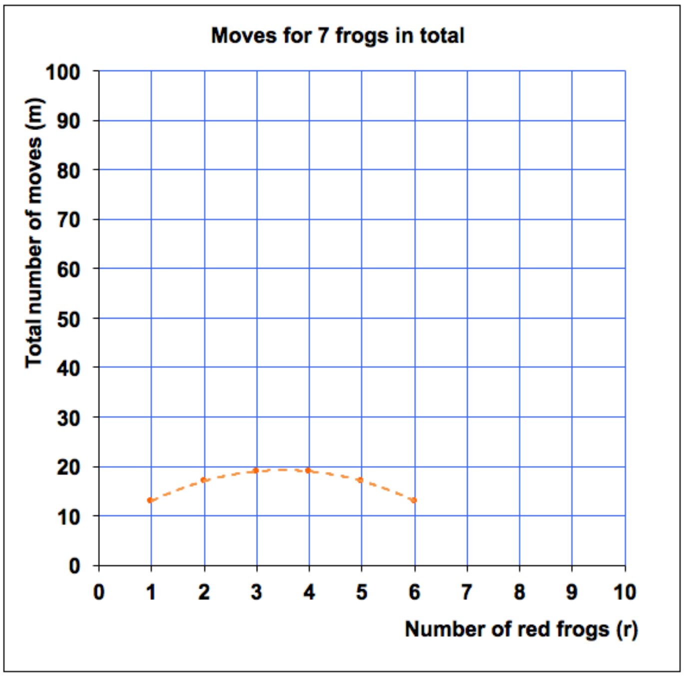
Relevant formulae are given in appropriate section above.

Total Number of Frogs is Fixed

Moves for 7 frogs in total

	7
1	13
2	17
3	19
4	19
5	17
6	13
7	
8	
9	
10	

Move scroll bar across to change number of fixed blue frogs



Formula hidden

Move scroll bar across to hide/show formula

Formula for number of moves for 7 frogs in total

where m = total number of moves and r = number of red frogs

	7	8	9	10	11	12	13	14
1	13							
2	17							
3	19							
4	19							
5	17							
6	13							
7								
8								
9								
10								

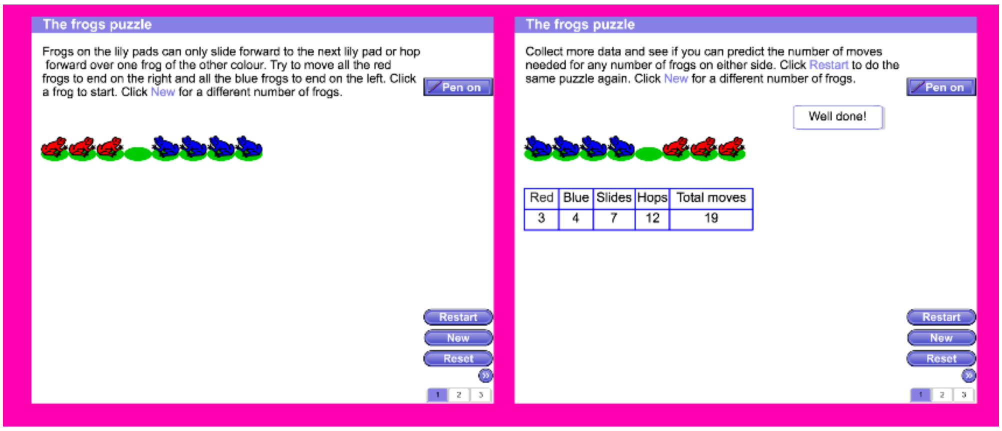
The Flash Files

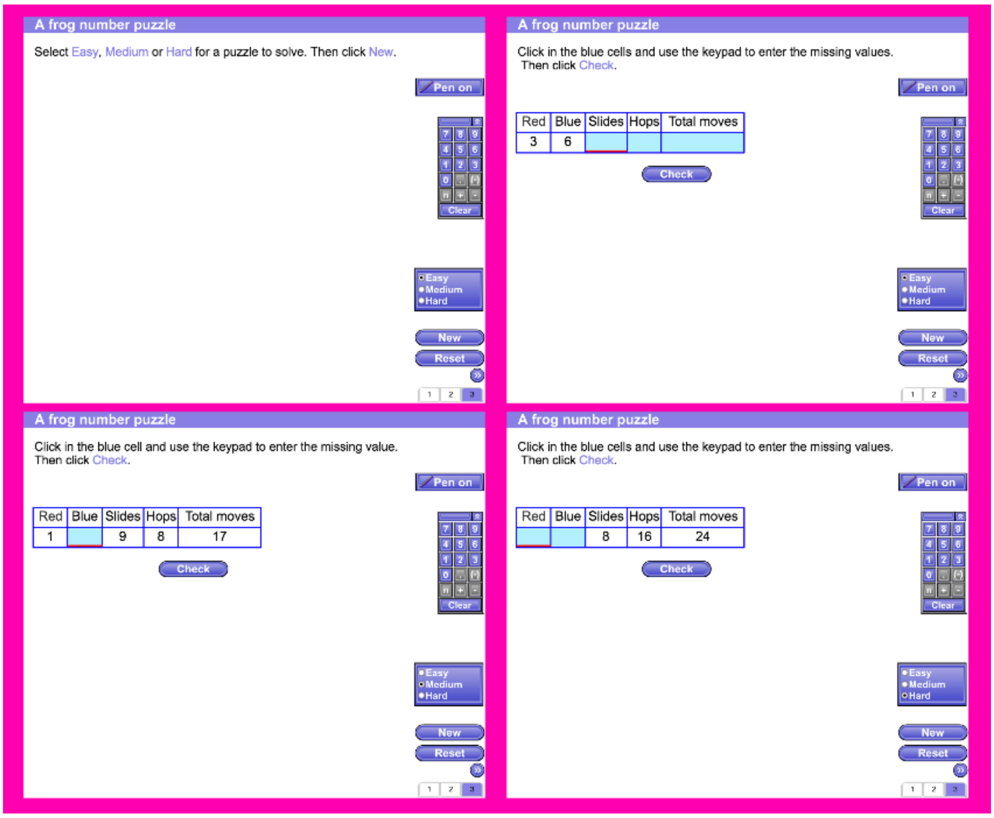
Main Activity

There are 3 main pages in this file. Each page is separated by blank lines.

These flash activities (so won't work on iPads and iPhones) found at:

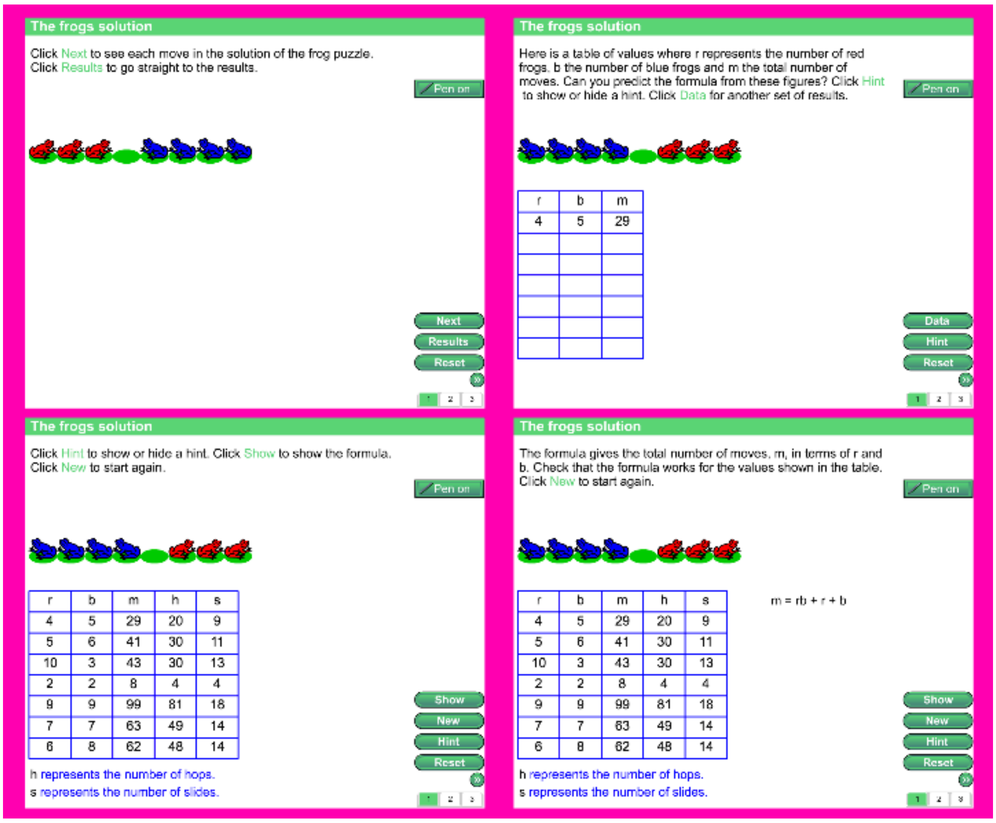
https://spiremaths.co.uk/wp-content/uploads/main_839.swf
https://spiremaths.co.uk/wp-content/uploads/plen_839.swf

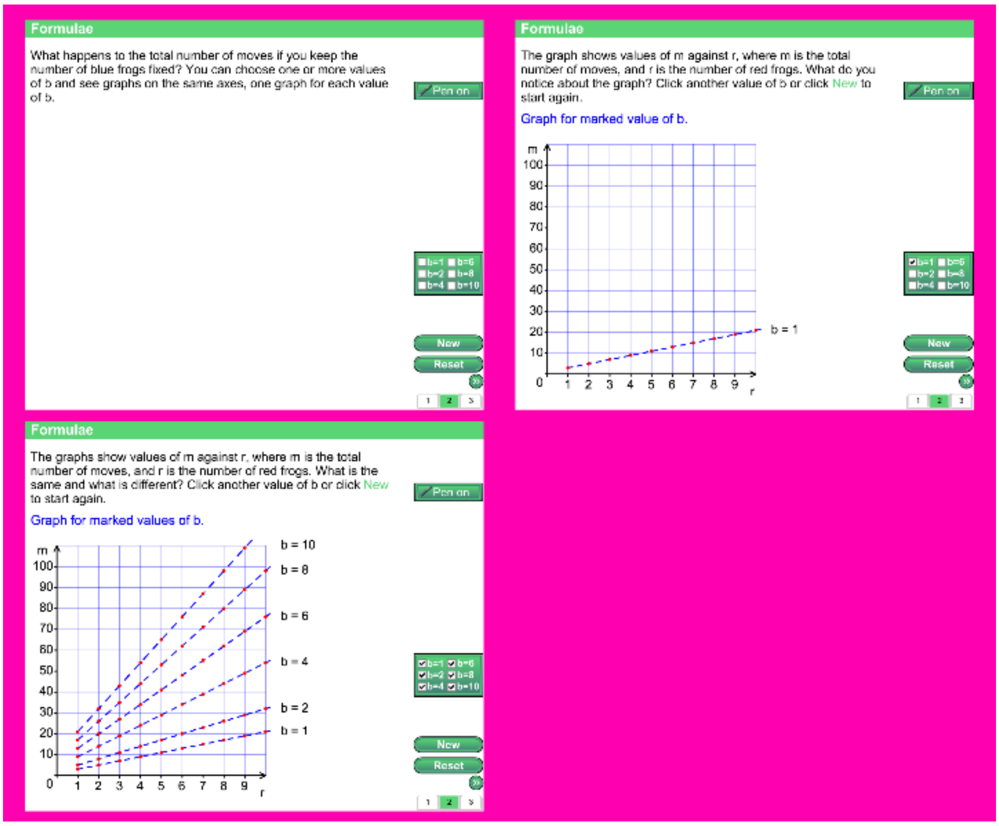




Plenary Activity

There are 3 main pages in this file. Each page is separated by blank lines.




Formulae
What happens to the total number of moves if you keep the number of blue frogs fixed? You can choose one or more values of b and see graphs on the same axes, one graph for each value of b.
Pen on
b=1 b=6 b=2 b=8 b=4 b=10
New
Reset
Formulae
The graph shows values of m against r, where m is the total number of moves, and r is the number of red frogs. What do you notice about the graph? Click another value of b or click New to start again.
Graph for marked value of b.
b = 1
Formulae
The graphs show values of m against r, where m is the total number of moves, and r is the number of red frogs. What is the same and what is different? Click another value of b or click New to start again.
Graph for marked values of b.
b = 10
b = 8
b = 6
b = 4
b = 2
b = 1


Vocabulary
Click on the top card to see a word.
Click on the card again to see its definition.
Symbol
A general name for something that is used in a special way in mathematics and has its own precise meaning.
Word
Definition
Reset
Vocabulary
Click on the top card to see a definition.
Click on the card again to see the word.
Algebraic expression
An algebraic mathematical statement, that does not include the = sign.

Our iPad and iPhone resources

Search for Jamtec on the AppStore. We also have other non-mathematics apps. Prices correct at 6 October 2015.


Age-ulator Free: Randomised £0.99

Directed Numbers £0.99: Equivalents £0.99: Multiplication Pairs £0.99

Maths Charts for Jenny Eather Free:
Maths Charts for Jenny Eather (Deluxe version) £4.99


Grids4Maths £0.99: GeoDraw £0.99 (iPad only)

Education APPs from Apple

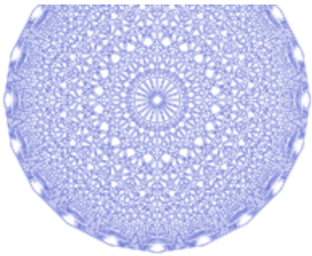
Half price for volume purchase of some Education APPs

Maths APPs for iPads and iPhones

GEODRAW

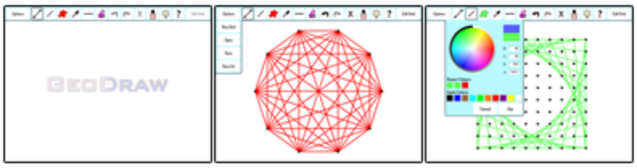
Available on iPad iOS 5.0 or later!

(iPad only)



Grids
Circular
Isometric: horizontal
Isometric: vertical
Polar
Square

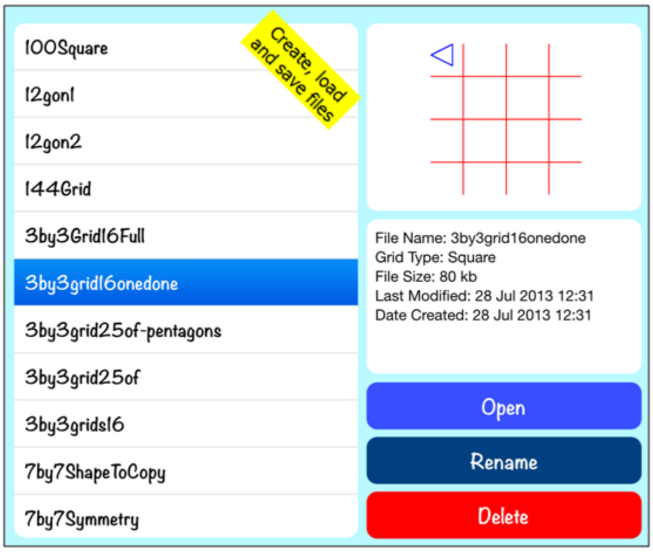
£0.99

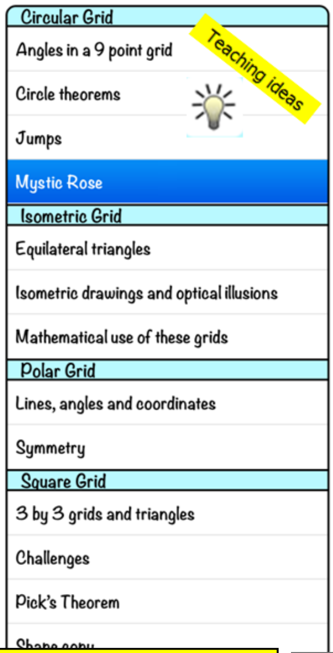


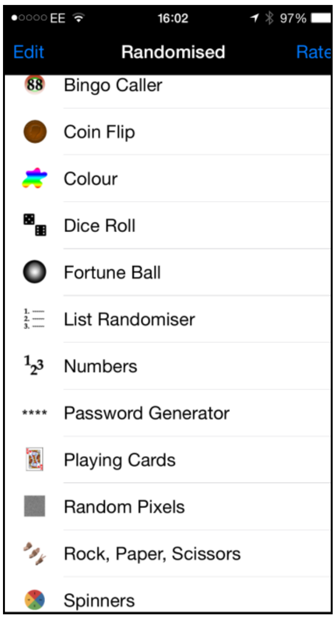
Change
Number of grid points
Grid point size
Line thickness
Line colour

- GeoDraw offers users a choice of 5 grids for use in mathematics and D&T lessons. Send/export images with/without grid using: Bluetooth, Email, Facebook, Twitter and into Pages or Keynote.

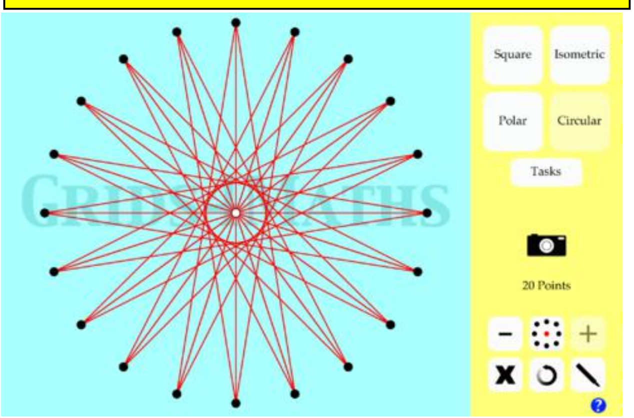
Eligible for VPP discount (see next page).

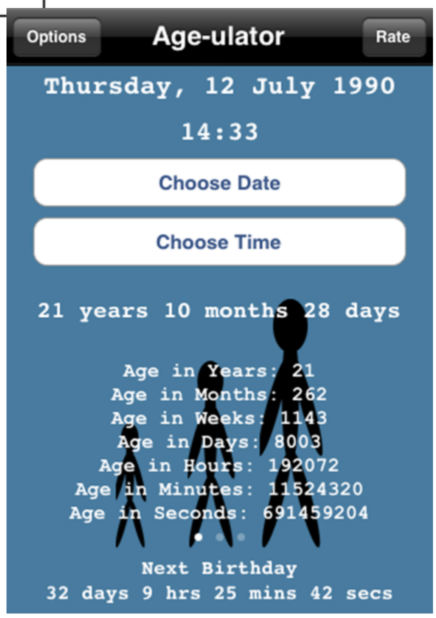




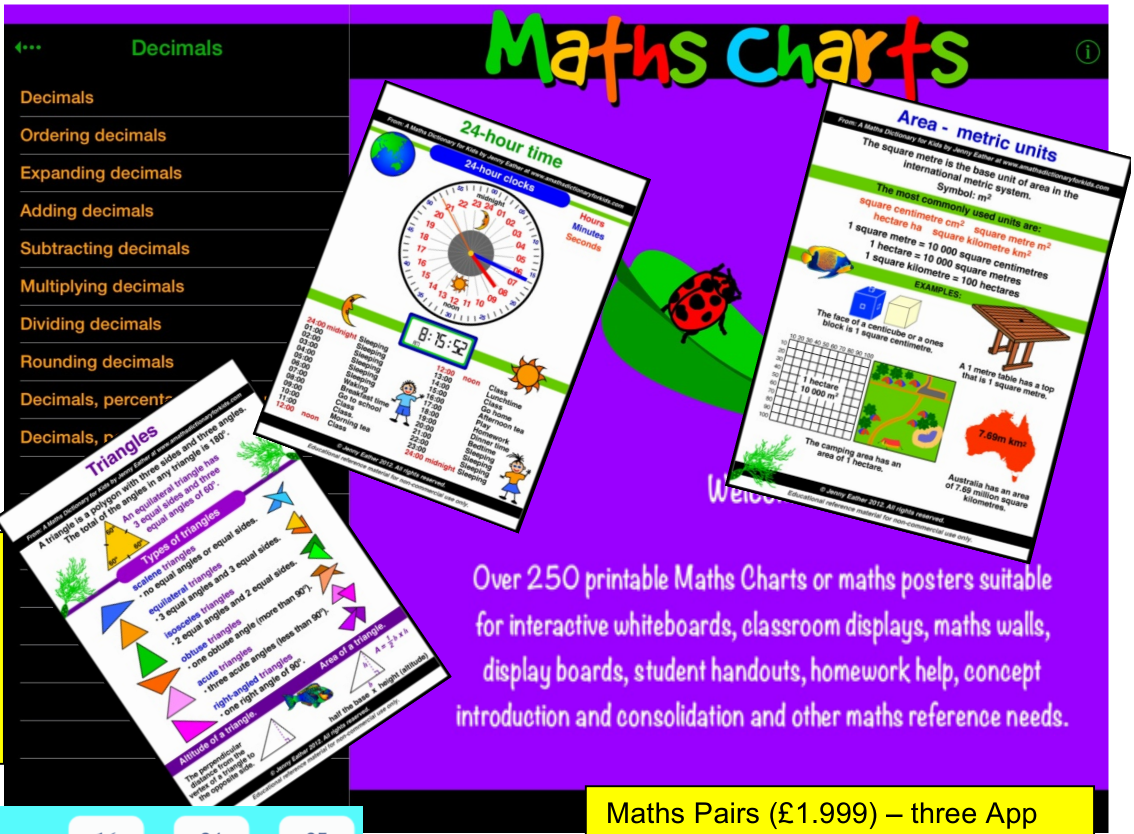


Randomised (99p): for probability lessons.
Age-ulator (free): for large number work and problem solving.
Grids4Maths (99p): much simpler version of GeoDraw for iPhones.

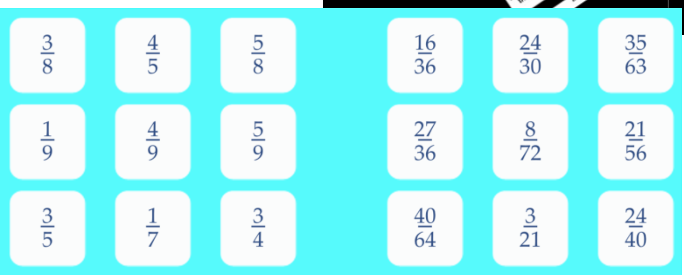




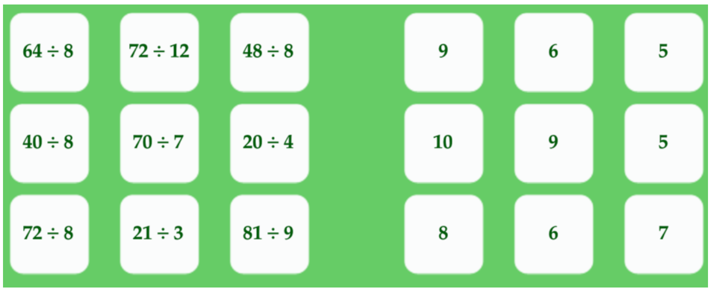
We've teamed up with Jenny Eather to bring her Maths Charts web resources to the iPad/iPhone. Try Maths Charts by Jenny Eather for free, then buy full Deluxe version for £4.99 (half this if you sign up for VPP with Apple and buy 20 or more copies).



Volume Purchase Programme (VPP) lets you buy Apple apps at discount rate of half price for 20 or more of the same app.



Maths Pairs (£1.999) – three App bundle: eligible for VPP discount Directed Number, Equivalents and Multiplication Pairs (or 99p each).



Contact and further details:
In school training can be arranged to support implementation. www.jamtecstoke.co.uk
contact@jamtecstoke.co.uk