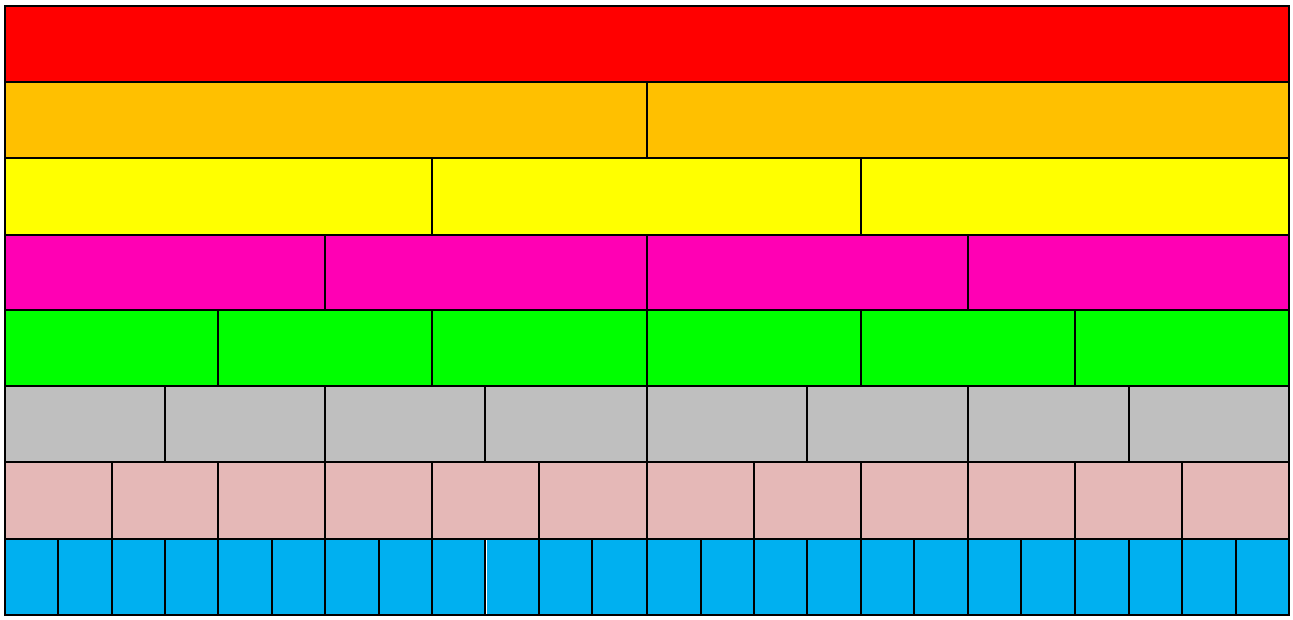
SPIRE MATHS

Stimulating, Practical, Interesting, Relevant, Enjoyable Maths for All

Factors (and Fraction) Activity

Factors image



Questions

1. What do you see?
2. What is the same and what is different?
3. How many rows? How many columns?

Possible answers and comments

Fractions or equivalent fractions. Not the target here but see later.
Halves, thirds, quarters, sixths, eighths, twelfths and twenty-fourths. See later.
Factors of 24 (less likely); 1, 2, 3, 4, 6, 8. 12, 24.

Factors work from the image

Further Questions

The 24-factor image has two odd rows and 6 even rows.

4. Will this always be the case for a different factor image?
5. Work out other factor images – what do you discover?

Results for Factor images

1. 1 is only number with exactly 1 row, all other numbers have two or more rows.
2. Prime numbers always have two rows, and except for the number 2, both are odd. [Reason: definition of a prime number.]
3. Square numbers always have an odd number of rows and all other numbers have an even number of rows. [Reason: factors are in pairs, except for the factor that is the square root of the number, which ‘pairs’ with itself to make the number.]
4. Powers of primes will always have one more row than the power of the number [Reason: each extra power just adds one more factor to the total number of factors; note that every other number is a square number.]

Examples: factors of powers of primes

Number	2	4	8	16	32	64	2^n
Power of number	1	2	3	4	5	6	n
Number of rows	2	3	4	5	6	7	n + 1

Number	3	9	27	81	243	729	3^n
Power of number	1	2	3	4	5	6	n
Number of rows	2	3	4	5	6	7	n + 1

5. Powers of non-primes will also alternate between odd and even numbers of factors. [Reason: every other power will be a square number.]

Number	10	100	1000	10000	100000	1000000	10^n
Power of number	1	2	3	4	5	6	n
Number of rows	4	9	14	19	24	29	4n + 1

6. Numbers that are the product of two primes will always have 4 rows: if one of the primes is 2 then two rows will be even and two will be odd, otherwise all the rows will be odd.

Examples: factors of multiples of two prime

6 has rows for 1, 2, 3, 6 – two odd, two even
15 has rows for 1, 3, 5, 15 – all odd
82 has rows for 1, 2, 41, 82 - two odd, two even
91 has rows for 1, 7, 13, 91 - all odd

7. Numbers that are the product of two primes will always have 4 rows: if one of the primes is 2 then two rows will be even and two will be odd, otherwise all the rows will be odd.
8. Generally, the number of factors of a number is found by sorting the number into its prime power factors, adding 1 to the highest power in each case and then multiplying these powers plus one together: an example is given below.

Examples: number of factors of a number (using powers of prime factors)

$2^1 \times 3^2 \times 5^3 = 2 \times 9 \times 125 = 2250$
$(1 + 1) \times (2 + 1) \times (3 + 1) = 2 \times 3 \times 4 = 24$
So: 2250 has 24 factors and these are: 1, 2, 3, 5, 6, 9, 10, 15, 18, 25, 30, 45, 50, 75, 90, 125, 150, 225, 250, 375, 450, 750, 1125, 2250

9. Generally, non-prime odd numbers will have only have odd rows. [Reason: all factors of an odd number are odd.]
10. Even number that are not multiples of 4 will have an equal number of odd and even rows. [Reason: in every factor pair of the number one factor of the pair will be even (since the 2 will be a factor of it) and the other will be odd (only made up of odd factors).]
11. The powers of two are the only numbers where it will have exactly 1 odd factor and the rest are all even. [Reason: all other numbers will have at least one odd factor which will add, at the very least one more odd row.]

Further reading/information

https://www.cut-the-knot.org/blue/NumberOfFactors.shtml
https://www.mathsisfun.com/numbers/factors-all-tool.html
http://mathforum.org/library/drmath/view/57151.html

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Fractions related activity

See

https://nrich.maths.org/4561

Equivalent fractions image