N5 • Understanding the laws of arithmetic

Mathematical goals

To enable learners to:

- interpret numerical expressions using words and area representations;
- recognise the order of operations;
- recognise equivalent expressions;
- understand the distributive laws of multiplication and division over addition (the expansion of brackets).

Starting points

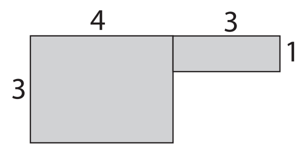
Most learners will have used the laws of arithmetic but some may have little understanding of the underlying principles. It is not enough to use a rote-learned rule like BODMAS or BIDMAS, as the introduction to the session will show.

The session assumes that learners are familiar with indices and with the area of simple compound shapes made up from rectangles joined together. However, you may need to check, through questioning, that learners understand these ideas:

What is the difference between 3×2 and 3^2?

Draw two different rectangles with an area of 36.

What is the area of this shape?



Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Calculations*;
- Card set B – *Areas*;
- Card set C – *Solutions*;
- the sheets for the brackets activity (optional).

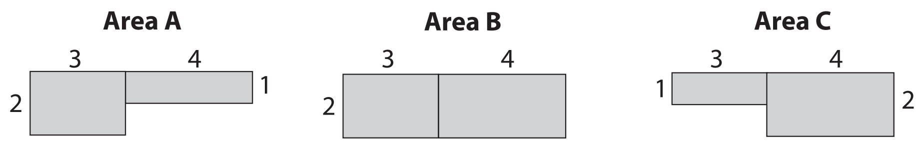
Time needed

Between half an hour and an hour.

Suggested approach

Beginning the session

Draw three compound shapes on the board.



Ask questions to probe learners' existing understanding.

> If you work out $3 + 4 \times 2$, which area are you working out? Explain how you know.
>
> If you work out $(3 + 4) \times 2$, which area are you working out? How do you know?
>
> What answers does your calculator give for these questions?
>
> Can you give me an expression for the other area?
>
> What is the difference between $(2 + 3)^2$ and $2^2 + 3^2$? Can you show me a diagram to explain the difference?

If learners struggle with any of these questions, explain that you will leave it for now and return to it later in the session.

Ask learners if they have heard of BIDMAS (or BODMAS) and ask them to explain what it means. Explain the danger of using such a rule without understanding it. For example, write the following and ask the group to tell you where you have gone wrong.

$$
\begin{aligned}
3 \times \frac{(3+5)^2}{4} - 5 + 9 &= 3 \times \frac{8^2}{4} - 5 + 9 && \text{(Brackets)}\\
&= 3 \times \frac{64}{4} - 5 + 9 && \text{(Indices – or 'powers')}\\
&= 3 \times 16 - 5 + 9 && \text{(Division)}\\
&= 48 - 5 + 9 && \text{(Multiplication)}\\
&= 48 - 14 && \text{(Addition)}\\
&= 34 && \text{(Subtraction)}
\end{aligned}
$$

Working in groups

Arrange learners in pairs or groups of three. Give each group Card sets A (*Calculations*), B (*Areas*) and C (*Solutions*).

Ask learners to place the cards face up on the table and take it in turns to match them. If they feel that cards belong together, they should place them side-by-side, so that they are all visible. (Cards should not be stacked as this makes it impossible for you to monitor their work as you go round the room.) Each time that a learner matches two or three cards, they should try to explain to their partner(s) why the cards belong together. Encourage learners to challenge their partner(s) if they think an explanation is not clear enough.

Learners will soon realise that there are more *Calculations* cards than *Areas* and *Solutions* cards – do not comment at this stage. Learners will soon find that some areas may be obtained by more than one calculation and that they need to provide additional answers for themselves. The blank cards are provided for this.

Encourage learners to explain how they can immediately see when a *Calculations* card matches an *Areas* card, without working out answers. Ask them to look for alternative ways of finding the areas.

Learners who struggle with this activity could cut the compound shapes into rectangles and find the area of each rectangle before finding the area of the compound shape.

Learners who match the cards quickly may be challenged to move towards generalisation.

What happens when we change the numbers?

Suppose we change the 4 in every card to a 5? Will the calculations cards still match in the same way?

Will this still be true when we change the 4 to a large number, a negative number or a decimal?

Do the area pictures help to explain why this happens?

Reviewing and extending learning

When learners have completed their matching, return to the questions asked at the beginning. What answers can the learners now give?

Using mini-whiteboards and whole group questioning, begin to generalise the learning:

Draw an area that requires this calculation: $3 \times (4 + 5)$.
Write a different calculation that gives the same area.

Draw an area that requires this calculation: $\frac{6 + 8}{2}$.

Write a different calculation that gives the same area.

Draw an area that requires this calculation: $(10 + 5)^2$.
Write a different calculation that gives the same area.

Draw out the general learning points that have emerged:

The equivalence of multiplying by $\frac{1}{2}$ and dividing by 2.

The order of operations:

- brackets first;
- then powers or roots;
- then multiplication or division;
- then addition or subtraction.

Equivalent expressions:

$2 \times (3 + 4) = 2 \times 3 + 2 \times 4$
(multiplication is distributive over addition);

$\frac{3 + 4}{2} = \frac{3}{2} + \frac{4}{2}$
(division is distributive over addition);

$(3 + 4)^2 = 3^2 + 4^2 + 2 \times 3 \times 4$.

What learners might do next

Session **A1 Interpreting algebraic expressions** may be used to generalise what has been learned in this session.

The brackets activity

The brackets activity may be used to consolidate what has been learned.

Arrange learners into teams of three or four. Give each team a felt tip pen and two copies of Sheet A, four copies of Sheet B and four copies of Sheet C. These can be A5 size.

Sheet A	Sheet B	Sheet C
$2 + 3 \times 4 + 5$	$2 \times 3 + 4 \times 5$	$2 + 3 \times 4^2$

Call out a sheet name and a target number. Teams have to show how they can reach this target by adding brackets to the sheet.

For example, if you call out "Sheet A, 25", the first team to show you Sheet A with brackets in the following position will gain a point.

Sheet A
(2 + 3) x 4 + 5

Here are some answers, but learners may come up with others.

Sheet	Target	Method
A	29	$2 + 3 \times (4 + 5)$
A	45	$(2 + 3) \times (4 + 5)$
B	26	$(2 \times 3) + (4 \times 5)$ or no brackets
B	46	$2 \times (3 + 4 \times 5)$
B	50	$(2 \times 3 + 4) \times 5$
B	70	$2 \times (3 + 4) \times 5$
C	80	$(2 + 3) \times 4^2$
C	146	$2 + (3 \times 4)^2$
C	196	$(2 + 3 \times 4)^2$
C	400	$((2 + 3) \times 4)^2$

Further ideas

This activity uses multiple representations to deepen understanding of number operations. This type of activity may be used in any topic where a range of representations is used. Examples in this pack include:

A1 Interpreting algebraic expressions;

SS6 Representing 3D shapes;

S4 Understanding mean, median, mode and range.

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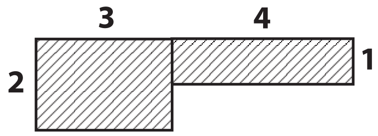
N5 Card set A – *Calculations*

A1 $3^2 + 4^2$	A2 $2 \times (3 + 4)$
A3 $(3 + 4)^2$	A4 3×4^2
A5 $(3 \times 4)^2$	A6 $\frac{3}{2} + \frac{4}{2}$
A7 $2 \times 3 + 4$	A8 $4 + 3 \times 2$
A9 $3^2 \times 4^2$	A10 $2 \times 3 + 2 \times 4$
A11 $\frac{1}{2}(3 + 4)$	A12 $3^2 + 4^2 + 2 \times 3 \times 4$
A13 $\frac{3 + 4}{2}$	A14 $\frac{3}{2} + 4$

N5 Card set B – *Areas*

B1



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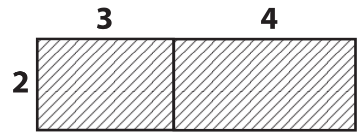

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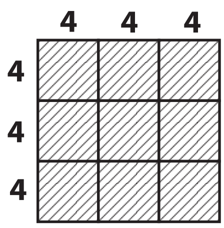

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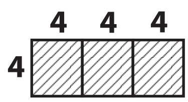

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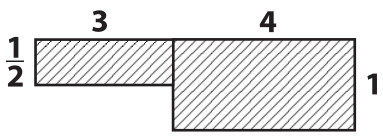

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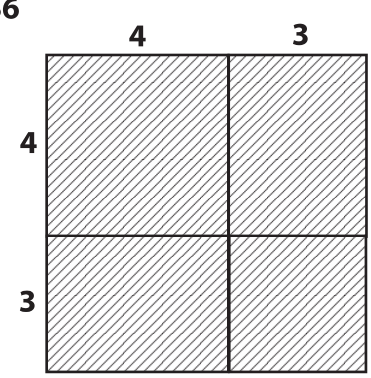

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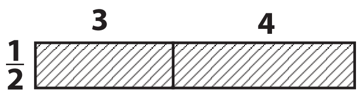

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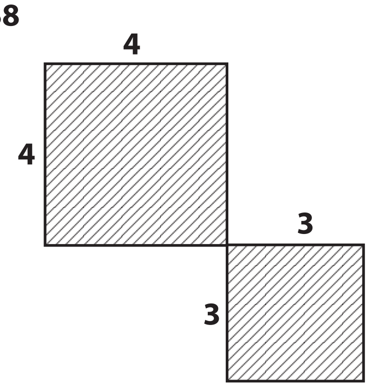

N5 Card set C – *Solutions*

144	48
49	3.5
5.5	14