SS8 • Developing an exam question: transformations

Mathematical goals

To help learners to:

- use past examination papers creatively;
- recognise and visualise transformations of 2D shapes;
- transform triangles and other 2D shapes by translation, rotation and reflection and combinations of these.

To develop learners' ability to:

- generalise and explore their own questions in this context.

Starting points

Most learners will have met the concepts of 'rotation', 'reflection' and 'translation' before, but may not have a deep understanding of them.

In this session, learners are provided with copies of a question similar to those found in GCSE examinations. They answer the question, then analyse the content and skills required to obtain the correct solution. They develop the task by asking further questions and by changing the task in various ways. They develop their own examination questions and attempt to answer the questions designed by other learners.

Materials required

- OHT 1 – *Transformations of a triangle*.

For each learner you will need:

- Sheet 2 – *Template for transformations*.

For each small group of learners you will need:

- Sheet 1 – *Transformations of a triangle*;
- a small supply of tracing paper or cut up blank acetate;
- pens that will write on acetate (if appropriate).

Time needed

Normally, this session will take about an hour.

Suggested approach

Beginning the session

Ask learners to work in pairs at the questions in Sheet 1 – *Transformations of a triangle*. Explain that these are similar to those found in GCSE examination papers. When everyone has had time to have a go, ask them to gather round for a whole group discussion about the questions.

Whole group discussion

(i) Answering the question

Begin by asking learners to define the terms used in the questions. As they do this, ask other learners to listen and suggest improvements. In particular, work on the meaning of the terms 'transformation', 'translation', 'reflection' and 'rotation'.

> A 'transformation' is a general term that describes a change in the position or shape of an object – in this case the shaded triangle. In this question we are considering three particular types of transformation.
>
> A 'translation' is the movement of a shape from one place to another in a straight line. Every point in the shape moves in the same direction and through the same distance.
>
> A 'reflection' is the movement of a shape so that it becomes a mirror image of itself. The reflection of each point in the original shape is perpendicular to the mirror line. If you fold your diagram along the mirror line, the original shape sits exactly on top of its reflection.
>
> A 'rotation' turns a shape about a fixed point called the 'centre of rotation'.

Collect suggestions for correct answers to each part of the question and write these on OHT 1 – *Transformations of a triangle*. If there are several different answers, ask learners to explain which they think is correct and why.

Although the question does not ask for explanations, it is important to get learners to describe their method for finding answers.

> How can you be sure that the reflection of the shaded triangle in the line $x = 3$ is triangle A?
> (If you fold the diagram along the line $x = 3$, the shaded triangle would sit exactly on top of triangle A.)
>
> How do you know that if the shaded triangle is rotated 90° clockwise about 0 it will sit exactly on top of triangle D?
> (I drew a line from the point (2, 4) to the right-angled corner of the shaded triangle. Then I rotated that line through 90° clockwise and reached the point (4, –2). This was the

right-angled corner of triangle D. I could repeat this for other points on the shaded triangle.)

Explanations may be helped by copying the shaded triangle onto a second, blank acetate and moving it over the first. In this way, translations, reflections and rotations can be shown dynamically.

Whole group discussion

(ii) Generating further questions

Ask learners to suggest alternative questions that could have been asked in this situation. Encourage learners to devise a range of questions, some easy and some more difficult. Also encourage them to devise questions that would test all the transformations. Make a list of these on the board.

For example:

> Write down the letter of the triangle after the shaded triangle is:
>
> • reflected in the line $y = 3$;
>
> • rotated by 90° anticlockwise about 0;
>
> • translated −3 units horizontally and −1 unit vertically.
>
> Describe fully the single transformation that will take:
>
> • triangle I to triangle J;
>
> • triangle H to triangle J.
>
> What triangles can be obtained from the shaded triangle by a single reflection, translation or rotation?
>
> You can get from triangle E to H in two steps; a reflection in $y = -3$ (taking it to D) followed by a rotation about 0 through 180° (taking it to H). Find some different ways of getting from E to H using two transformations.

Working in groups

Ask learners to work in pairs to choose and answer questions from those listed on the board. Learners may like to compare their ideas using overhead transparencies or by writing on the board.

Whole group discussion

(iii) Developing the situation

Hand out Sheet 2 – *Template for transformations* to each learner. Ask learners to write a new transformation question by completing

the sheet using diagrams and words. They should try to produce questions that are challenging, but which they think they can get right. They should write the answers on the back of the sheet.

For example, learners may suggest that the shaded triangle should be in a different position; they may also suggest that they would like to ask questions about particular kinds of transformations. They will then need to ensure that correct triangles appear on their diagram, together with some triangles in incorrect positions.

While doing this task, learners have to construct their own triangles to represent the outcomes of different transformations. This therefore extends the work that has gone before.

Working in groups

The new questions should be passed round the group to be answered by other learners. In cases of difficulties in answering questions, the question-writers should explain what they intended and act as a teacher helping other learners to answer the questions.

Alternatively, some of the new questions can be photocopied for further sessions or for homework.

Reviewing and extending learning

Finally, hold a whole group discussion on what has been learned, drawing out any common misconceptions. You should include a discussion of the level of difficulty of the new questions.

What learners might do next

Ask learners to choose another question from an exam paper and follow the process adopted in this session.

(i)   Answer the question.

(ii)   Generate new questions about the same situation (and answer them).

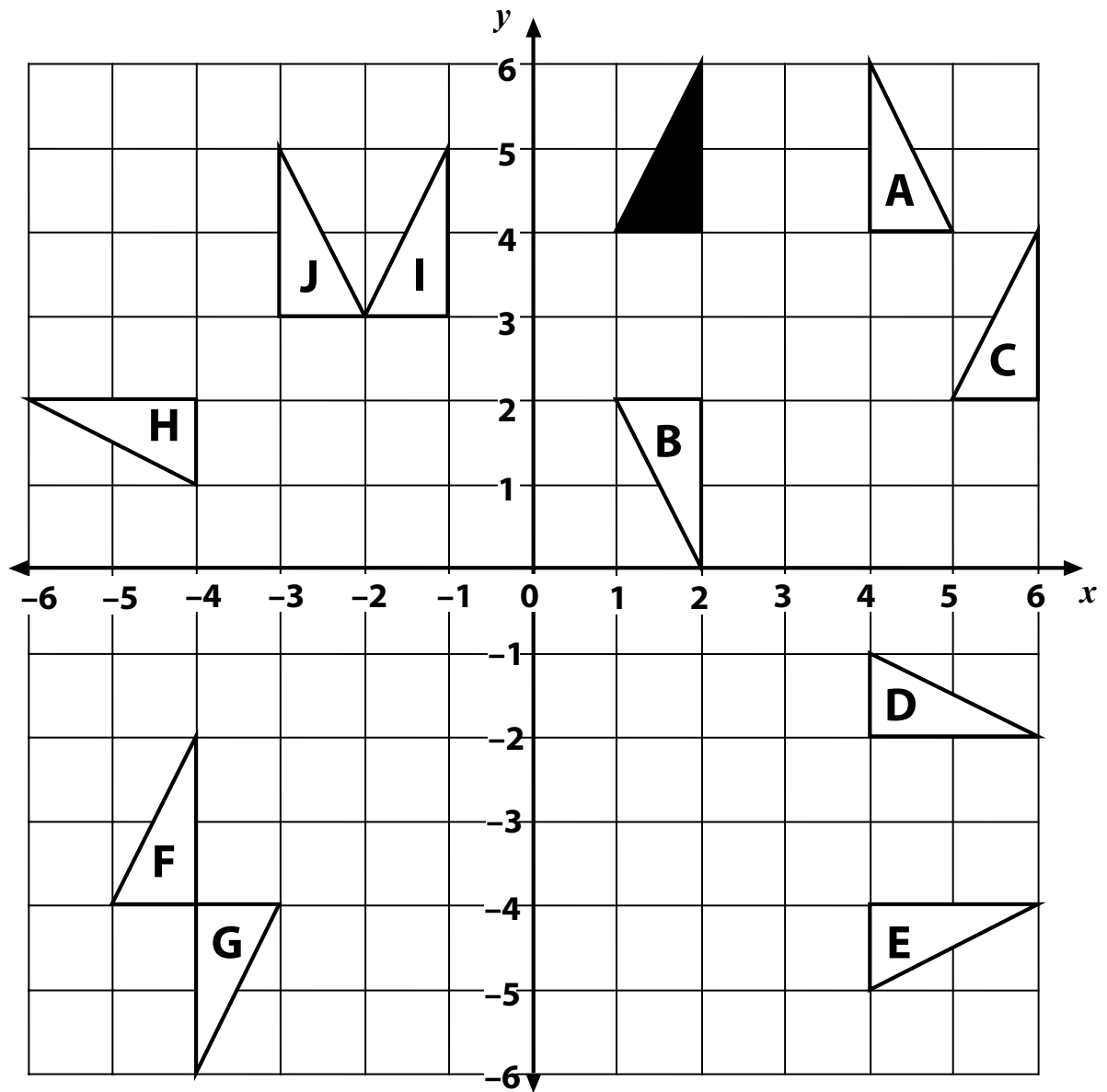(iii)   Change the situation and make up a new question.

Further ideas

This method for developing exam questions may be used in any topic. Examples in this pack include:
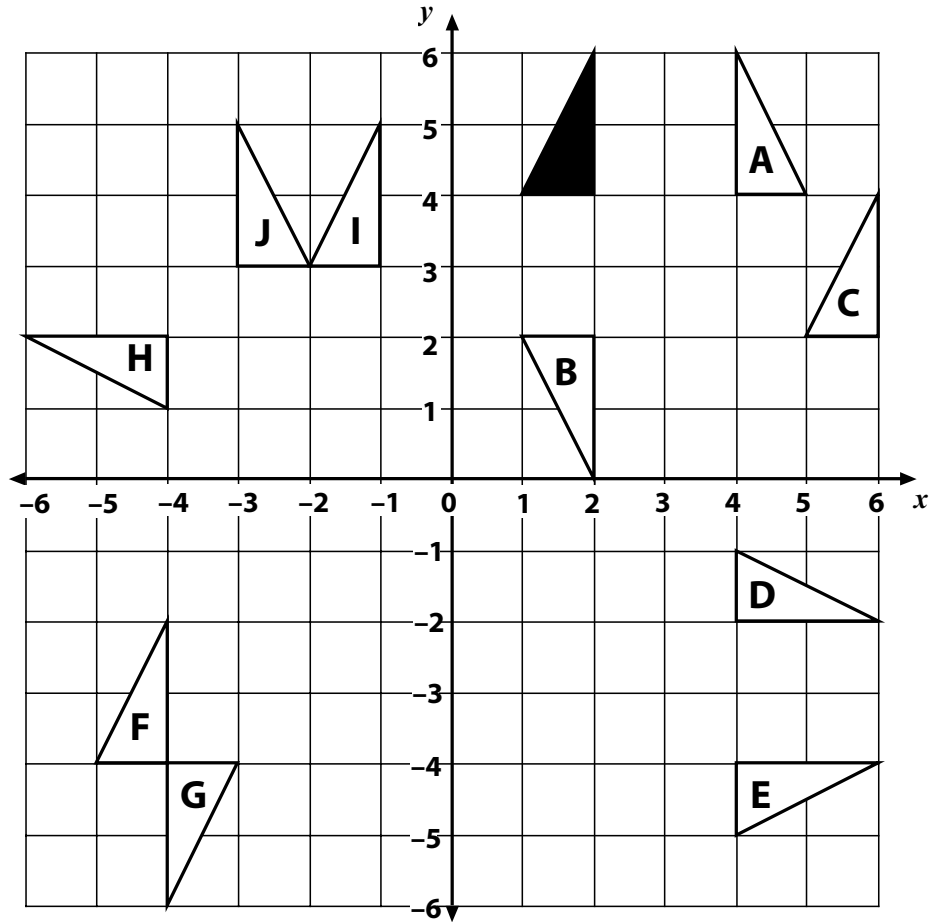
N10   **Developing an exam question: number;**

A8   **Developing an exam question: generalising patterns;**

S7   **Developing an exam question: probability.**

SS8 OHT 1 – *Transformations of a triangle*

SS8 Sheet 1 – *Transformations of a triangle*

1. Write down the letter of the triangle:

    (a) after the shaded triangle is reflected in the line $x = 3$;

    (b) after the shaded triangle is translated by 4 squares to the right and 2 squares down;

    (c) after the shaded triangle is rotated 90° clockwise about 0.

2. Describe fully the single transformation that takes triangle F onto triangle G.

What other questions could have been asked?

SS8 Sheet 2 – *Template for transformations*
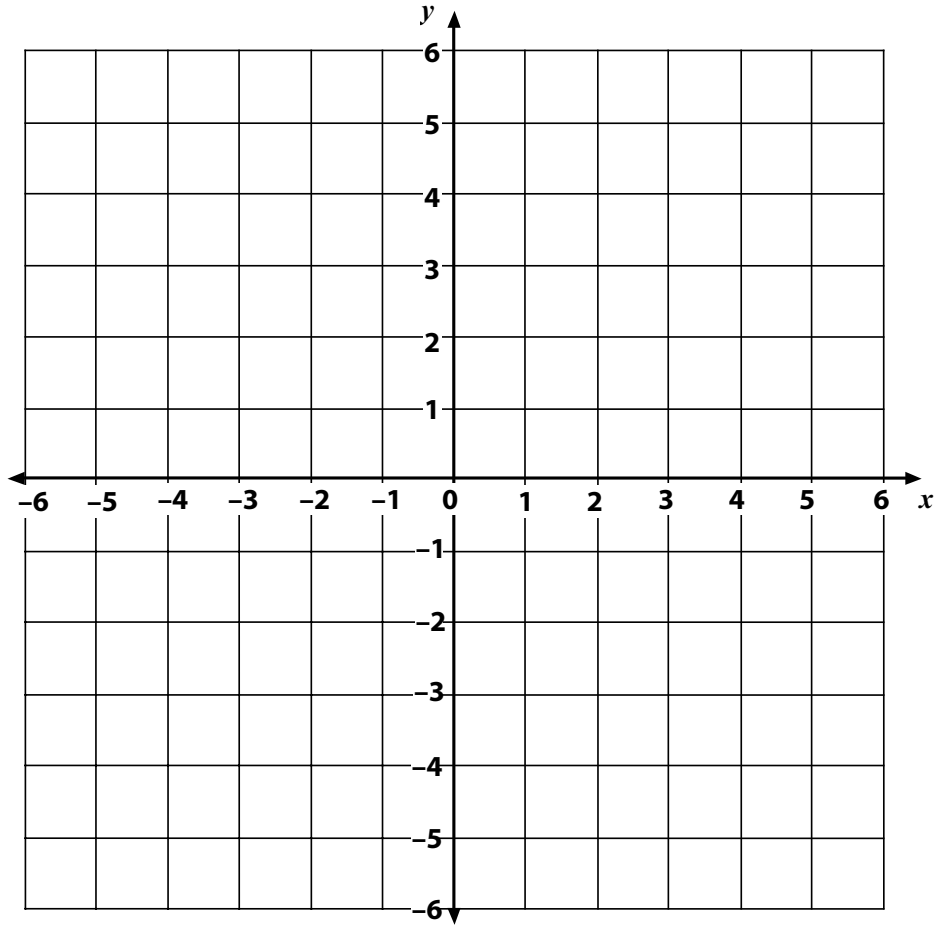
1. Write down the letter of the shaded triangle:

   (a) after it has been translated .... horizontally and .... vertically;

   (b) after it has been rotated through ....;

   (c) after it has been reflected in the line ....

2. Describe fully the single transformation that takes shape ...... onto shape ....

3. (Write your own harder question here)

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