

S3 • Using probability computer games

Mathematical goals

To enable learners to:

- confront and overcome common misconceptions about probability;
- count equally likely outcomes, using diagrams;
- discuss relationships between theoretical probabilities, observed outcomes and sample sizes;
- calculate probabilities of dependent and independent events.

These goals may be adapted for learners aiming at lower levels. For example, you may decide to focus only on the first three goals.

Starting points

Learners will not need any prior knowledge about probability in order to use and analyse the games. This knowledge will be developed through the activity and the discussion that follows it.

It would however be helpful to use session **S2 Evaluating probability statements** before this one. This session will allow you to revisit some of the misconceptions that are described in that session.

Materials required

For each small group of learners you will need:

- Access to a computer loaded with the programs *Coin races* and *Dice races*;
- Sheet 1 – *Coin races*;
- Sheet 2 – *Recording sheet for coin races*;
- Sheet 3 – *Dice races*;
- Sheet 4 – *Recording sheet for dice races*.

Learners aiming at lower levels may find it more helpful to use just the first one or two games in each set.

Time needed

This is flexible, depending on the number of games used. As a rough guide, allow 1 hour for *Coin races* and 1 hour for *Dice races*. It is not essential that learners try to complete all the games. It is better to tackle two or three in depth than to cover them all superficially.

Suggested approach **Beginning the session**

If you have a data projector or interactive whiteboard, it is very helpful to work through one race with the whole group. Before doing so, however, ask learners to make predictions about who will win and ask them to explain their reasoning.

As you work through a race, show learners how to fill in the recording sheet for that race. In particular, show them how to record the positions of the horses as the winner crosses the finishing line. Rather than simply writing the finishing order (1, 2, 3 etc.) it is more interesting to write the positions (the number of crosses in each row shows this) as the winner finishes the course.

Working in groups

Ask learners to work in pairs at each computer, making sure they have enough room to write down their results and their reasoning.

Ask learners to work on one of the two situations, using the computer programs provided: *Coin races* or *Dice races*. The procedure for each set of situations is similar (see Sheets 1–4).

Learners are asked to:

1. Predict the outcome of each race before starting it. You may need to re-emphasise this as learners often find it hard to stop and think when they are working at a computer.
2. Run the race on the computer and record the outcome in a table. They should record the position of each horse when the winner reaches the finish. This should be repeated at least three times for each situation.
3. Reflect on what happened and try to explain this. For example:
 - Does the outcome vary very much from race to race?
 - Why does a particular horse seem to win more often than the others?
 - Why does a particular horse come last more often than the others?
 - Could the winning distance have been predicted?

Learners aiming at lower levels may find it helpful to work through some games using real dice and coins before they try to use the simulations.

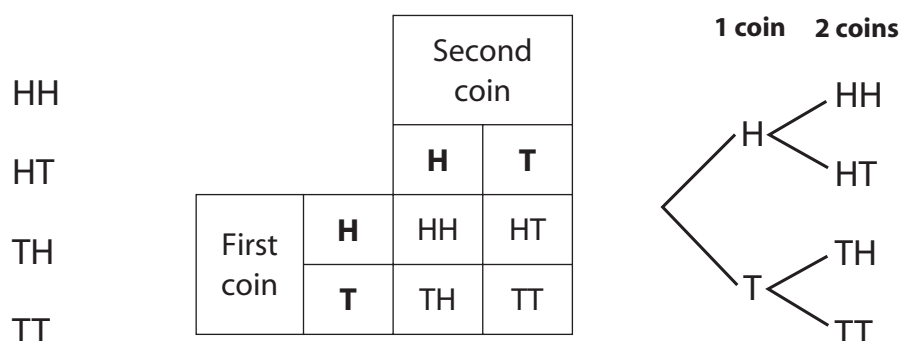
Whole group discussion

After a set of races, collect together the results from the whole group and hold a discussion about the results. In the notes that follow, we have outlined one direction in which the discussion might go.

(i) Coin races

For the two-coin race, learners will have found that horse 1 wins most often, though the results vary for each race. If the group results are aggregated and average positions are found as the winner finishes, learners will find that, when horse 1 wins, the others have reached approximately the half-way stage. The race seems unfair. Ask learners why this should be so.

At this point, someone may mention the equally likely outcomes HH, HT, TH and TT and that this shows why horse 1 is twice as likely to move as the other horses. Explain that these outcomes may be shown using a list, a two-way sample space table or a tree diagram:



Learners aiming at lower levels may find discussion of these representations helpful even if they do not go on to calculate probabilities.

Explain how this relates to the probabilities of each horse moving. Note that this is not the same as the probability of each horse winning. The longer the course, the more chance horse 1 has of winning. This relates to the 'sample size is irrelevant' misconception that is discussed in session **S2 Evaluating probability statements**.

Ask learners to draw tree diagrams for the three-horse situation in a similar way. They should then count the number of ways of getting 0, 1, 2 . . . heads for each number of coins and thus explain the likelihood of each horse winning.

They may like to list their observations and observe the patterns that emerge:

		Number of ways of getting						Total outcomes
		0 Heads	1 Heads	2 Heads	3 Heads	4 Heads	5 Heads	
Number of coins	1	1	1					2
	2	1	2	1				4
	3	1	3	3	1			8
	4	1	4	6	4	1		16
	5	1	5	10	10	5	1	32

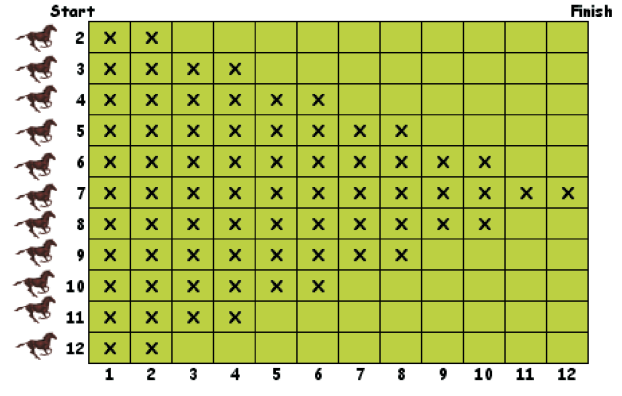
The probability of each horse winning can be calculated directly from this table. Thus with five coins, the probability that horse number 2 ('2 heads') wins is $\frac{10}{32} = \frac{5}{16}$. Learners may be able to predict further rows of this table from those shown here.

Relate this to the outcomes observed. Was it true that, for three coins, horses 2 and 3 won most often and were approximately three times as far down the course as the other two horses? What if the results of the whole group were aggregated?

(ii) Dice races

In the Sums race, learners should notice that the horses in the centre of the field tend to win more often than those at the outside. Can they explain this? Refer again to the sample space diagrams that display equally likely outcomes:

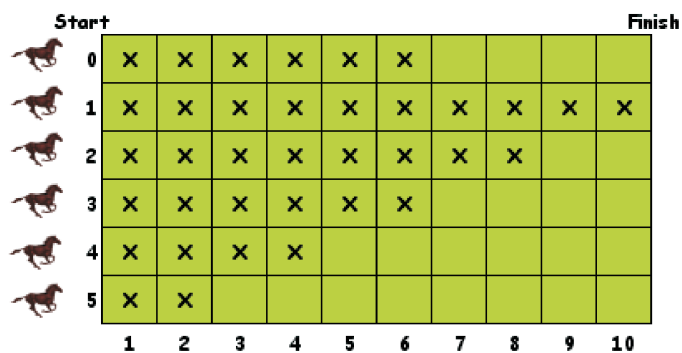
Sums race		First die					
		1	2	3	4	5	6
Second die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12



The diagram on the left shows, for example, that we would expect horse 7 to move six times more often than horse 2. Show learners how to calculate, from this diagram, the probabilities that particular horses will move. Can learners describe how this table gives a prediction of the positions of horses when horse 7 finishes? (This is shown in the diagram on the right.)

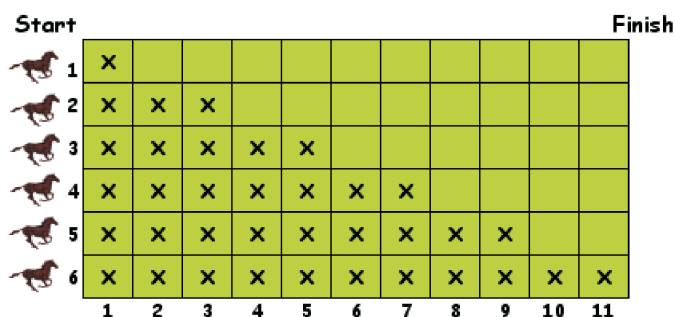
Now learners can be asked to analyse the remaining two situations in a similar manner, using sample space diagrams. They should find that the results from the Differences race give an asymmetric distribution, with horse 1 the most likely to win.

Differences race		First die					
		1	2	3	4	5	6
Second die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0



They should find that the results from the Max dice race also give an asymmetric distribution, with horse 6 the most likely to win.

Max race		First die					
		1	2	3	4	5	6
Second die	1	1	2	3	4	5	6
	2	2	2	3	4	5	6
	3	3	3	3	4	5	6
	4	4	4	4	4	5	6
	5	5	5	5	5	5	6
	6	6	6	6	6	6	6



The Multiples dice race is slightly more complicated as a particular throw of the dice may result in two or more horses moving at one time. Thus a throw of $3 \times 4 = 12$ will result in horses 2, 3, 4 and 6 all moving. Ask learners to tell you which of horses 6 and 2 will be the faster and why.

The easiest place to begin is by simply writing out a list of possible products. Learners can then count the multiples systematically.

Multiples race		First die					
		1	2	3	4	5	6
Second die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

There are:

$$27 \text{ multiples of } 2 \text{ and so prob (mult of } 2) = \frac{27}{36}$$

$$20 \text{ multiples of } 3 \text{ and so prob (mult of } 3) = \frac{20}{36}$$

$$15 \text{ multiples of } 4 \text{ and so prob (mult of } 4) = \frac{15}{36}$$

$$11 \text{ multiples of } 5 \text{ and so prob (mult of } 5) = \frac{11}{36}$$

$$15 \text{ multiples of } 6 \text{ and so prob (mult of } 6) = \frac{15}{36}$$

Note that the probabilities do not add up to 1.

This enables us to predict the outcome: horse 2 should win, followed by horse 3, then 4 and 6 (together), with 5 bringing up the rear.

In each of the above situations, learners may question why their own results do not correspond to the theory. This is a good time to consider the issue of sample size. Small samples may not correspond to these results but, aggregated over the whole group, the results should correspond more closely.

Reviewing and extending learning

Ask learners to analyse a game that they haven't yet used. For example, you could ask them to analyse a game where they have one coin and one die. If they throw a head, they double the number thrown on the die, otherwise the number stands. Can learners work out the possible outcomes and their probabilities?

What learners might do next

Learners may enjoy developing their own GCSE probability question. A suitable session, **S7 Developing an exam question: probability**, is provided in this pack.

Further ideas

This activity uses multiple representations to deepen understanding of probability. Learners may find it helpful to devise sample space and tree diagrams for their own dice, coin and spinner situations.

S3 Sheet 1 – Coin races

You will be using the computer for these tasks.

Start with the two-coin race and work through the other races in order.

You should write your results in the tables on your recording sheet.

- Before you start each race, write down the positions you predict horses will be in as the winner crosses the finishing line.

Why do you think this?

Write down your reason next to the table in the space provided.

Coin Races

Keep pressing the **Toss coins** button,


The computer works out how many **heads** there are.

It puts a cross in the corresponding row of the grid.

When a row of crosses passes the finishing line, that horse wins.

Toss Coins [T]

Start again [A]






2 coins [2]

3 coins [3]

4 Coins [4]

5 Coins [5]

	Start															Finish									
0																									
1																									
2																									
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16								

- Run each race several times.
Each time, record the positions of the horses as the winner crosses the finishing line.
(Count how many crosses there are in each row).

- Describe and explain any patterns you find in your data.

Repeat steps 1–3 for the three-, four- and five-coin races.

S3 Sheet 2 – Recording sheet for coin races

2 coin tosses	Horse number (number of heads thrown)		
	0	1	2
<i>Your prediction</i>			
First race actual finishing positions			
Second race actual finishing positions			
Third race actual finishing positions			

Space for your reasoning

3 coin tosses	Horse number			
	0	1	2	3
<i>Your prediction</i>				
First race actual finishing positions				
Second race actual finishing positions				
Third race actual finishing positions				

4 coin tosses	Horse number				
	0	1	2	3	4
<i>Your prediction</i>					
First race actual finishing positions					
Second race actual finishing positions					
Third race actual finishing positions					

5 coin tosses	Horse number					
	0	1	2	3	4	5
<i>Your prediction</i>						
First race actual finishing positions						
Second race actual finishing positions						
Third race actual finishing positions						

In order to draw sensible conclusions, you will need to put these results together with those from the rest of the group.

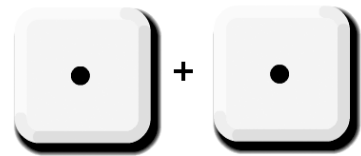
S3 Sheet 3 – Dice races

You will be using the computer for these tasks.
 Start with the Sums race and then work through the other races in order.
 You should write your results in the tables on your recording sheet.

- Before you start each race, write down the positions you predict the horses will be in as the winner crosses the finishing line.
 Use the row of the table that is labelled 'Your prediction'.
 Why do you think this?
 Write down your reason next to the table in the space provided.

Sums dice race

Keep pressing the **throw dice** button,
 The computer works out the **sum** of the numbers on the dice.
 It puts a cross in the corresponding row of the grid.
 When a row of crosses passes the finishing line, that number wins.



Throw dice [T]

Start again [A]

Choose the type of race:

Sum [S]

Difference [D]

Max [X]

Multiples [M]

	Start											Finish				
2																
3																
4																
5																
6																
7																
8																
9																
10																
11																
12																
		1	2	3	4	5	6	7	8	9	10	11	12			

- Run each race several times.
 Each time, record the positions of the horses as the winner crosses the line. (Count how many crosses there are in each row.)
- Try to explain any patterns you find in your data.

Repeat steps 1, 2 and 3 for the Difference, Max and Multiples races.

S3 Sheet 4 – Recording sheet for dice races

Addition race	Horse number											
	2	3	4	5	6	7	8	9	10	11	12	
<i>Your prediction</i>												
First race actual finishing positions												
Second race actual finishing positions												
Third race actual finishing positions												

Difference race	Horse number					
	0	1	2	3	4	5
<i>Your prediction</i>						
First race actual finishing positions						
Second race actual finishing positions						
Third race actual finishing positions						

Max race	Horse number					
	1	2	3	4	5	6
<i>Your prediction</i>						
First race actual finishing positions						
Second race actual finishing positions						
Third race actual finishing positions						

Multiples race	Horse number				
	2	3	4	5	6
<i>Your prediction</i>					
First race actual finishing positions					
Second race actual finishing positions					
Third race actual finishing positions					

Space for your
reasoning

In order to draw sensible conclusions you will need to put these results together with those from the rest of the group.