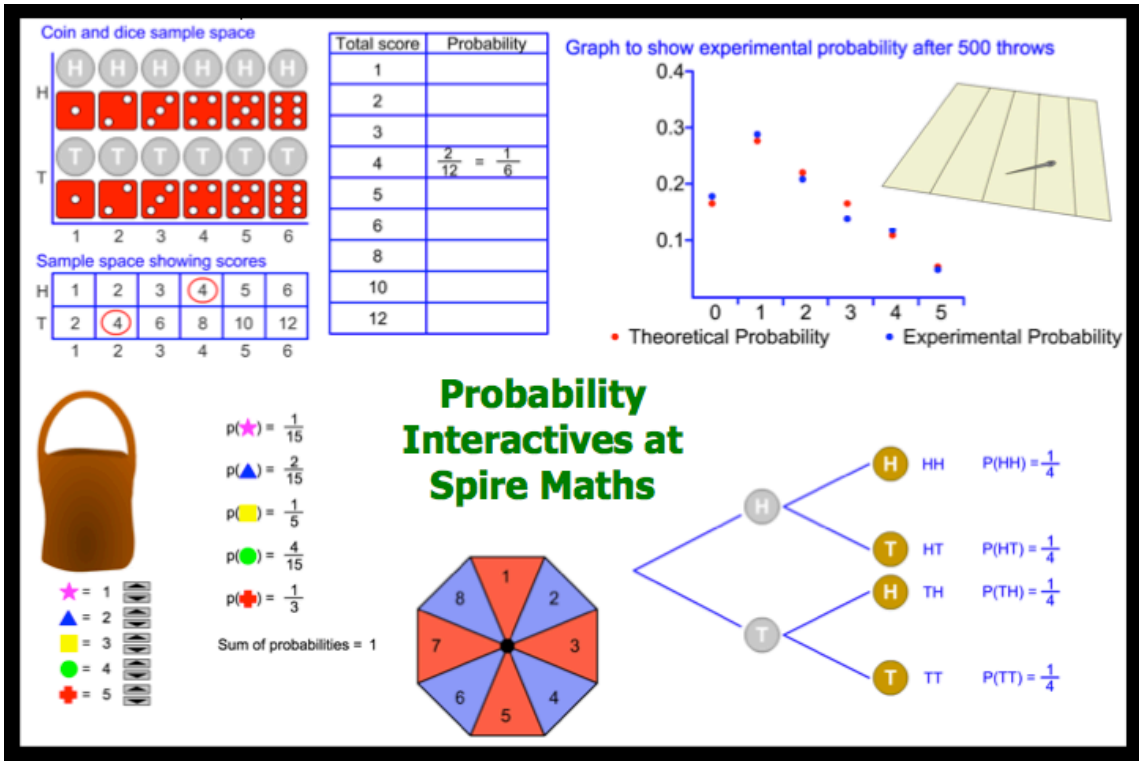
Probability Interactives from Spire Maths



A Spire Maths Activity

https://spiremaths.co.uk/ia/

There are 12 sets of Probability Interactives: each contains a main and plenary flash file. Titles are shown below together with screens for each of the different pages in the flash files. Additional titles indicate worksheet support pages. Teacher notes included for each of the screens.

Unfortunately flash files will not work on iPads or iPhones.

Table of Contents

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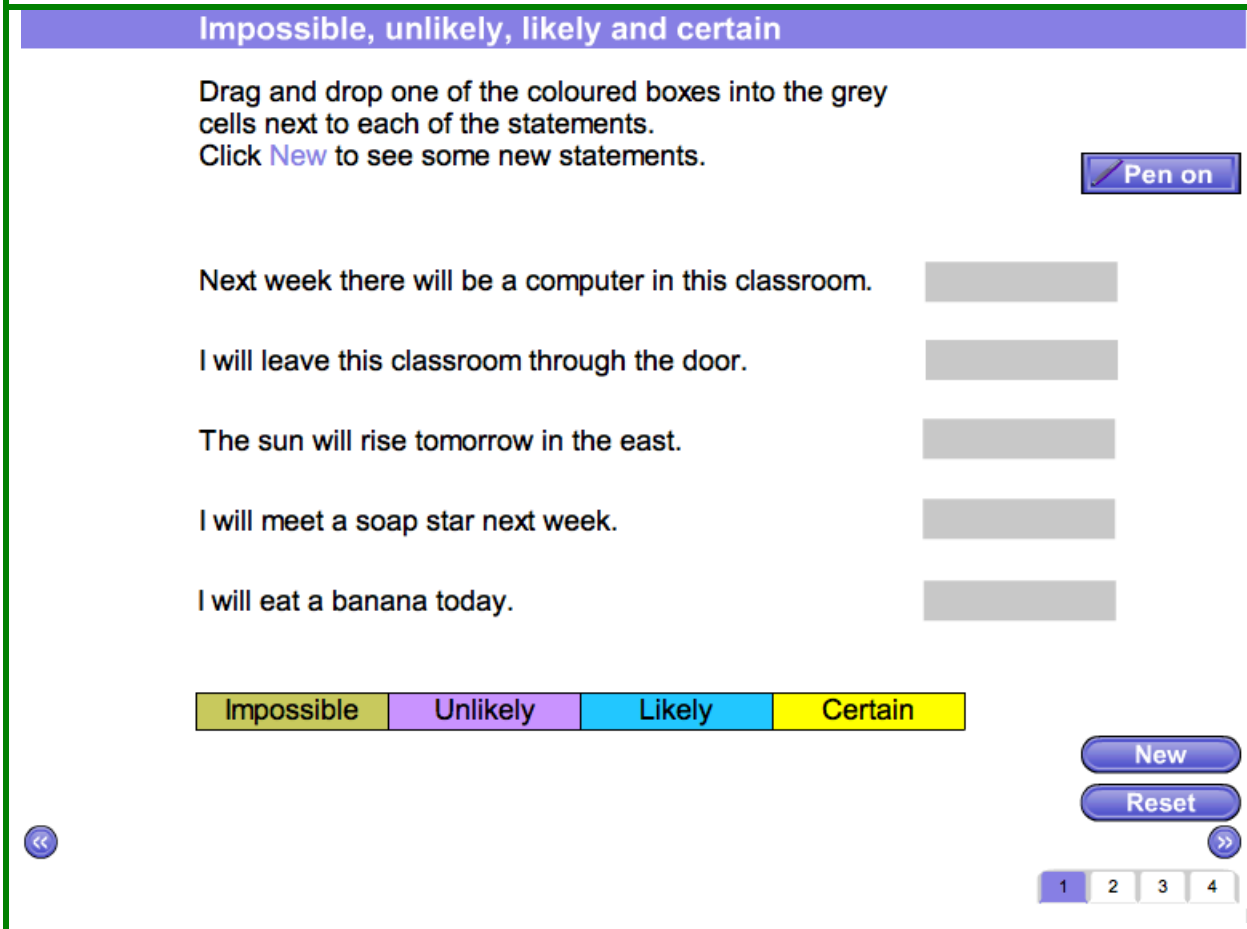
Beginning probability

TYPE:	Main
OBJECTIVE(S):	Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; use vocabulary and ideas of probability, drawing on experience.
DESCRIPTION:	4 screens. 1 shows 5 statements and asks you to match them with Impossible, Unlikely, Likely or Certain. 2 is as screen 1 but you can add your own statements. 3 lets you decide whether a game is fair or not. 4 is a variation on a higher or lower theme.
OVERVIEW:	Starting probability and looking at simple 'games' involving chance.
EQUIPMENT:	Teacher notes include a photocopiable master of the probability shapes found on screen 3 (there are 27 of each shape).

TYPE:	Plenary
OBJECTIVE(S):	Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; use vocabulary and ideas of probability, drawing on experience.
DESCRIPTION:	3 screens. 1 lets you make your own simple probability game. 2 is a version of higher or lower and can show probabilities. 3 is the vocabulary screen.
OVERVIEW:	Starting probability and looking at simple 'games' involving chance.
EQUIPMENT:	None specific.

Main Whiteboard and Screen information

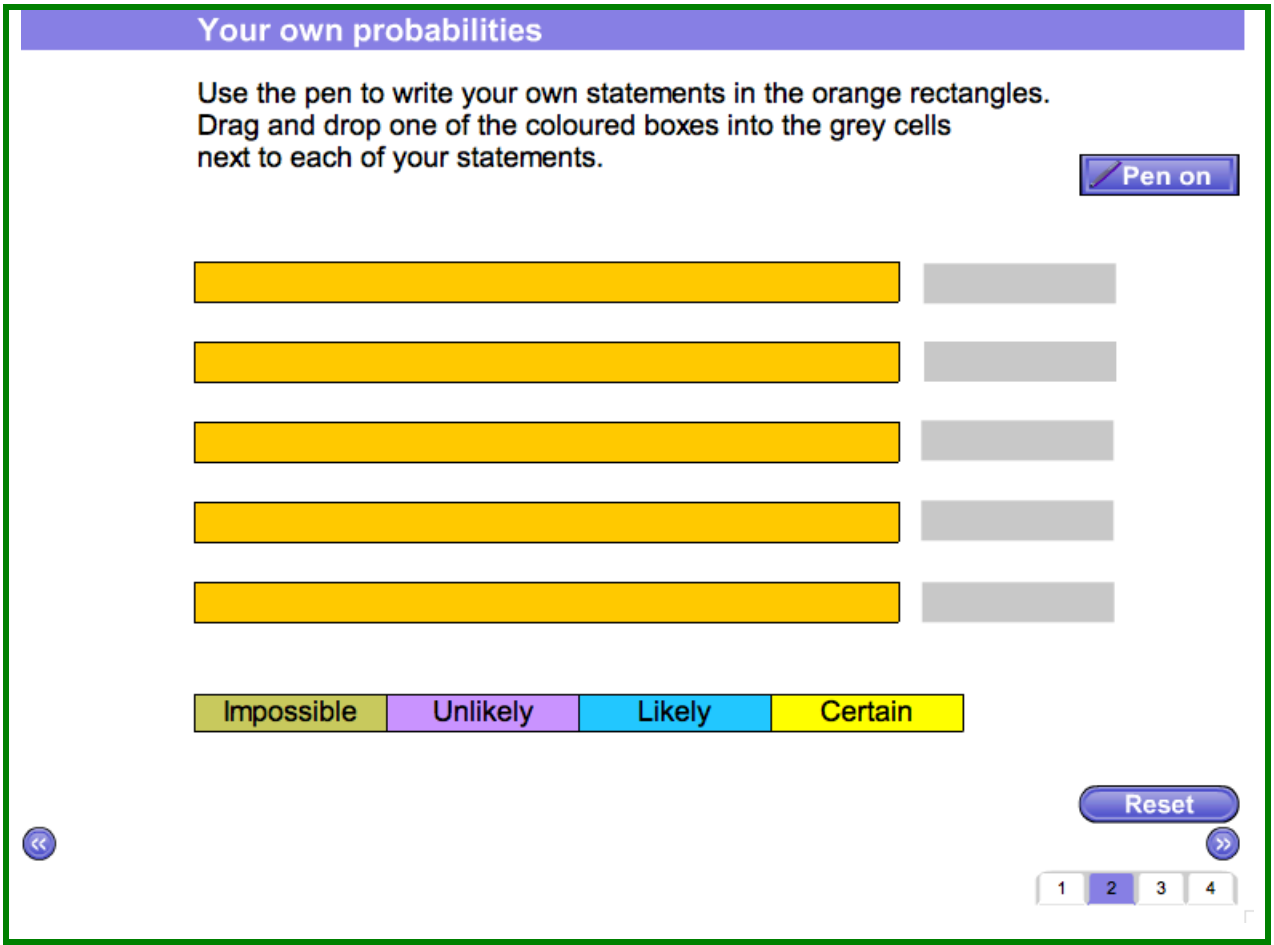


Screen 1: Certain, likely, unlikely and impossible

A discussion page with cross curricular opportunities.

You are given five statements and can move one four words (Impossible, Unlikely, Likely or Certain) to match the statement. In many cases there are no correct answers and this is quite deliberate.

Key points: many of the statements will elicit a different response from pupils in different places, at different times of the day, in different seasons; some statements will be Certain or Impossible for everyone (though pupils may consider exceptions); consider "who, if anyone, would answer this Certain/Impossible?"; cross curricular or citizenship opportunities can be addressed by consideration of other people's response (older people or anyone living in another country); you may also wish to extend to more precise notions of probability with "how likely, 50-50?".

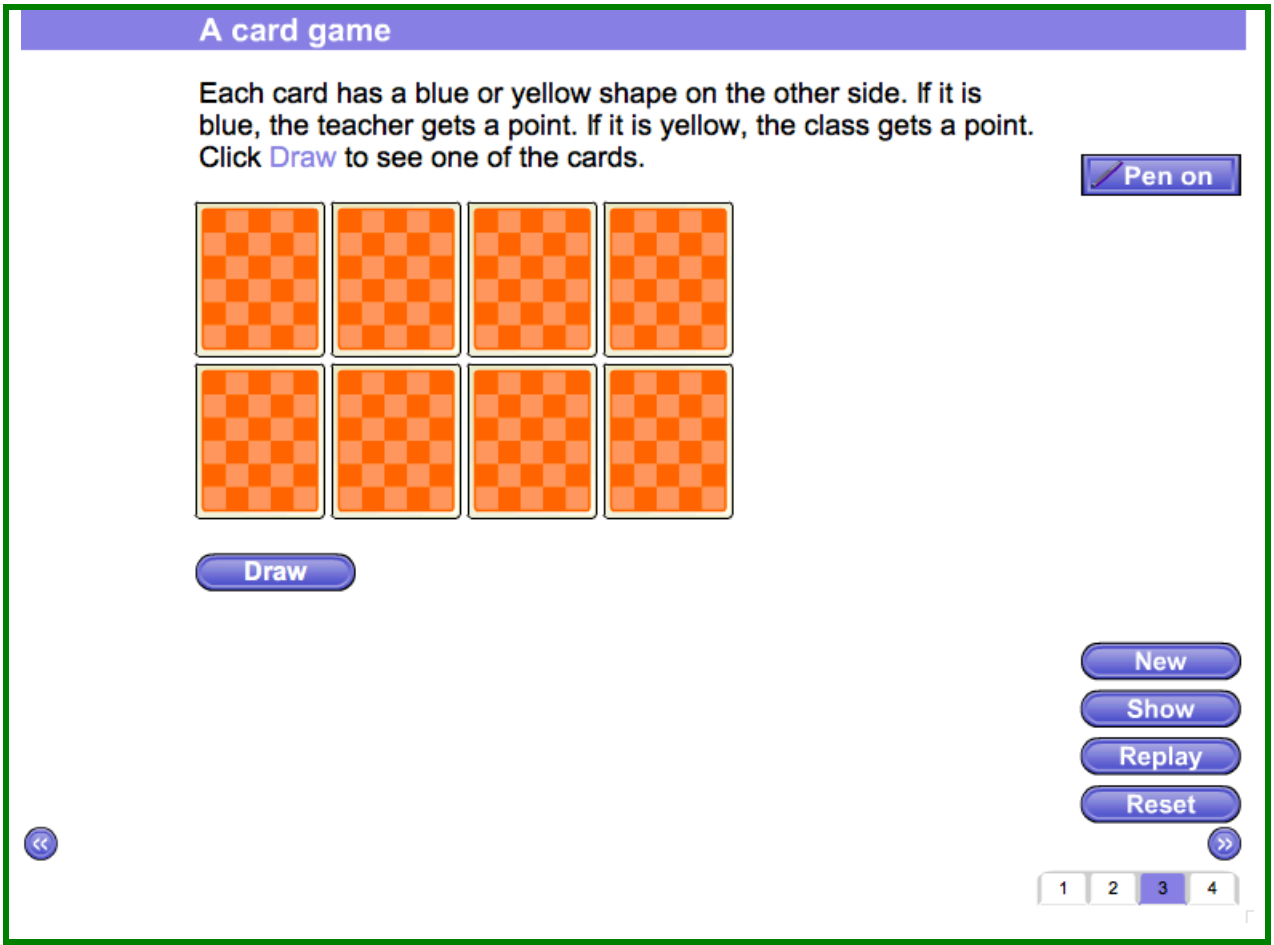


Screen 2: Your own probabilities

A discussion page with cross curricular opportunities.

You can use this page either way round: you collect statements and then have one of the four words (Impossible, Unlikely, Likely or Certain) match each statement or place one of the words in a grey cell and then ask for a statement to match.

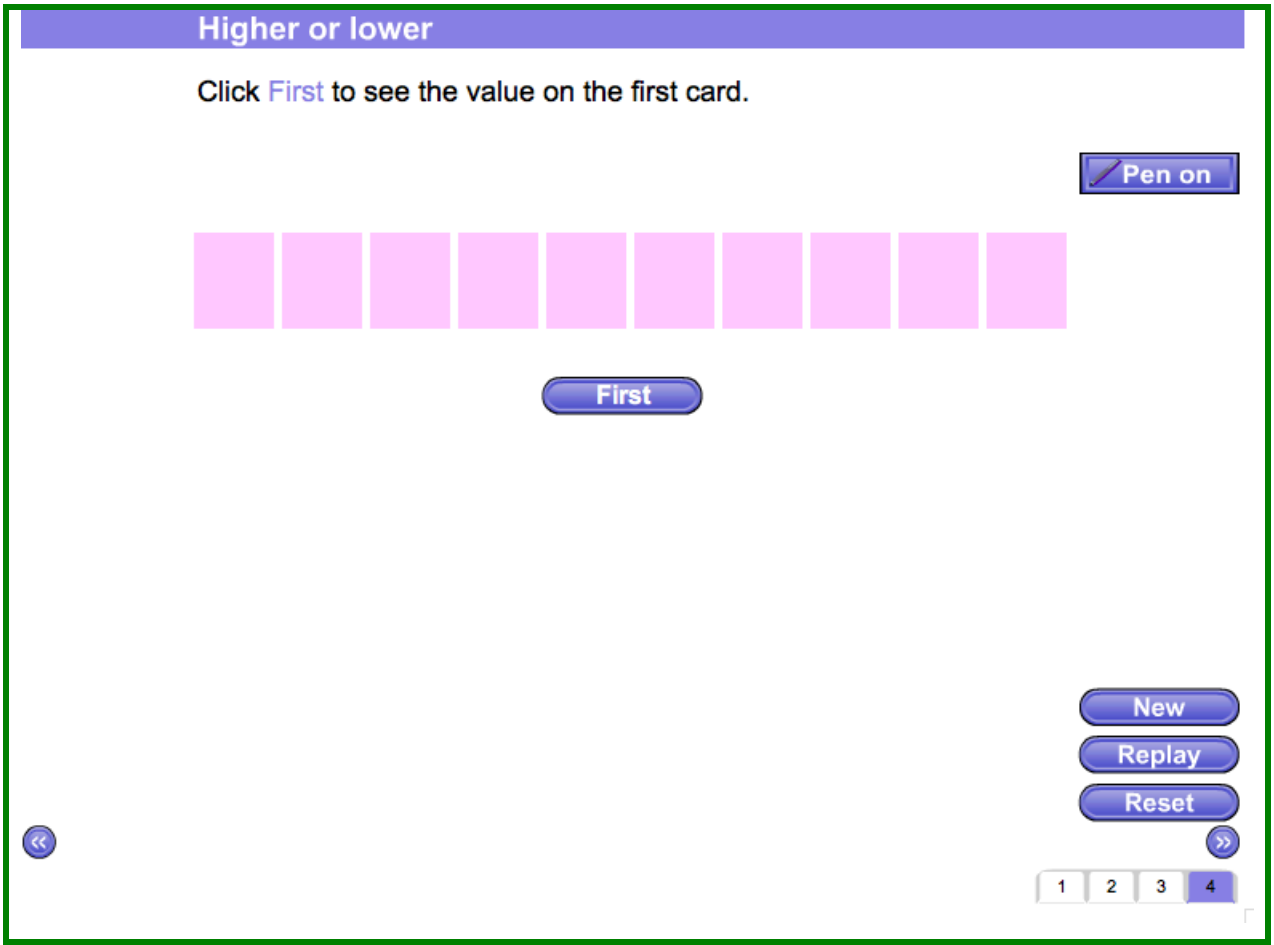
Key points: consider the universality of a statement or whether it is true for an individual, a class, only certain people etc.; consider the extension to more precise notions of probability with "how likely, 50-50?".



Screen 3: A card game

You have 8 cards face down. On the reverse of each one is a blue or yellow triangle. You have to imagine a game where points are scored according to the colour drawn (with replacement after each draw). You can draw a random card as many times as you want (shuffling in between) and have to decide whether the game is fair. At any time it is possible to inspect the cards and carry on with the same set. There are 9 different sets of cards that you can start with (ranging from all blue to all yellow) and they are all equally likely.

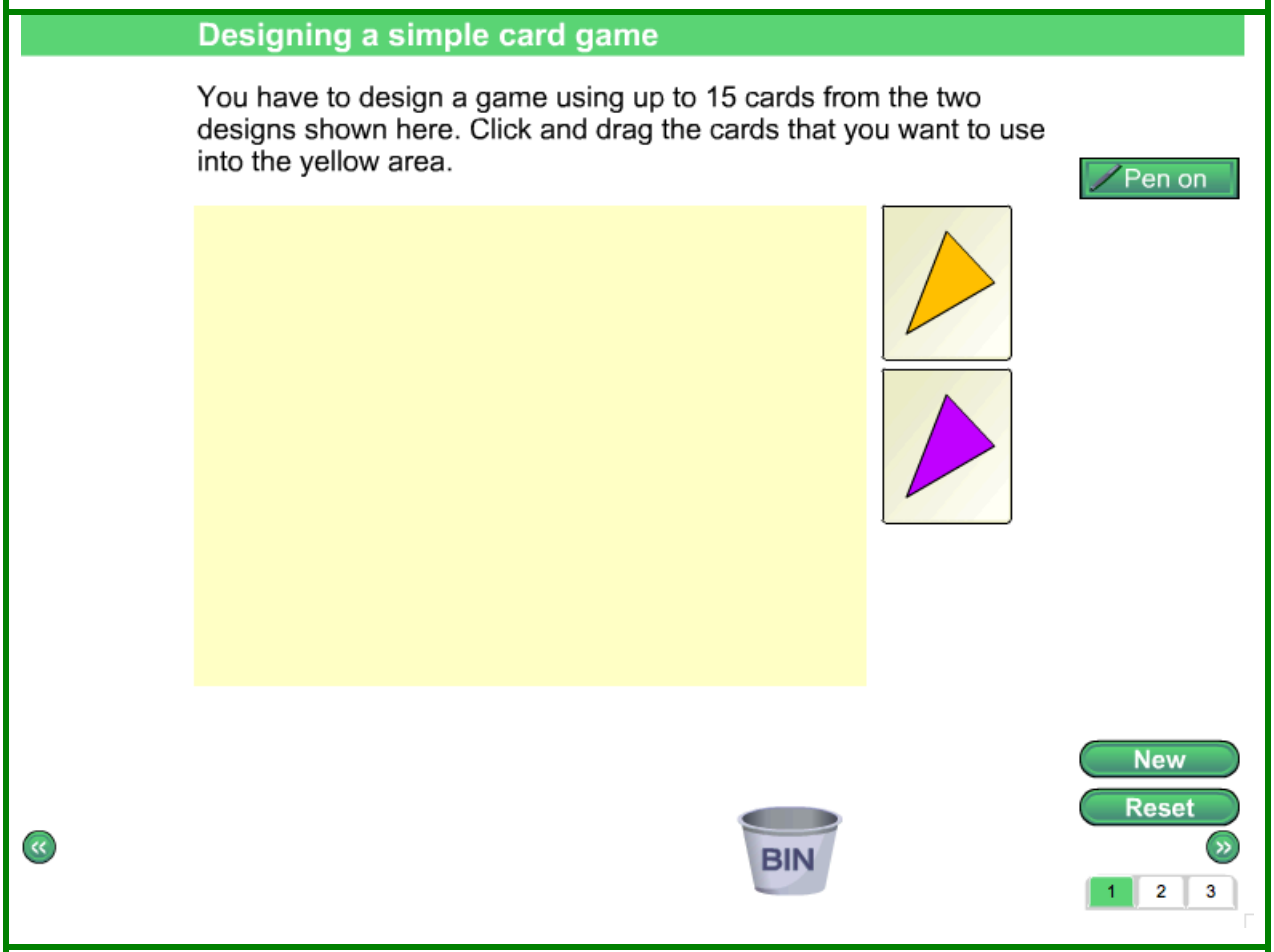
Key points: it helps if you only think of whether the game favours one side or the other (rather than try to guess the correct number of each colour); discuss when you might know that there is only one colour triangle present; consider how many times you need to draw a card before you know with confidence who will have the greater chance of winning.



Screen 4: Higher or lower

A higher or lower game variation. Randomly generated (by the computer).
Ten digit cards, numbered 1 to 10, are shown face down. You can turn them over one at a time. You are asked to decide whether the next is higher or lower. Once you have turned over all the cards, you can use the same set again (so you could set this up ahead of time).
Key points: first few times through it helps to let pupils use their instinct whether the next is higher or lower; gradually some pupils may start considering 'the odds' and be using a form of probability; extend such informal methods into a formal structure; introduce a definition of probability if appropriate.

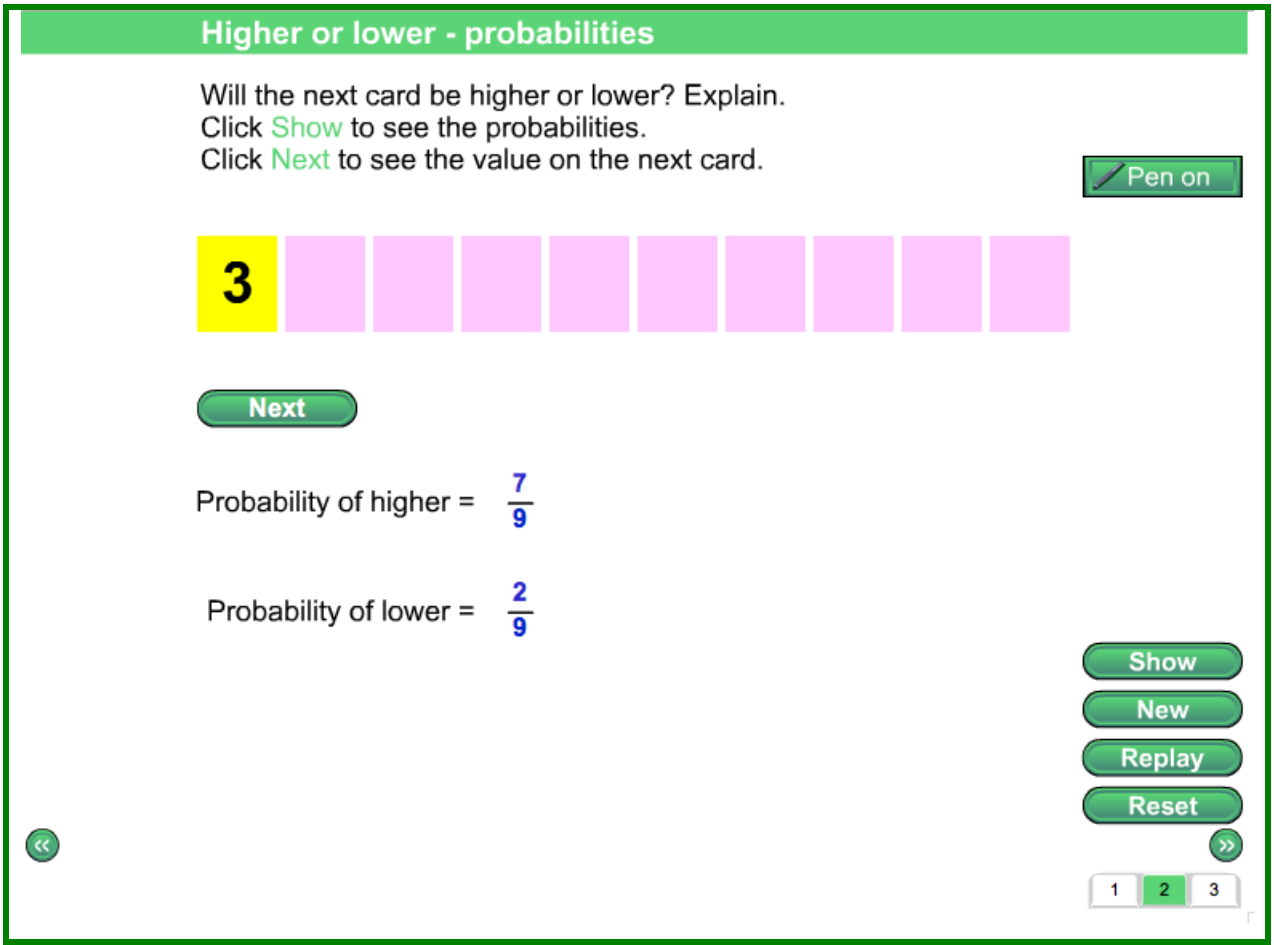
Plenary Whiteboard and Screen information



Screen 1: Designing a simple card game

You can make your own game based on up to 15 cards that are either orange or purple. When you have placed one card you are asked what it the probability of drawing an orange or purple card.

Key points: a discussion screen where you might want pupils to make a fair game (or not) or a game that is completely unfair; consideration of 'how unfair' a game might be and 'impossible' odds may be appropriate for a few pupils; let pupils offer ideas as it may help bring misconceptions to the surface.

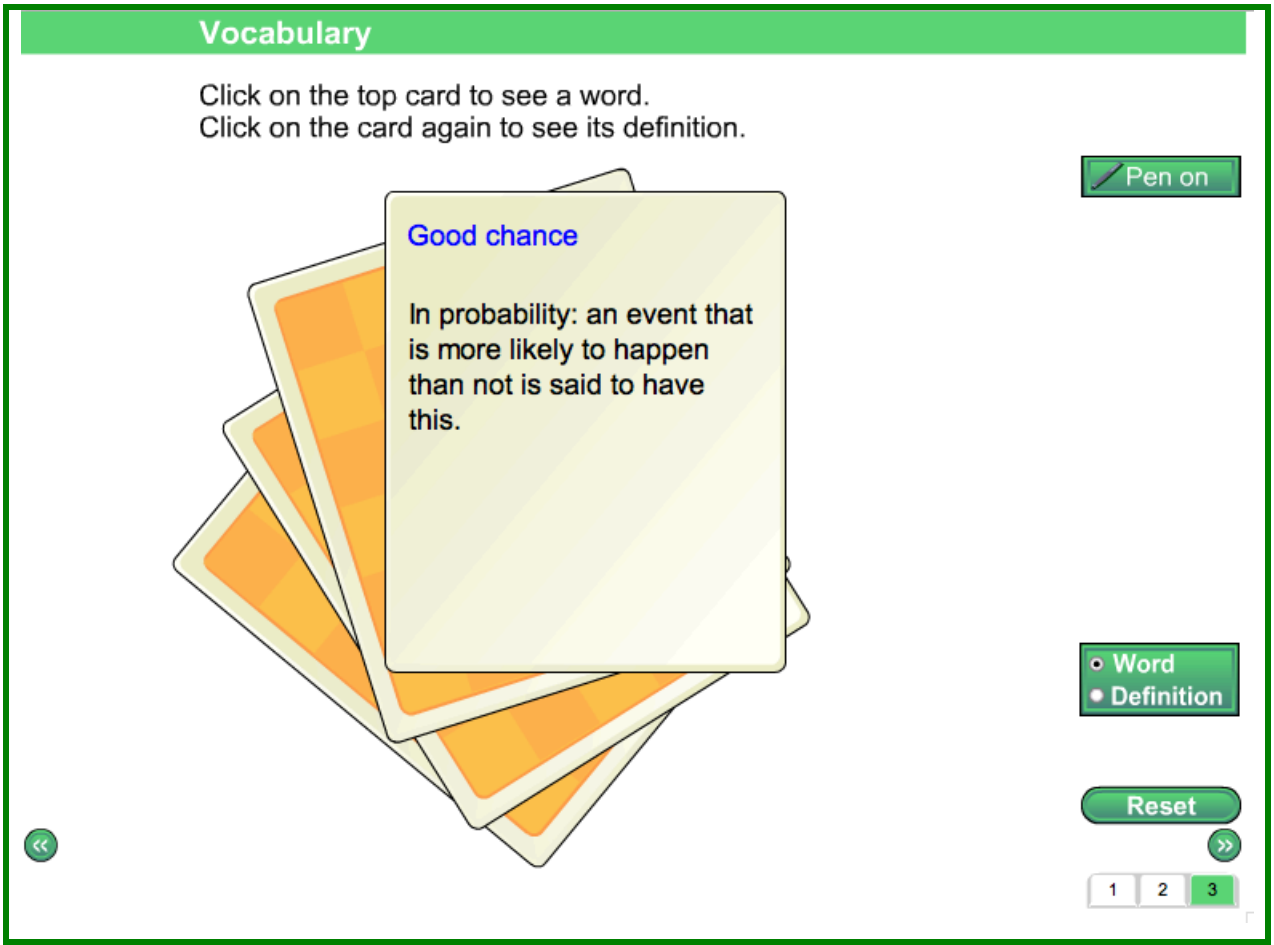


Screen 2: Higher or lower - probabilities

A higher or lower game variation. Randomly generated (by the computer).

Ten digit cards, numbered 1 to 10, are shown face down. You can turn them over one at a time. You are asked to decide whether the next is higher or lower. Once you have turned over all the cards, you can use the same set again (so you could set this up ahead of time). The 'Show' feature will show the probabilities for the next turn, these have not been reduced to their lowest terms (to help reinforce the probability idea).

Key points: have pupils discuss the probabilities before you use 'Show'; meaning of probability and what happens when the card is 'wrong'; maximising your chances of success and 'instinct'; 'good luck' and probability; introduce a definition of probability if appropriate.



Screen 3: Vocabulary

Vocabulary present: Average, Certain, Chance, Doubt, Equally likely, Even chance, Fair, Fifty-fifty chance, Good chance, Impossible, Interval, Likelihood, Likely, Mean, Median, Modal class/group, Mode, No chance, Outcome, Poor, Possible, Probability, Probable, Random, Range, Risk, Statistic, Uncertain, Unfair, Unlikely.

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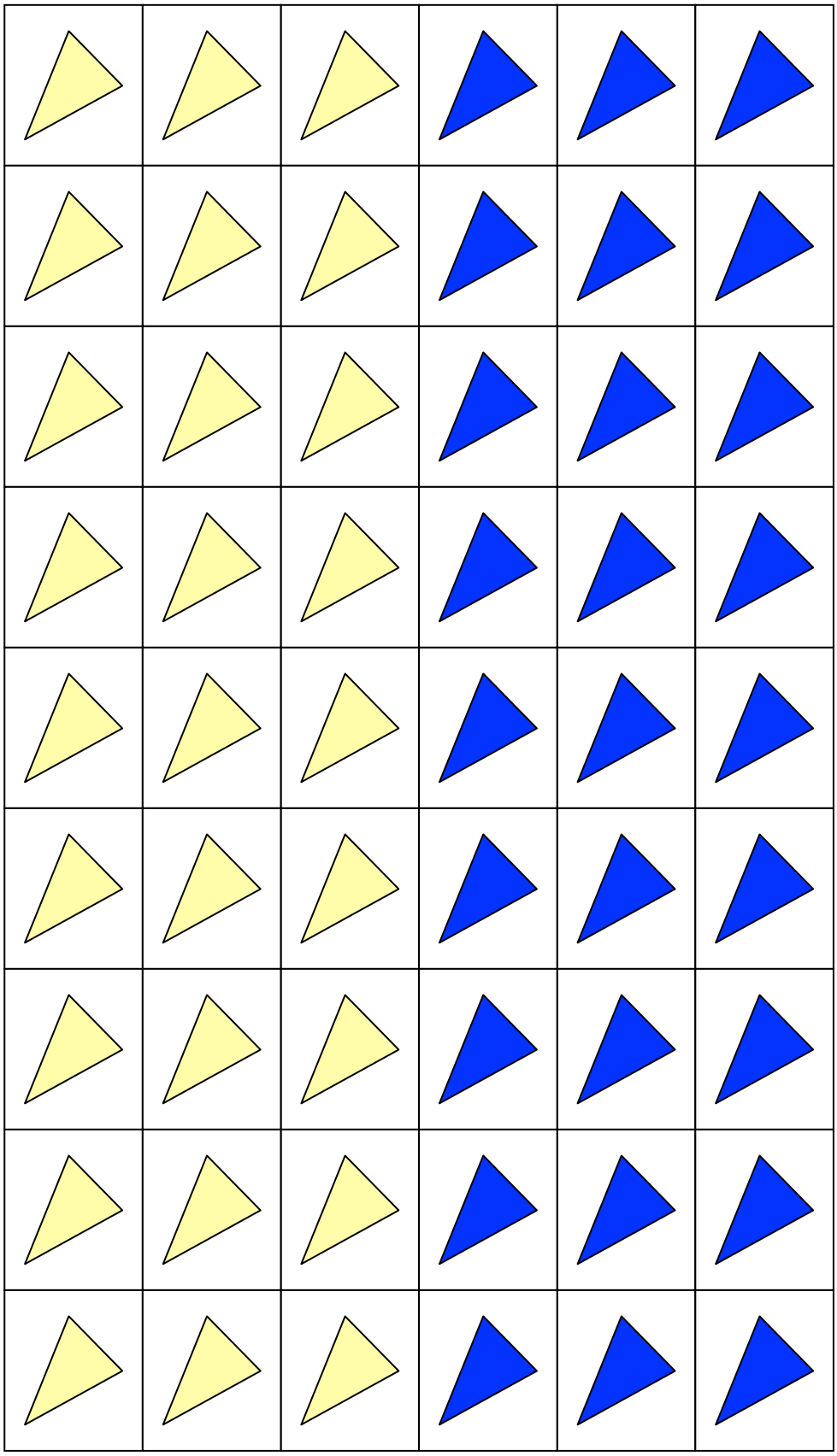
Probability shapes

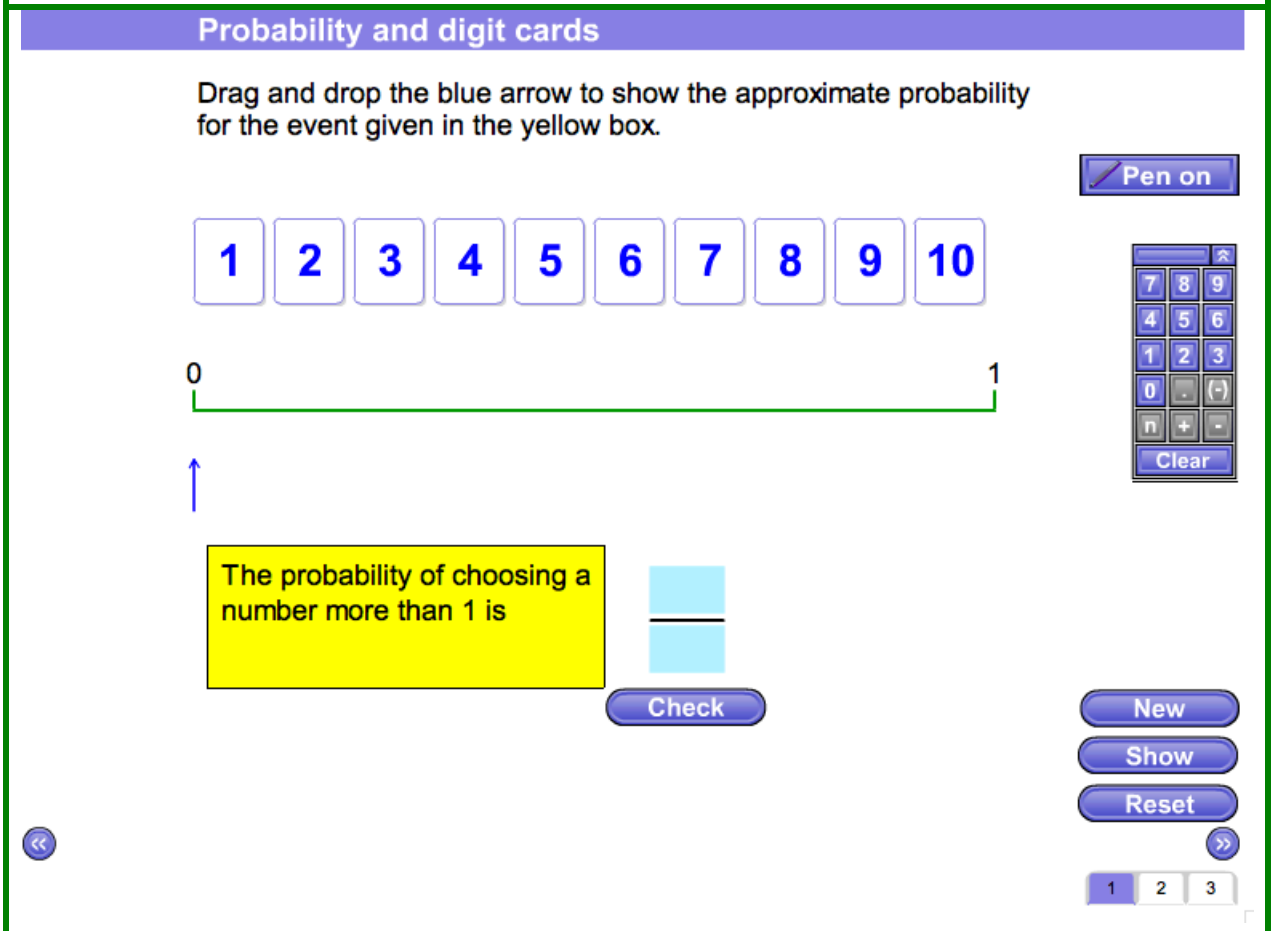


Probabilities with digit cards, words and dice

TYPE:	Main
OBJECTIVE(S):	Use vocabulary and ideas of probability, drawing on experience; find and justify probabilities based on equally likely outcomes in simple contexts; simplify fractions by cancelling all common factors.
DESCRIPTION:	3 screens. 1 shows 10 digit cards and asks for probabilities based on these. 2 is similar but with words. 3 is similar but with a normal dice.
OVERVIEW:	Using the probability scale to assign probabilities and finding probabilities with digit cards, words and dice.
EQUIPMENT:	Teacher notes include 2 photocopiable masters. The first includes the 10 digit cards and 5 probability scales with space to write down two events per scale. The second includes 6 probability scales with space to write down a word and an event.

TYPE:	Plenary
OBJECTIVE(S):	Use vocabulary and ideas of probability, drawing on experience; find and justify probabilities based on equally likely outcomes in simple contexts; simplify fractions by cancelling all common factors.
DESCRIPTION:	2 screens. 1 animates solutions to 10 digit card probability questions. 2 is the vocabulary screen.
OVERVIEW:	Using the probability scale to assign probabilities and finding probabilities with digit cards, words and dice.
EQUIPMENT:	None specific.

Main Whiteboard and Screen information

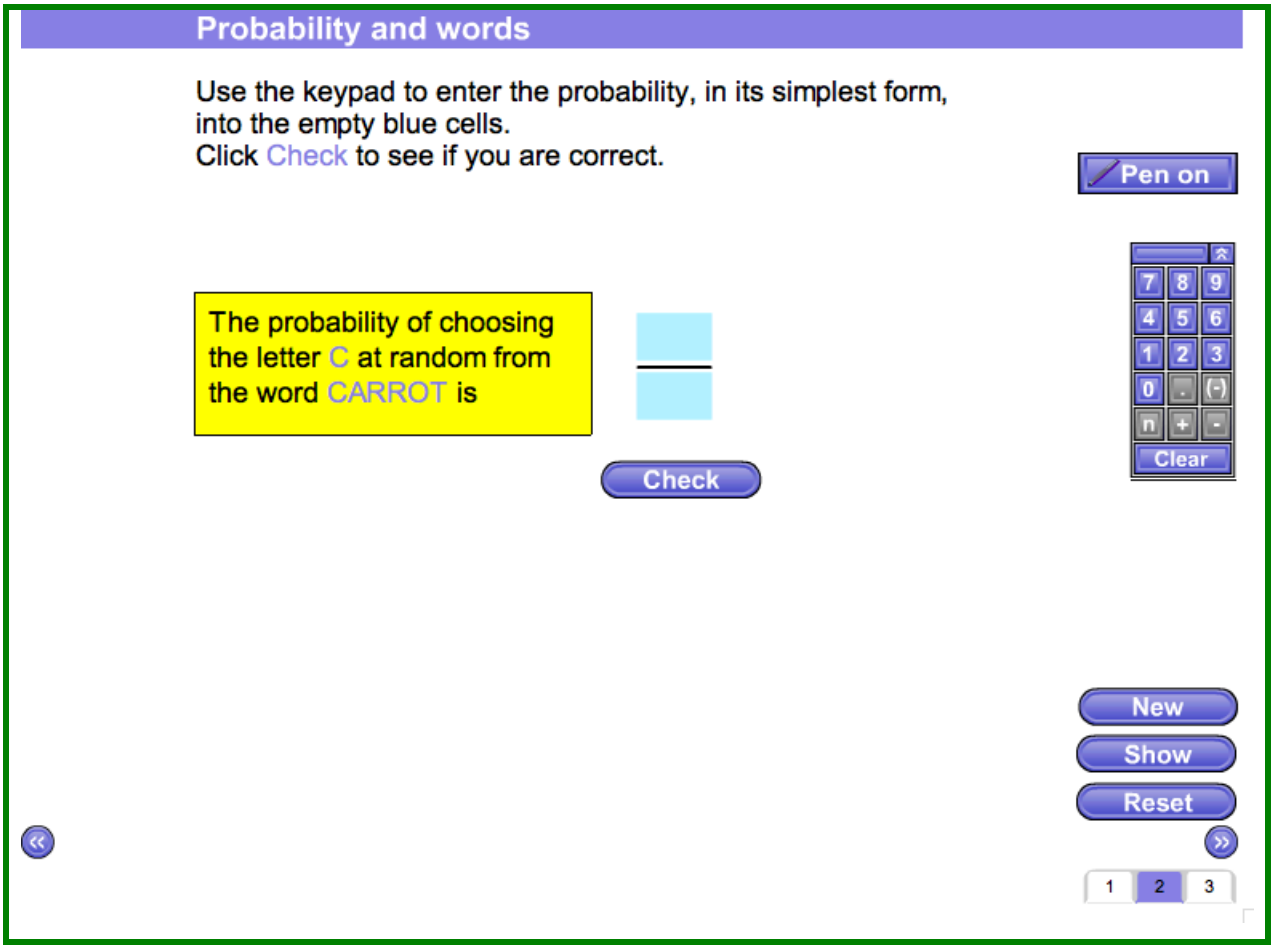


Screen 1: Probability and digit cards

Ten digit cards numbered 1 to 10 are shown on screen. You are then asked to find the probability of an event, such as randomly drawing a prime number, from the cards. You have to show the answer by dragging a blue arrow to the appropriate point on a probability scale and then use the keypad to enter the probability. If you do not enter the probability in its simplest form you are given a message "Correct, but you can simplify your answer." You are given some leeway on the probability scale so if you click 'Show' the blue arrow may move to the exact position.

Difficulties arise from the event as much as from the probability (e.g. knowing which numbers are prime).

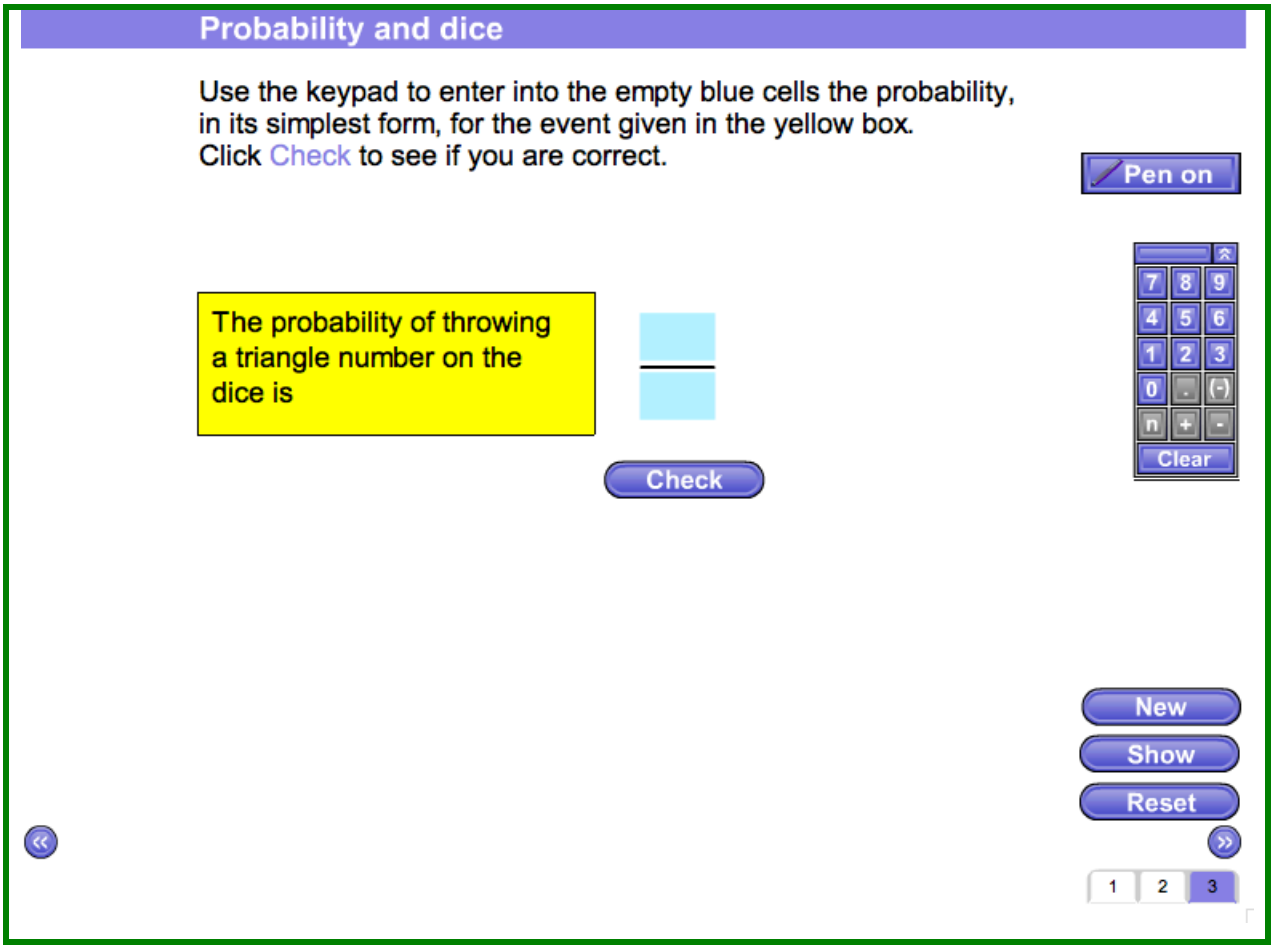
Key points: informal notion of probability and link of this to the probability scale; it helps to establish ideas of probabilities on this line and link to the actual fraction; pupils use the digit cards to help with the probability; reducing fractions to their lowest terms may create minor problems.



Screen 2: Probability and words

You are given a word and asked to find the probability of selecting a given letter from the word. You use the keypad to enter the probability. If you do not enter the probability in its simplest form you are given a message "Correct, but you can simplify your answer."

Key points: informal notion of probability should continue to be the focus; reducing fractions to their lowest terms may create minor problems.



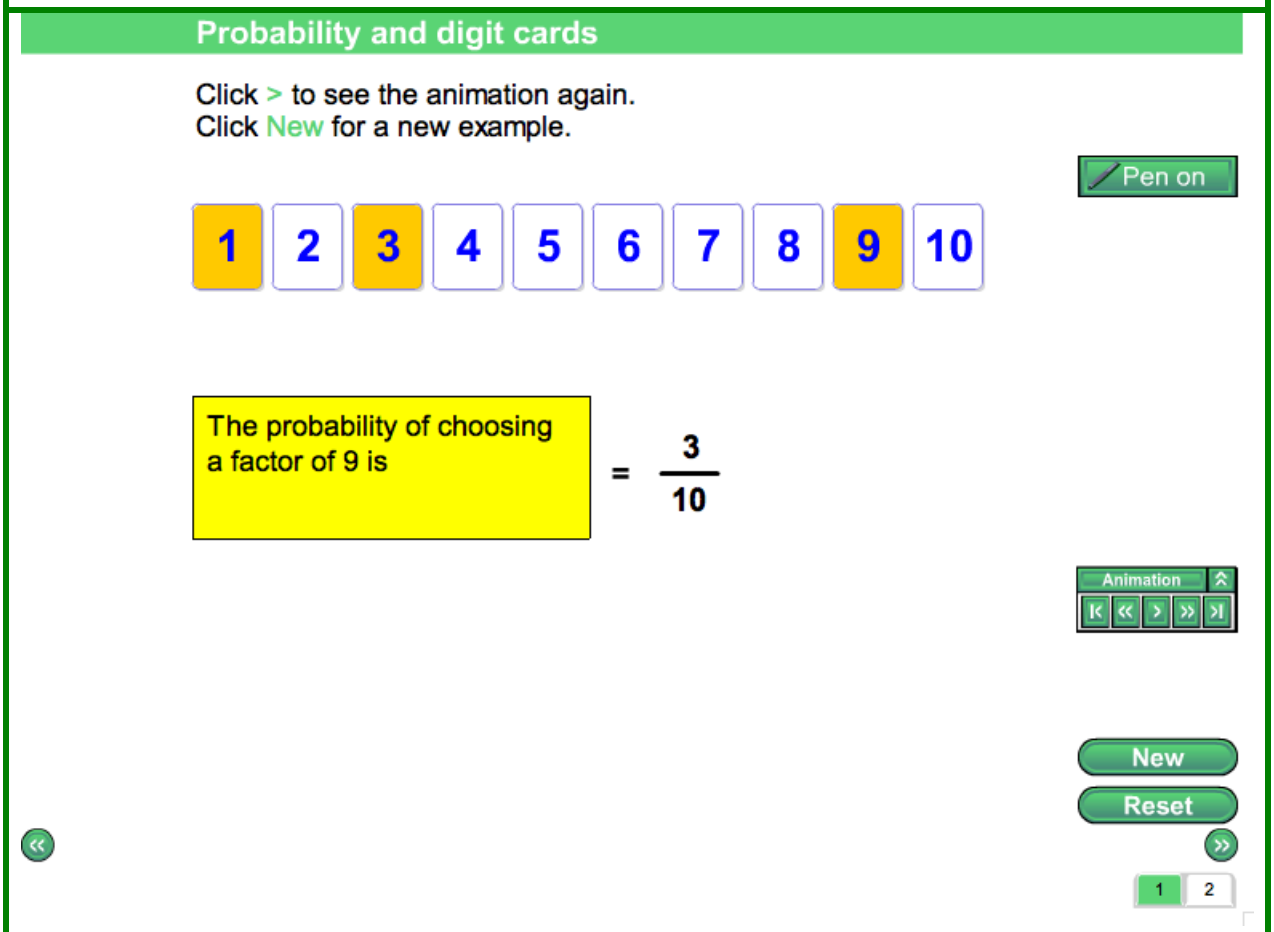
Screen 3: Probability and dice

You are asked to find the probability of an event based on throwing a normal cubical dice, such as throwing a triangle number. You use the keypad to enter the probability. If you do not enter the probability in its simplest form you are given a message "Correct, but you can simplify your answer."

Difficulties arise from the event as much as from the probability (e.g. knowing which numbers are the triangle numbers).

Key points: informal notion of probability should continue to be the focus; reducing fractions to their lowest terms may create minor problems.

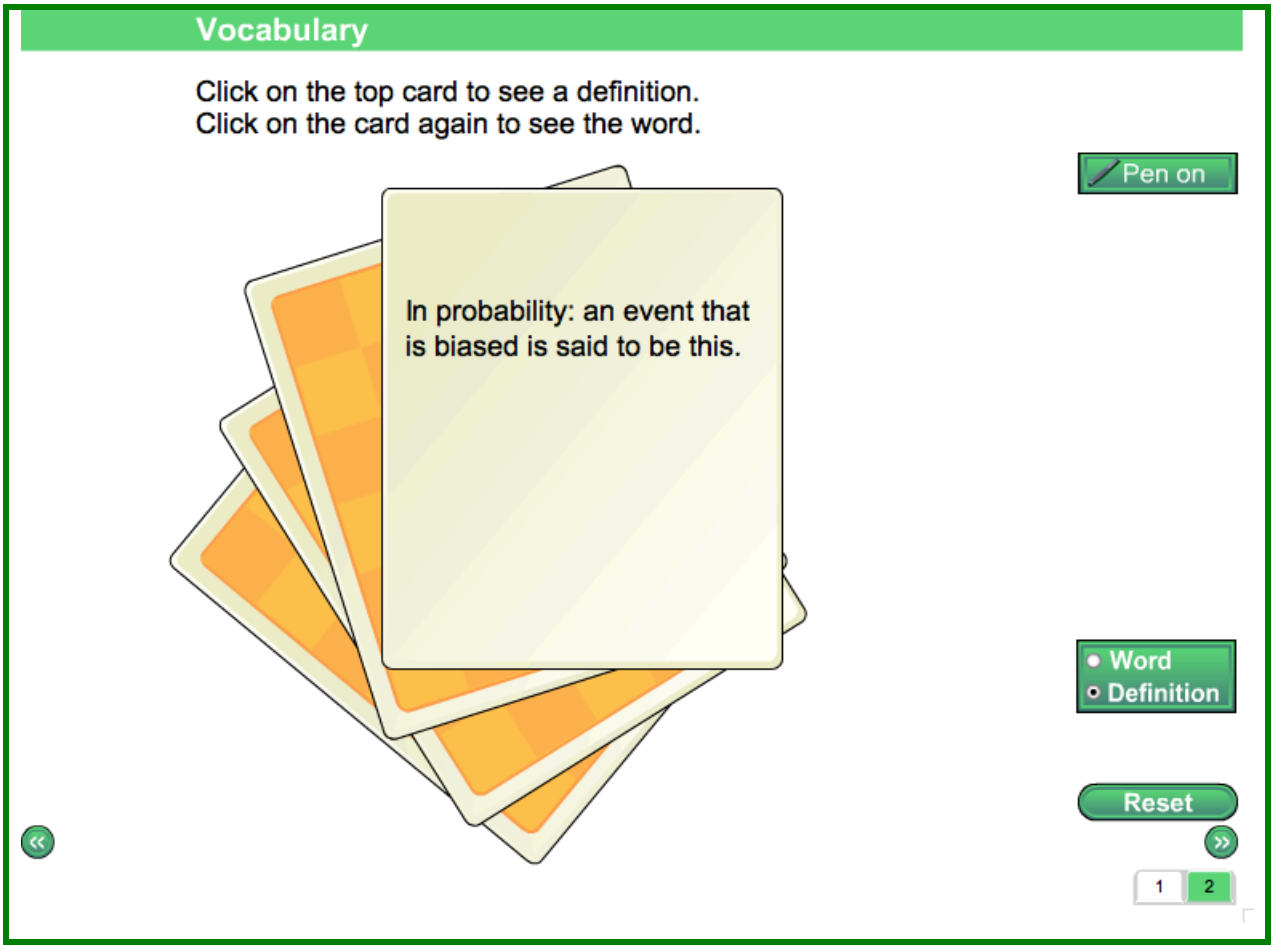
Plenary Whiteboard and Screen information



Screen 1: Probability and digit cards

Ten digit cards numbered 1 to 10 are shown on screen. You are given an event, such as randomly drawing a prime number, from the cards and shown an animation of how to work out the probability, which includes shading the relevant cards and reducing the fraction to its lowest terms.

Key points: a more formal notion of probability can now be introduced; have pupils discuss events so that misconceptions can be addressed; reducing fractions to their lowest terms by dividing numerator and denominator by common factor (and to check for this).



Screen 2: Vocabulary

Vocabulary present: Average, Certain, Chance, Doubt, Equally likely, Even chance, Fair, Fifty-fifty chance, Good chance, Impossible, Interval, Likelihood, Likely, Mean, Median, Modal class/group, Mode, No chance, Outcome, Poor, Possible, Probability, Probable, Random, Range, Risk, Statistic, Uncertain, Unfair, Unlikely.

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Probability and digit cards

Draw an arrow on the scale to show the probability and write the exact probability next to it.

1	2	3	4	5	6	7	8	9	10

Event 1:	Event 2:

0 1

Event 1:	Event 2:

0 1

Event 1:	Event 2:

0 1

Event 1:	Event 2:

0 1

Event 1:	Event 2:

0 1

Probability and words

Draw an arrow on the scale to show the probability and write the exact probability next to it.

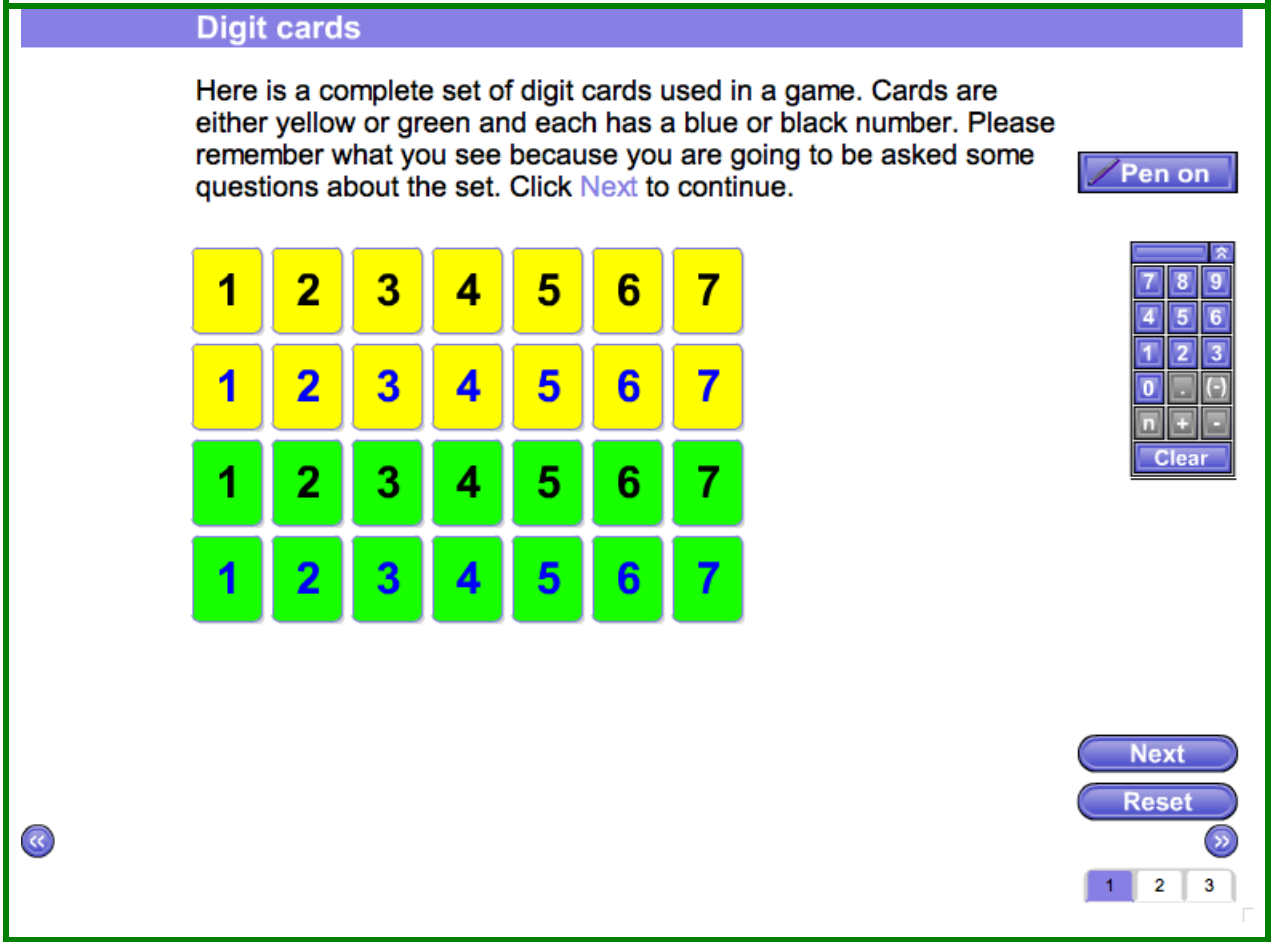
Word:	Event:
0	1
Word:	Event:
0	1
Word:	Event:
0	1
Word:	Event:
0	1
Word:	Event:
0	1
Word:	Event:
0	1

Probability: choices

TYPE:	Main
OBJECTIVE(S):	Probability of not an event is 1–p; estimate probabilities from experiments; repeat an experiment usually means different outcomes.
DESCRIPTION:	1 estimate probabilities based on pack of digit cards; 2 from balloons (numbers in a table); 3 from skill of 'game' player.
OVERVIEW:	Probability from pack of digit cards, balloons and a game.
EQUIPMENT:	None.

TYPE:	Plenary
OBJECTIVE(S):	Probability of not an event is 1–p; estimate probabilities from experiments; repeat an experiment usually means different outcomes.
DESCRIPTION:	1 shows how to answer the digit card questions. 2 and 3 do likewise for the balloons and disc questions respectively. 4 is vocabulary.
OVERVIEW:	Probability from pack of digit cards, balloons and a game.
EQUIPMENT:	None.

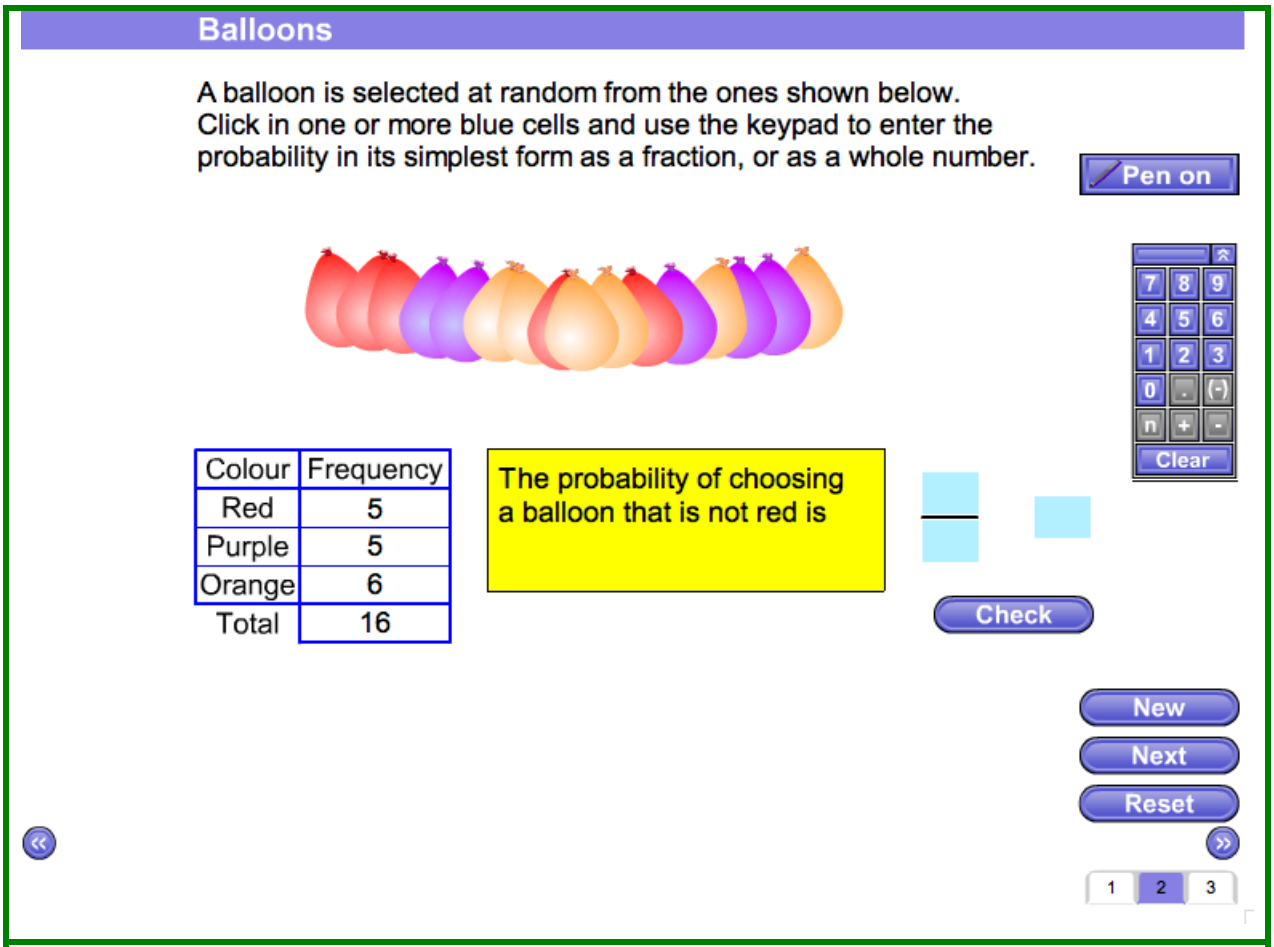
Main Whiteboard and Screen information



Screen 1: Digit cards

You are shown a complete set of digit cards and asked to find the probability of a given event based on the cards you have seen. The 'complete' pack varies each time. The maximum number of cards in a 'complete' set is 36 made up of the numbers 1 to 9 in four different combinations: black or blue numbers on a yellow or green background. In any 'complete' set numbers go from 1 to n, where n is 5 or more with 1, 2 or 4 of the different combinations present. Probabilities are concerned with a number, a background colour, a number colour and mixtures of these (and the 'not' versions). Answers have to be given as a fraction in its simplest form. When the question is asked the cards are not shown, however they can be viewed again. The same set can be kept for the next question.

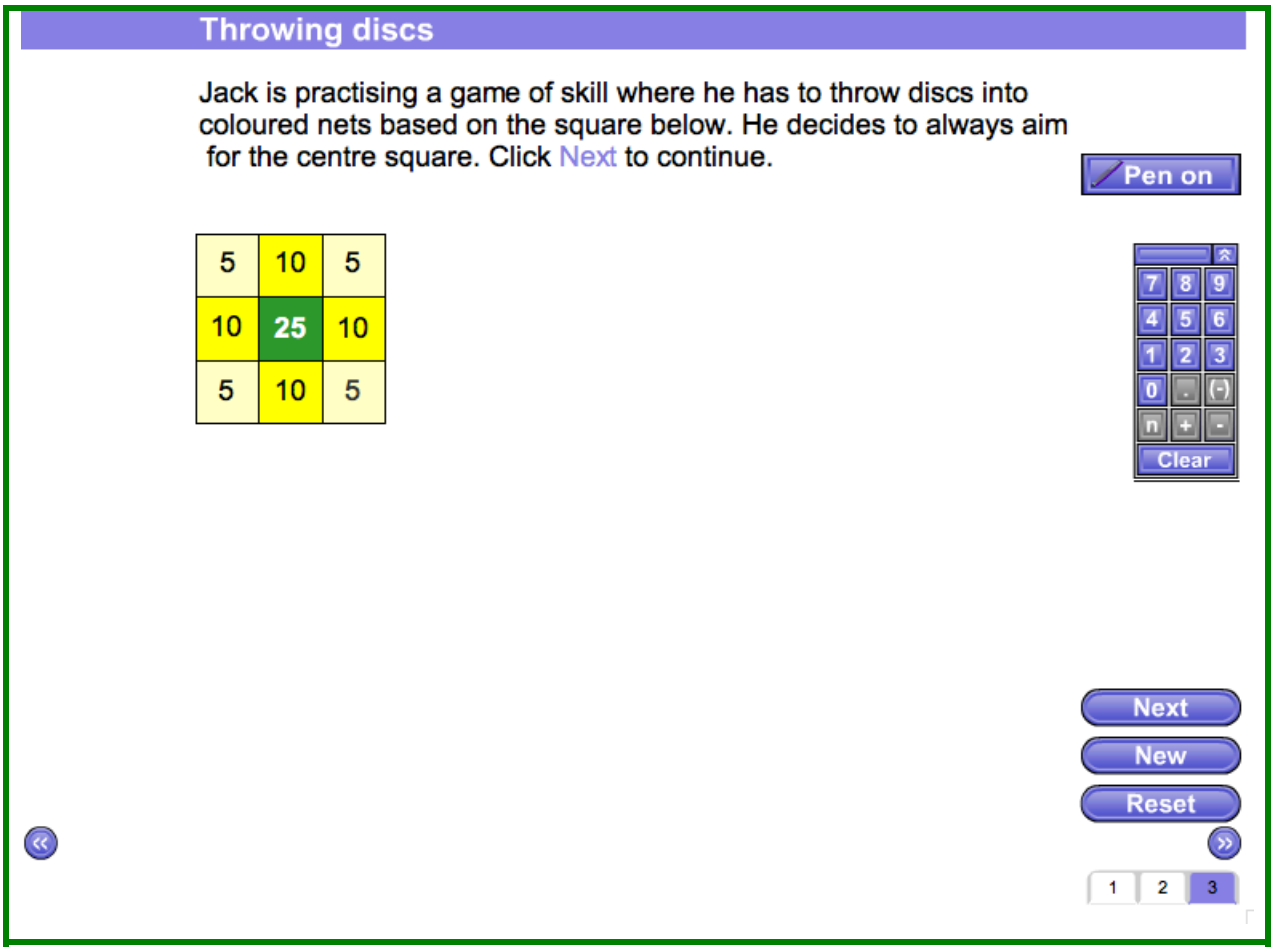
Key points: pupils should discuss probabilities of events and the probability of 'not' an event and note that probability of not an event is equal to one take away the probability of the event; pupils should be encouraged to calculate probabilities rather than just count the number of cards.



Screen 2: Balloons

Between 12 and 20 balloons are shown. There are three colours and the number of each colour is shown in a table. A probability question is asked in terms of choosing one balloon that may be a single colour (including a colour not present), or one of two colours or not these things. Answers have to be given as a fraction in its simplest form. When you get a question correct you can keep the same set for another question, but a random balloon is removed (it bursts).

Key points: pupils should discuss probabilities of events and the probability of 'not' an event and note that probability of not an event is equal to one take away the probability of the event; pupils should be encouraged to use the table rather than just count the balloons.

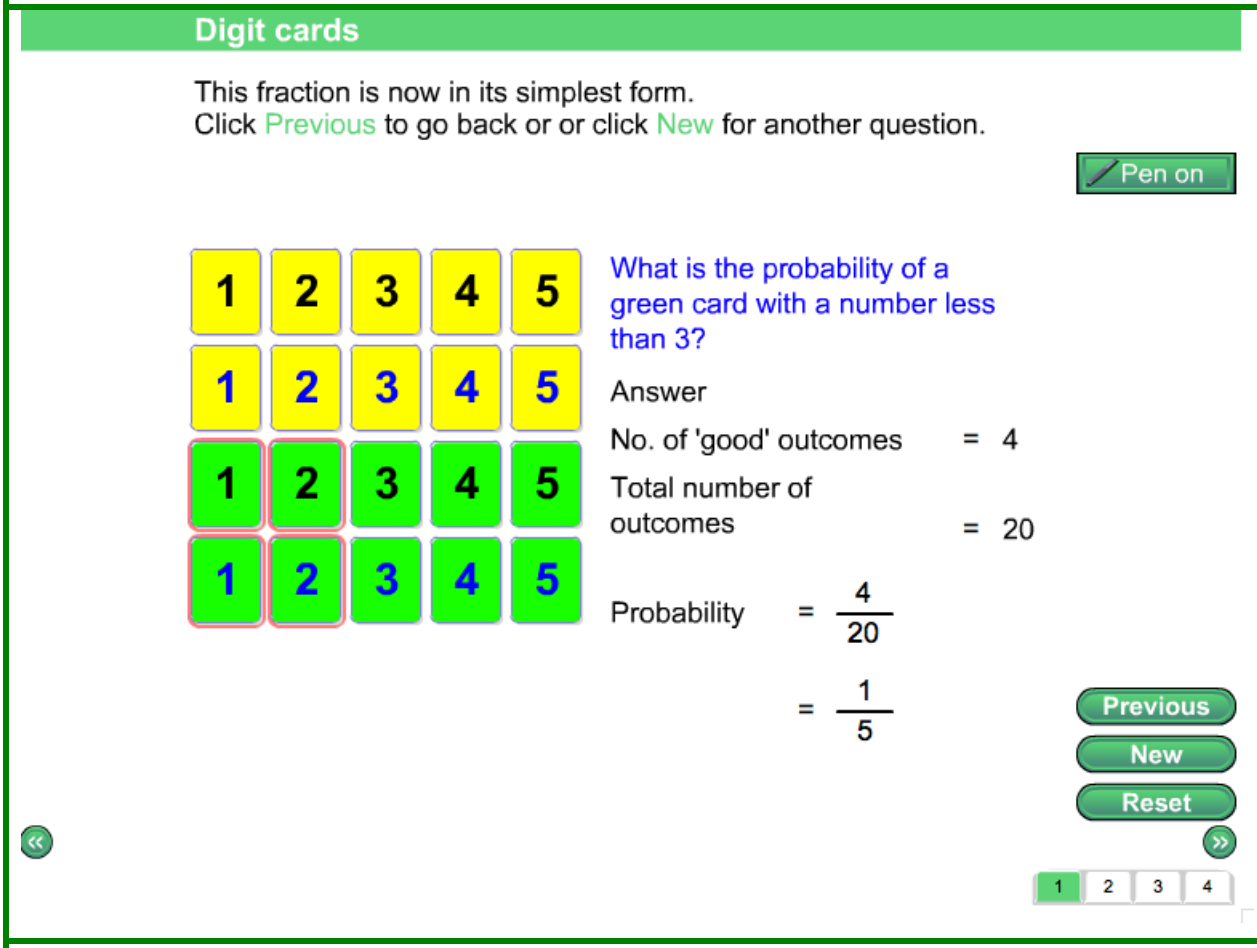


Screen 3: Throwing discs

A pupil is practising a game and her/his level of success to date is given in terms of how many times each possibility has happened. The board for the game is also shown. Pupils are asked to enter the probability of a given event (including an impossible or certain event) on the pupil's next throw. The answer must be given in its simplest form.

Key points: this activity has been designed so that it may lead to considerable discussion in terms of how to find the probability since some pupils will want to ignore the previous results and use the board as the means to find the probability (usually assuming that all events are equally likely); some pupils will also have difficulty at not wanting to put probabilities to events that have not happened in the earlier practice (like missing the board).

Plenary Whiteboard and Screen information



Screen 1: Digit cards

Screens 1 to 3 work in a similar way. You are shown a complete set of digit cards and shown one way to find the answer to a probability question. The 'complete' pack consists of yellow cards numbered 1 to 5 with the numbers in both black and blue and the same for green cards, making 20 cards in total. A question is shown and you can 'step' through the solution by clicking on Next (or using Previous to go back a step). The first step shows the 'good' outcomes by surrounding the relevant cards; the second shows the total number of outcomes (always 20 here); the third shows the probability as a fraction (and pupils are told when it is not in its simplest form) and the fourth has the fraction in its simplest form.

Key points: pupils should discuss probabilities of events and the probability of 'not' an event and note that probability of not an event is equal to one take away the probability of the event; pupils should be encouraged to calculate probabilities rather than just count the number of cards.

Balloons

The probability is given as a fraction.
Click Previous to go back to the last step.
Click New for another question.

Pen on

Colour	Frequency
Red	3
Purple	5
Orange	3
Total	11

What is the probability of choosing a balloon that is not orange nor red?

Answer

No of 'good' outcomes = 5

Total number of outcomes = 11

Probability $= \frac{5}{11}$

Previous
New
Reset

1 2 3 4

Screen 2: Balloons

Screens 1 to 3 work in a similar way. Between 9 and 14 balloons are shown. There are three colours and the number of each colour is shown in a table. You are then able to 'step' through a probability question which is asked in terms of choosing one balloon that may be a single colour (including a colour not present), or one of two colours or not these things. The answers to these are given as a fraction in its simplest form. The first step shows the 'good' outcomes by shading in yellow the appropriate cells in the table; the second shows the total number of outcomes by shading in orange the total cell in the table; the third shows the probability as a fraction (but pupils are not told if it is in its simplest form, though Next disappears if it is) and, if there is a fourth, it has the fraction in its simplest form.

Key points: pupils should discuss probabilities of events and the probability of 'not' an event and note that probability of not an event is equal to one take away the probability of the event; pupils should be encouraged to see how using the table is easier, quicker and likely to be more accurate than just counting the balloons.

Throwing discs

Click Previous to go back to the last step, or click New for another question.

Pen on

5	10	5
10	25	10
5	10	5

Points	Frequency
25	3
10	9
5	6
Total	18

What is the probability that Frank scores 25 or 5 with his next throw?

Answer

Number of 'good' outcomes = 9

Total number of outcomes = 18

Probability $= \frac{9}{18}$

$= \frac{1}{2}$

Previous

New

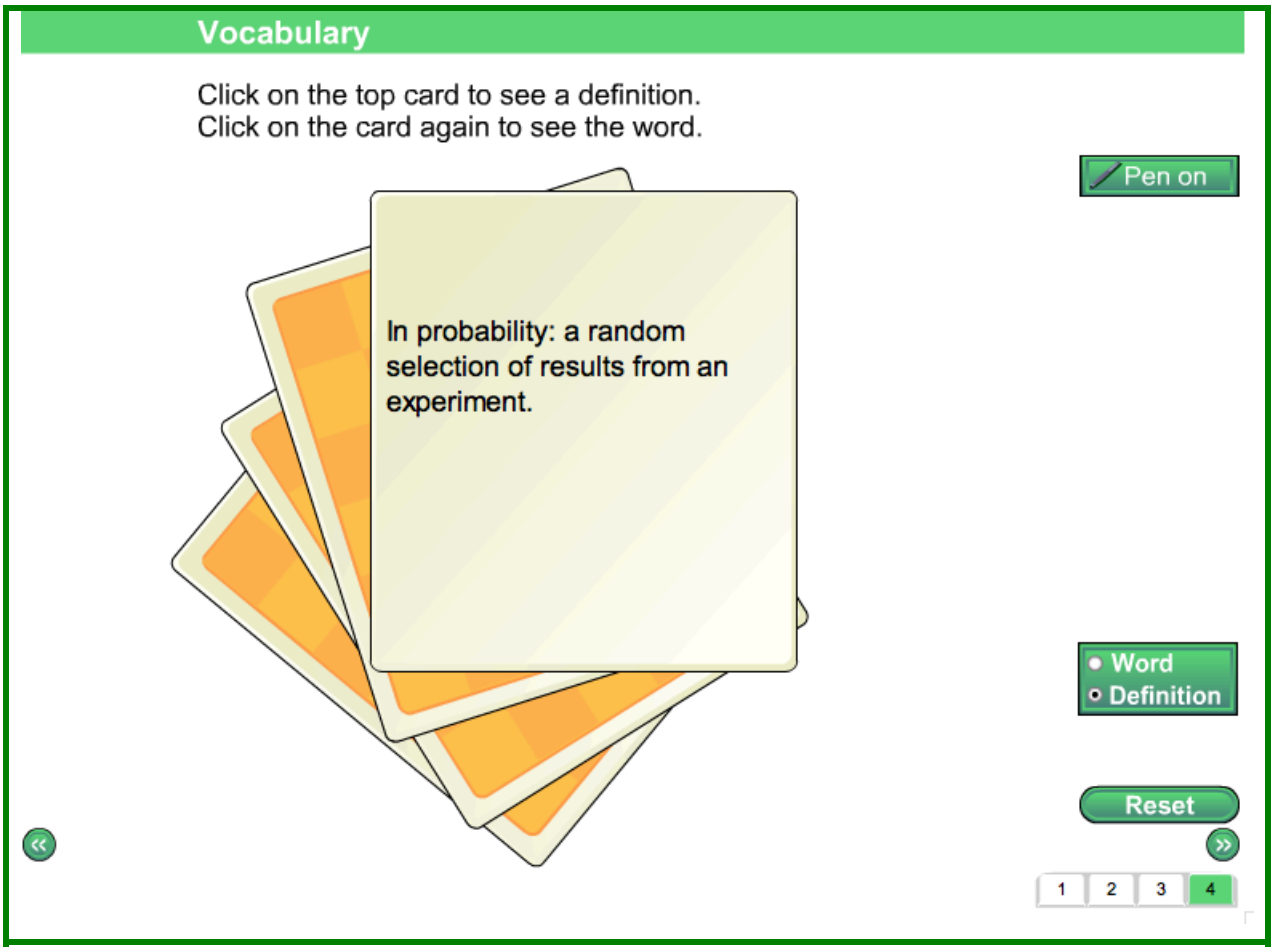
Reset

1 2 3 4

Screen 3: Throwing discs

Screens 1 to 3 work in a similar way. A pupil is practising a game and her/his level of success to date is given in terms of how many times each point scoring possibility has happened. The board for the game is also shown. A blank two column table is shown where the first column is labelled points and the second frequency. You can then be shown step by step how to find the answer to a question posed about the pupil's level of success at the game. You click Table and the table is completed based on the success of the pupil to date. The rest of the steps repeat the same process shown on screen 2.

Key points: this activity has been designed so that it may lead to considerable discussion in terms of how to find the probability since some pupils will want to ignore the previous results and use the board as the means to find the probability (usually assuming that all events are equally likely); some pupils will also have difficulty at not wanting to put probabilities to events that have not happened in the earlier practice (like missing the board); some pupils will have difficulty completing the table.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Doubt, Equally likely, Even chance, Event, Fair, Fifty-fifty chance, Good chance, Impossible, Interval, Likelihood, Likely, No chance, Outcome, Poor, Possible, Probability, Probable, Random, Risk, Sample, Sample space, Statistic, Theory, Uncertain, Unfair, Unlikely.

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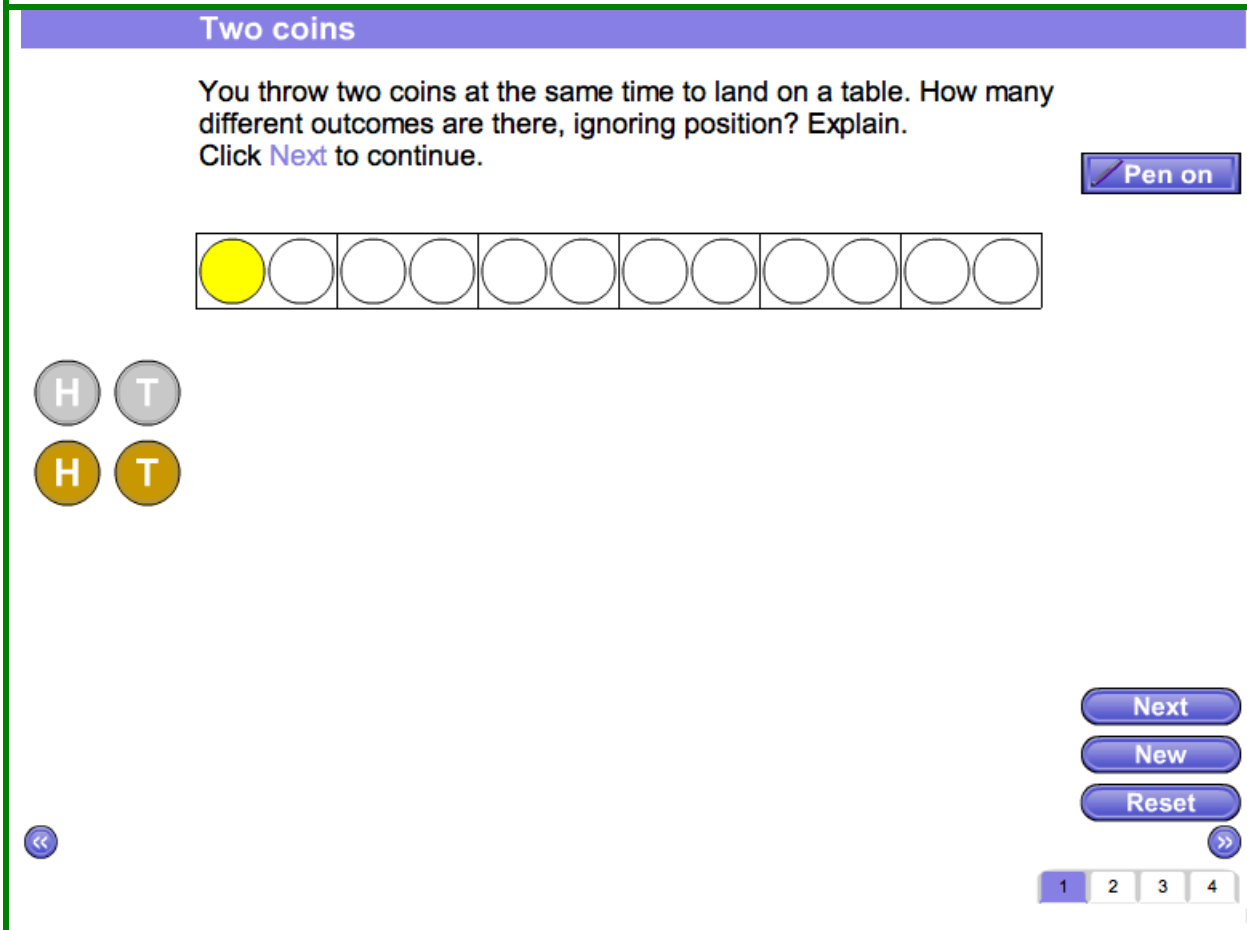
Probability: two events

TYPE:	Main
OBJECTIVE(S):	Find and record all possible mutually exclusive outcomes for 1 or 2 events in a systematic way, using diagrams and tables.
DESCRIPTION:	1 is a sample space for two coins. 2 and 3 are sample spaces for two dice. 3 asks probability questions for two dice.
OVERVIEW:	Sample space for two coins and two dice.
EQUIPMENT:	It often helps to have coins or dice available so that pupils might consider all the different combinations. Two photocopy masters are available, the first contains copies of sample space diagrams for two coins and two dice. The second contains a data collection sheet to for two coin and two dice experiments.

TYPE:	Plenary
OBJECTIVE(S):	Find and record all possible mutually exclusive outcomes for 1 or 2 events in a systematic way, using diagrams and tables.
DESCRIPTION:	1 and 2 concern throwing 2 coins and associated probabilities. 3 is similar for the sum of two dice. 4 is vocabulary.
OVERVIEW:	Sample space for two coins and two dice.
EQUIPMENT:	None.

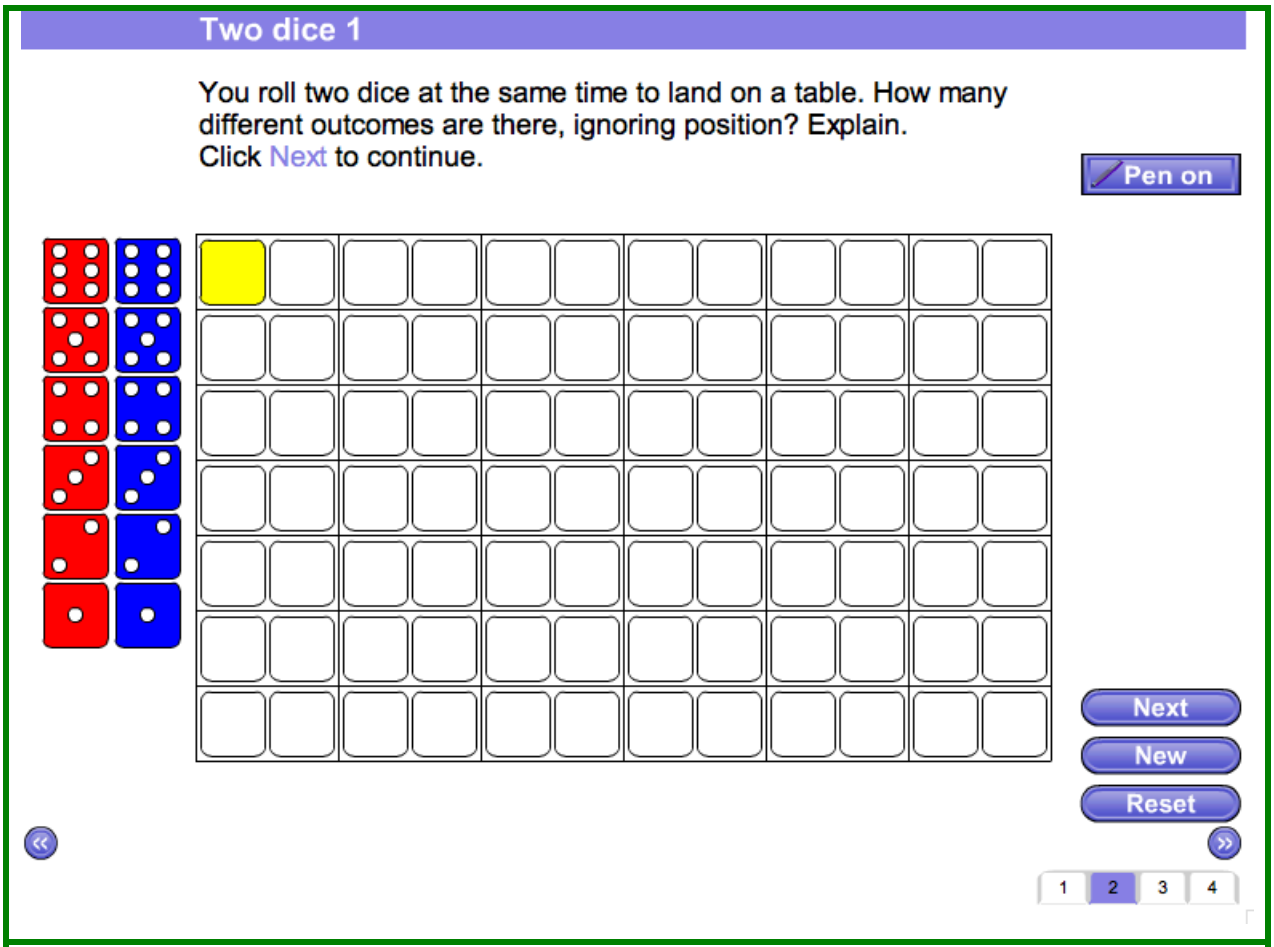
Main Whiteboard and Screen information



Screen 1: Two coins

You are told that you have two coins and that they land on a table. You are asked to show the different ways (in terms of heads and tails only) that the coins can land. You may need to clarify this so pupils are aware that the order of placing coins, in terms of silver and bronze, on the screen does not matter. Six pairs of circles are shown across the screen and a yellow filled in circle is the left-hand circle of a pair of circles, all the rest are empty. Under this are two silver circles, one containing H (for head) and the other T (for tail) and below these are two more bronze circles similarly labelled. These represent the ways that a silver and bronze coin can land on a table. If you click one of these four circles a copy of it is placed in the position of the yellow circle, and the yellow circle moves across to the right. In this way you can place silver and bronze H and T in position. It helps pupils understanding if they collect results of their own, especially looking at number of heads or tails showing.

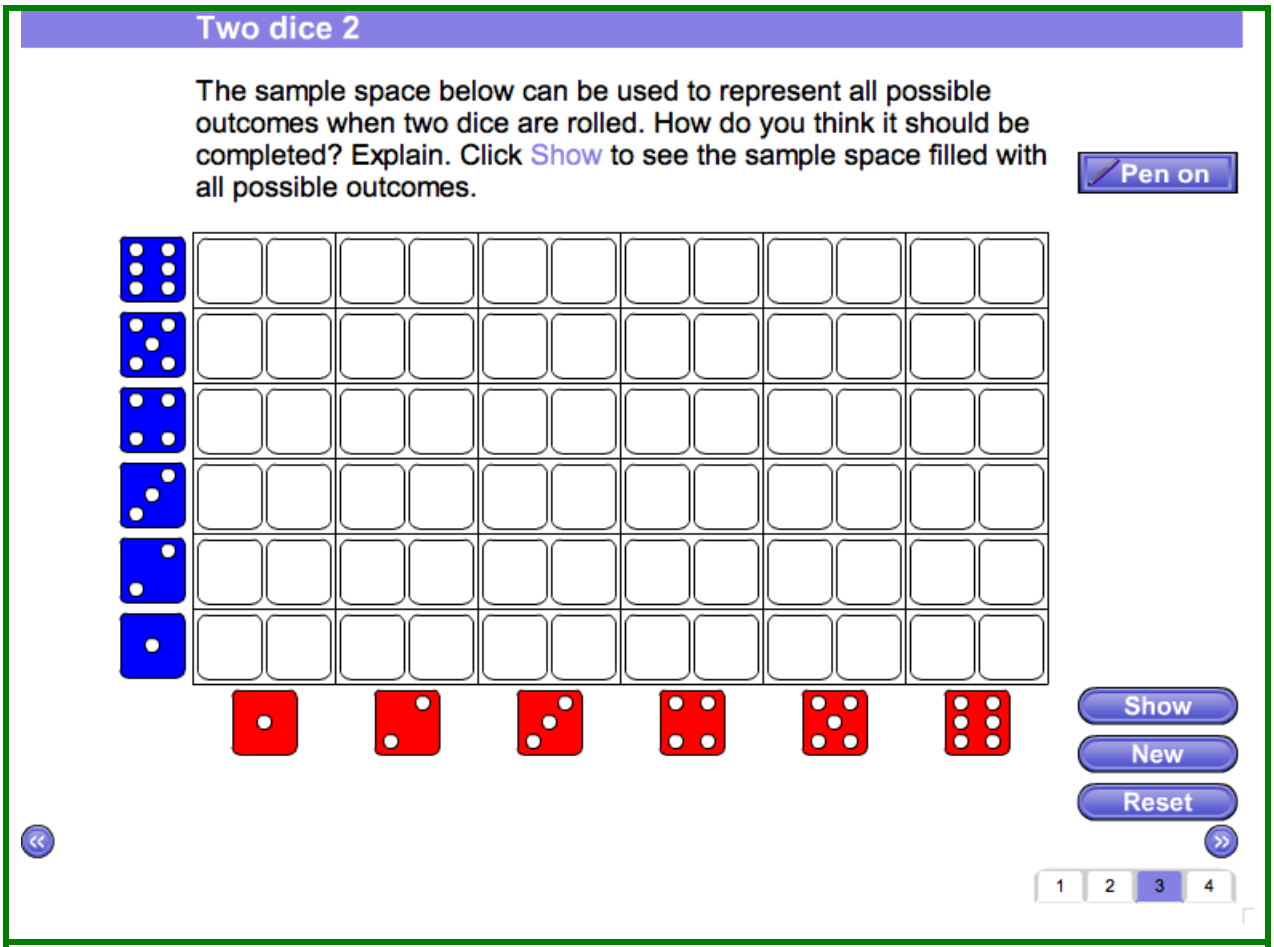
Key points: we have deliberately given too many circle pairs; remind pupils that order of coins in terms of silver/bronze is not relevant; pupils should discuss possible head and tail combinations and note that a bronze H with silver T is the same as a silver T with a bronze H, but that a bronze H with silver T is NOT the same as a silver H with a bronze T; pupils should persuade each other why the 3 results of 0, 1 and 2 heads are not equally likely; you might like to have pupils talk about the sample space and ways that this might be represented; for some it may be appropriate to consider 3 coins.



Screen 2: Two dice 1

You are told that you roll a red and blue dice and that they land on a table. You are asked to show the different ways (in terms of numbers1 to 6) that the dice can land. You may need to clarify this so pupils are aware that the order of placing dice, in terms of red and blue, on the screen does not matter. Six pairs of 'domino-type' shapes are shown across the screen and there are seven rows of these. A yellow filled in shape is in the left-hand shape of the first domino, all the rest are empty. In the margin on the left are the six possible outcomes on the red and blue dice. If you click one of these twelve dice faces a copy of it is placed in the position of the yellow shape, and the yellow shape moves across to the right. In this way you can place the red and blue dice scores to make all the possible 'domino' combinations.

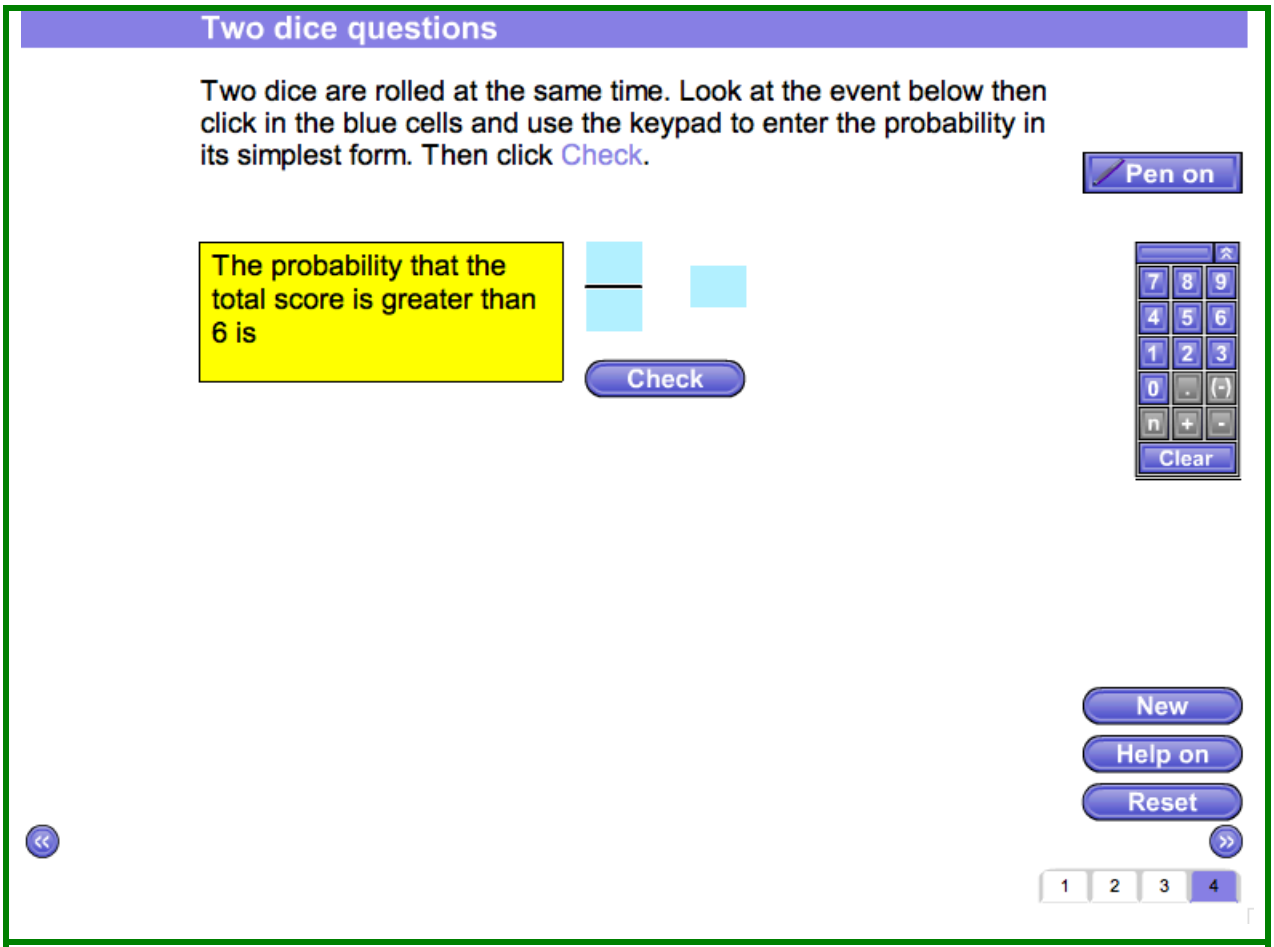
Key points: we have deliberately given too many 'dominoes'; remind pupils that order of dice in terms of red/blue is not relevant; pupils should discuss possible combinations, how they might be systematic and how they might know when they have all the possible combinations (which is a lead into screen 3); pupils should also consider how they can tell whether they have missed any combination out or counted any twice; you might like to have pupils talk about the sample space and different ways that this might be represented.



Screen 3: Two dice 2

The sample space of all the possible outcomes with the red and blue dice is to be represented. At the start the red dice faces are placed in order along the x-axis and the blue dice faces up the y-axis. There are 36 blank 'dominoes' in this coordinate area in an obvious 6 by 6 array. When you click Show the 36 possible combinations are put in place in the sample space in the 'conventional' way with (1, 1) in the bottom left and (6, 6) in the top right, where (1, 1) is the coordinate position with the 'domino' of the red dice showing 1 followed by the blue dice showing 1. When Show is clicked again the two dice in each 'domino' are added, removed and replaced instantly by a yellow square containing the sum of the two dice (clicking Previous will take you back to the dice). A further click of Show 'loses' the white space to give the 'conventional' sample space diagram. Clicking Previous takes you back a step.

Key points: it will help if pupils have considered different ways in which one might put together a sample space and seen the advantages of a systematic approach (see screen 2); you may wish to discuss with pupils why we might have displayed the sample space in the way we have when there are a number of other 'sensible' ways that could also have been used; although not made evident here you could use the pen to start using the sample space to think about probabilities (see screen 4).

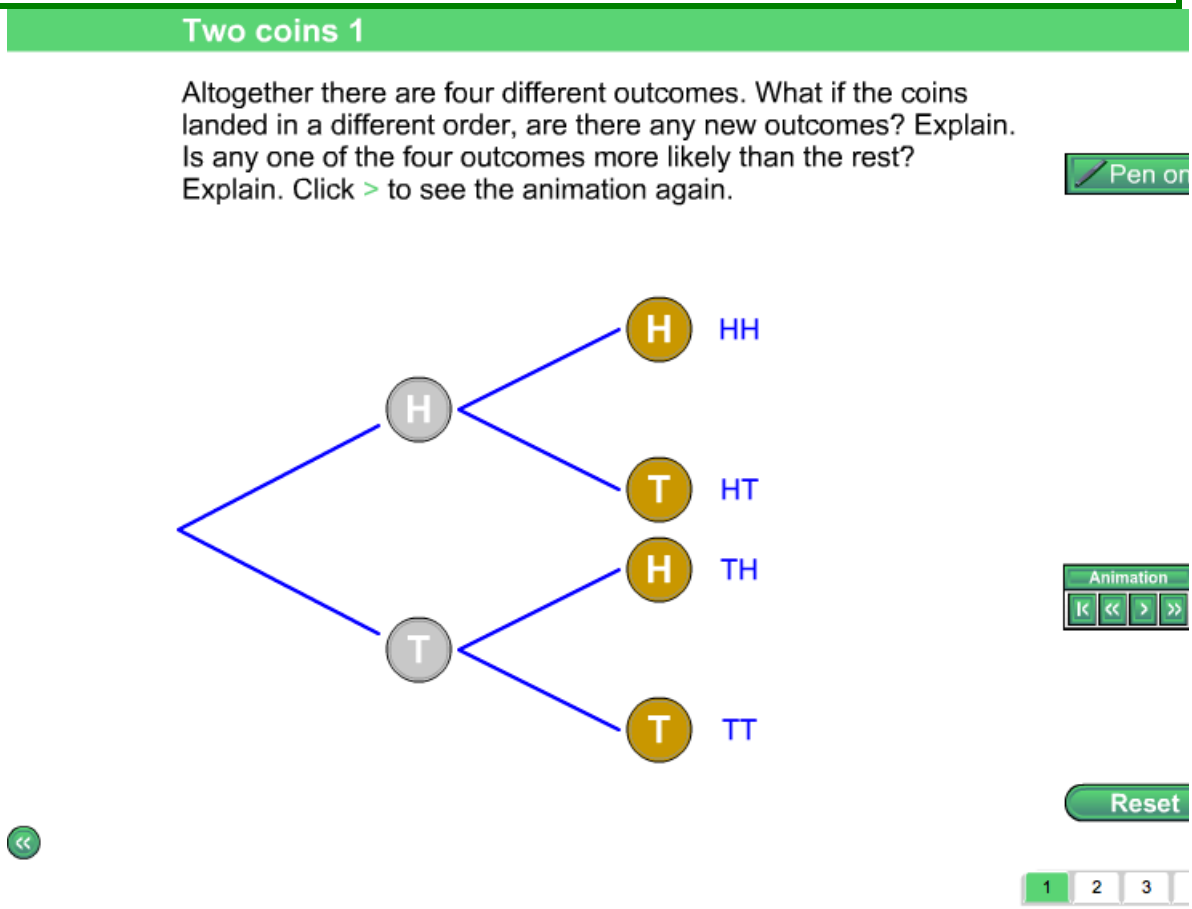


Screen 4: Two dice questions

You are asked to find a probability based on rolling two dice at once using the keypad. You can turn 'Help on' by clicking this button which will show a small version of the sample space from screen 3. The sample space stays on until you click 'Help off'. Solutions are expected to be in their simplest form, including 0.

Key points: you may want to encourage pupils to visualise the sample space and consider the probability before showing the sample space; when the sample space is displayed you could ask the pupils to use the pen to help mark the appropriate totals (see the plenary for this as well); pupils may need to be reminded to look carefully at what the question is asking (greater than, equal to); pupils should remember to give answers in their simplest form.

Plenary Whiteboard and Screen information



Screen 1: Two coins 1

An animation is available showing what can happen when two coins, one silver and one bronze are thrown together. The four outcomes of HH, HT, TH, TT are shown one at a time (with it assumed that the silver coin lands first) and then altogether in a 'tree diagram' type arrangement, but without any probabilities. The 'coding' of HH etc. is then added. Two further questions are posed asking whether there are any more outcomes if the bronze coin lands first and if any of the outcomes are more likely than the rest. The second screen deals with assigning probabilities to the outcomes.

Key points: pupils should discuss whether the order of coins landing means that there are more outcomes, or whether we just have to look at the coins after they have both landed; some pupils may think that the position of the coins makes a difference; there may also be confusion when considering the difference between HT and TH since they are different if the first coin in both is silver and the second bronze, but the same if the coins are listed as silver, bronze then bronze, silver; a few pupils may believe that one of the outcomes is more likely and some may believe that it is likely that one coin will land on its edge; you may like to use the term 'tree diagram'.

Two coins 2

Look at the outcomes in terms of the number of tails showing. The probabilities are not the same. Explain.
Click > to see the animation again.

Pen on

Outcomes	Probability
0 Tails	$\frac{1}{4}$
1 Tail	$\frac{1}{2}$
2 Tails	$\frac{1}{4}$

HH $P(HH) = \frac{1}{4}$

HT $P(HT) = \frac{1}{4}$

TH $P(TH) = \frac{1}{4}$

TT $P(TT) = \frac{1}{4}$

Animation

Reset

1 2 3 4

Screen 2: Two coins 2

This screen continues from where screen 1 finished. It shows the 'tree diagram' but without the probabilities. It then states that all the four outcomes are equally likely and then shows the probabilities associated with HH etc. one at a time (all equal to a quarter). Terminology used is "p(HH) = " to represent "the probability associated with the coins landing HH is equal to ". The tree diagram is removed and replaced with a table where outcomes in terms of number of tails (0, 1 or 2) are given of the left and a column on the right is blank with a heading 'Probability' (i.e. the probabilities associated with these outcomes). These probabilities are added one at a time (a quarter, a half and a quarter). The final screen asks pupils to explain why these three outcomes do not have equal probabilities.

Key points: pupils should be encouraged to discuss the precise meaning of the terminology as well as consider the probabilities involved; you may wish to follow this through in terms of language and mathematical 'symbolisation'; many pupils find the notion of 'non-symmetrical' outcomes very difficult as it almost seems to be common sense that with three outcomes of 0, 1 and 2 tails that they should be equally likely (this is why it is important that pupils do some experiments for themselves.

Two dice

Click another number between 2 and 12 in the table on the right to see the probability of the two dice giving exactly that total. The score will be highlighted in the sample space. Click Show to see all the probabilities. Click New to start again.

Pen on

+						
	7	8	9	10	11	12
	6	7	8	9	10	11
	5	6	7	8	9	10
	4	5	6	7	8	9
	3	4	5	6	7	8
	2	3	4	5	6	7

Total score	Probability
2	
3	
4	
5	
6	$\frac{5}{36}$
7	
8	
9	
10	
11	
12	

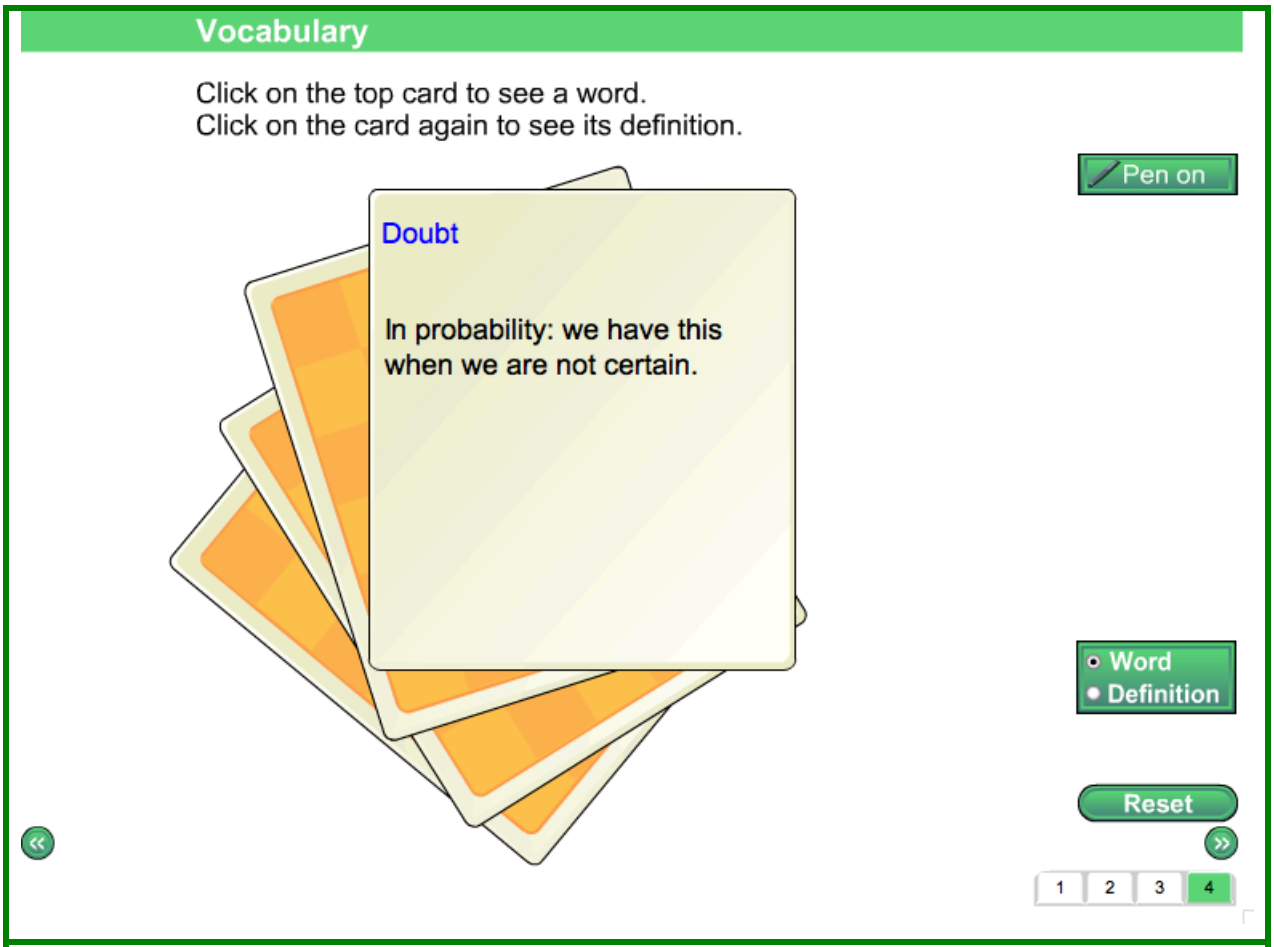
Show
New
Reset

1 2 3 4

Screen 3: Two dice

The sample space for the sum of two dice is shown on the left of the screen. On the right of the screen is a table that contains the possible 'Total scores' from 2 to 12 and a blank column where the probabilities will be placed. When you click a number in the table a corresponding probability is put in place (in 36ths and simplest form) and the numbers in the sample space are ringed. Any number of 'Total scores' can be selected or de-selected (by a second click). A Show option puts all the probabilities into the table.

Key points: pupils should be encouraged to observe the patterns in the table and discuss the probabilities; you could consider the most likely score, whether you are more likely to get an odd or even score; some pupils might still think that it is harder to get a double six than any other double.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Doubt, Equally likely, Even chance, Event, Fair, Fifty-fifty chance, Good chance, Impossible, Interval, Likelihood, Likely, No chance, Outcome, Poor, Possible, Probability, Probable, Random, Risk, Sample, Sample space, Statistic, Theory, Uncertain, Unfair, Unlikely.

Spire Maths interactive files available in a flash format at: https://spiremaths.co.uk/ia/

Unfortunately they will not work on iPads or iPhones.

Two coin data collection

Event	Tally	Frequency
Head, Head		
Head, Tail		
Tail, Head		
Tail, Tail		
	Total	

Experiments completed

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

Two dice data collection

Event	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
	Total	

Experiments completed

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10

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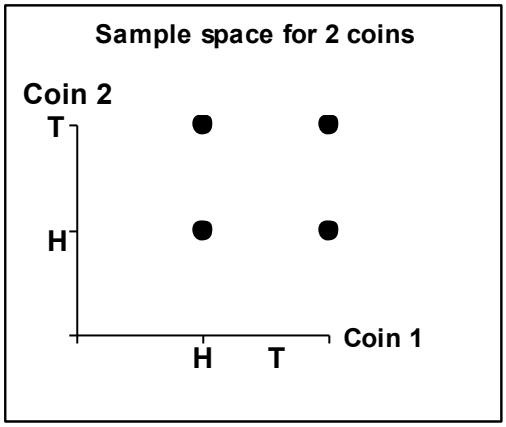
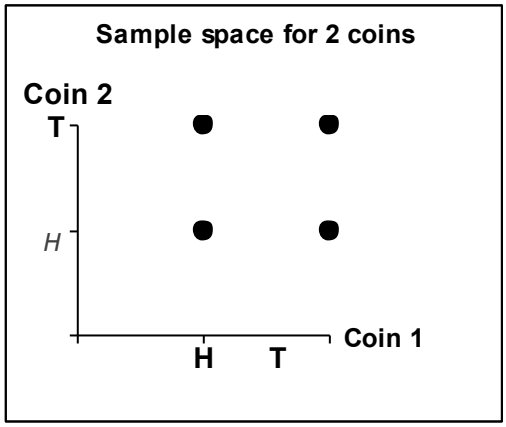
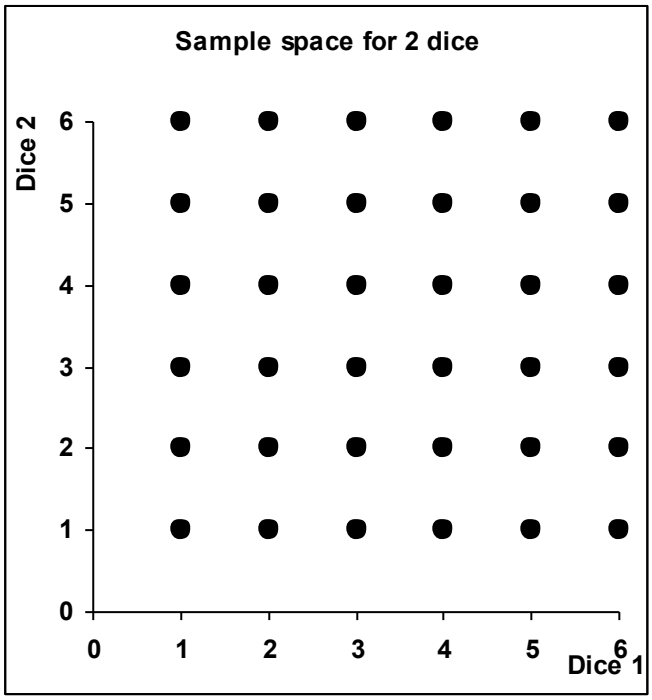
1 2 3 4 5 6 7 8 9 10

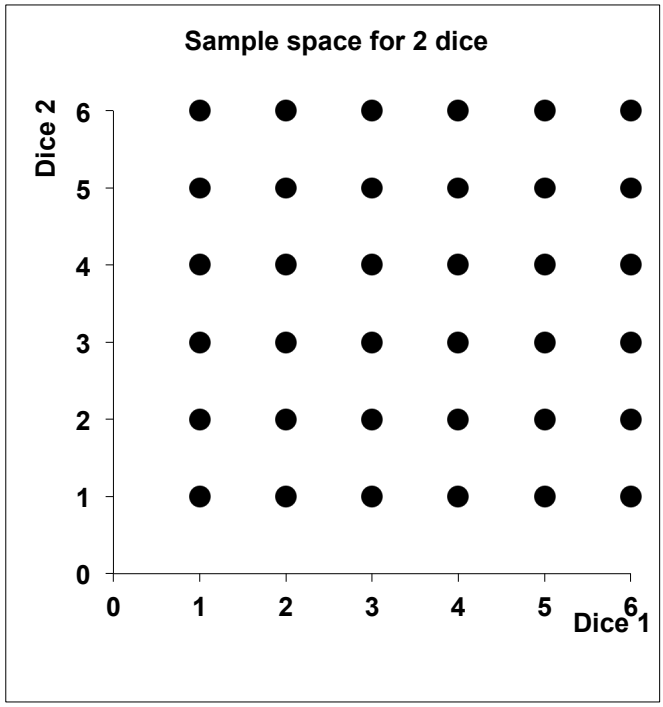
1 2 3 4 5 6 7 8 9 10

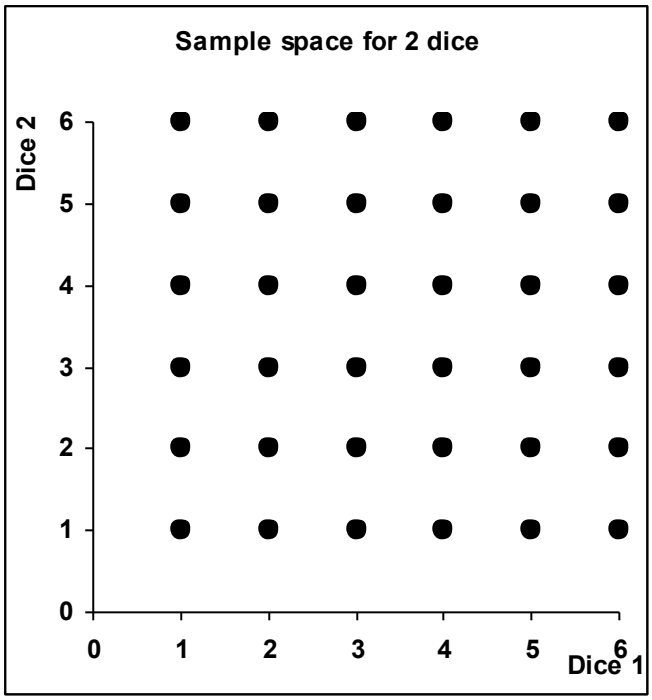
Probability experiments: Sample spaces

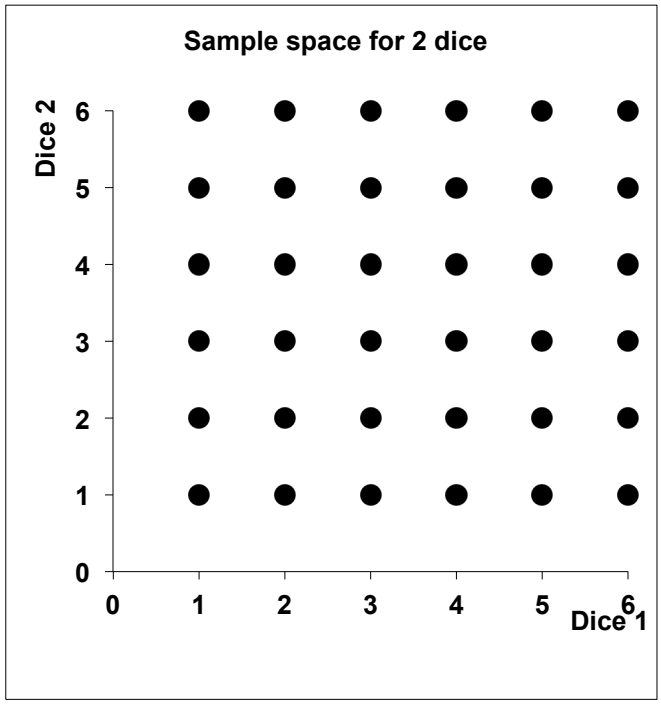

Sample space for 2 coins


Sample space for 2 coins


Sample space for 2 dice


Sample space for 2 dice


Sample space for 2 dice


Sample space for 2 dice

Probability experiments

TYPE:	Main
OBJECTIVE(S):	Probability from experiments; repeating experiments gives different outcomes; and more generally means better probability estimates.
DESCRIPTION:	1 is experimental probability about football goals. 2 is about a dice and coin experiment. 3 is the difference between two dice.
OVERVIEW:	Experimental probability for two objects.
EQUIPMENT:	It often helps to have coins or dice available so that pupils might consider all the different combinations. A photocopy master is available, it contains copies of sample space diagrams for one coin and one dice and two dice.

TYPE:	Plenary
OBJECTIVE(S):	Probability from experiments; repeating experiments gives different outcomes; and more generally means better probability estimates.
DESCRIPTION:	1, 2 are about a coin and dice. 1 is theoretical probability. 2, 3 are experimental probability. 3 with two dice. 4 is vocabulary.
OVERVIEW:	Experimental probability for two objects.
EQUIPMENT:	None.

Main Whiteboard and Screen information

Football goals

The total number of goals scored in each football match for one team in a season is given in the table below. Click Next to continue.

Pen on

Number of goals per game	Frequency
0	2
1	13
2	7
3	5
4	8
5	1
6	1
7	1
8	0
Total	38

7 8 9
4 5 6
1 2 3
0 . (-)
n + -
Clear

Next
Reset

1 2 3

Screen 1: Football goals

A table on screen shows the frequency of goals scored per game for a season for a football team (the data used are taken from premiership results for 2002-03). You are then asked a probability question based on the table.

Key points: pupils should discuss how they might interpret the table; pupils should consider what might influence the result of an individual game and consider when and how this would influence any probabilities; also pupils should consider how this probability is different from theoretical probability (arising from, for example, the symmetry of an object); you could ask pupils to find other similar sorts of probabilities (you could do this for one weekend during the football season).

The dice and coin experiment

The table shows the recorded scores for 100 throws of the coin and dice. What do you expect to happen if the number of throws is increased? Explain. Click Probability to see the experimental probability results. Click Next for another 100 throws.

Pen on

Score	1	2	3	4	5	6	7	8	9	10	11	12	Total
Frequency	12	18	2	15	13	17	0	11	0	8	0	4	100

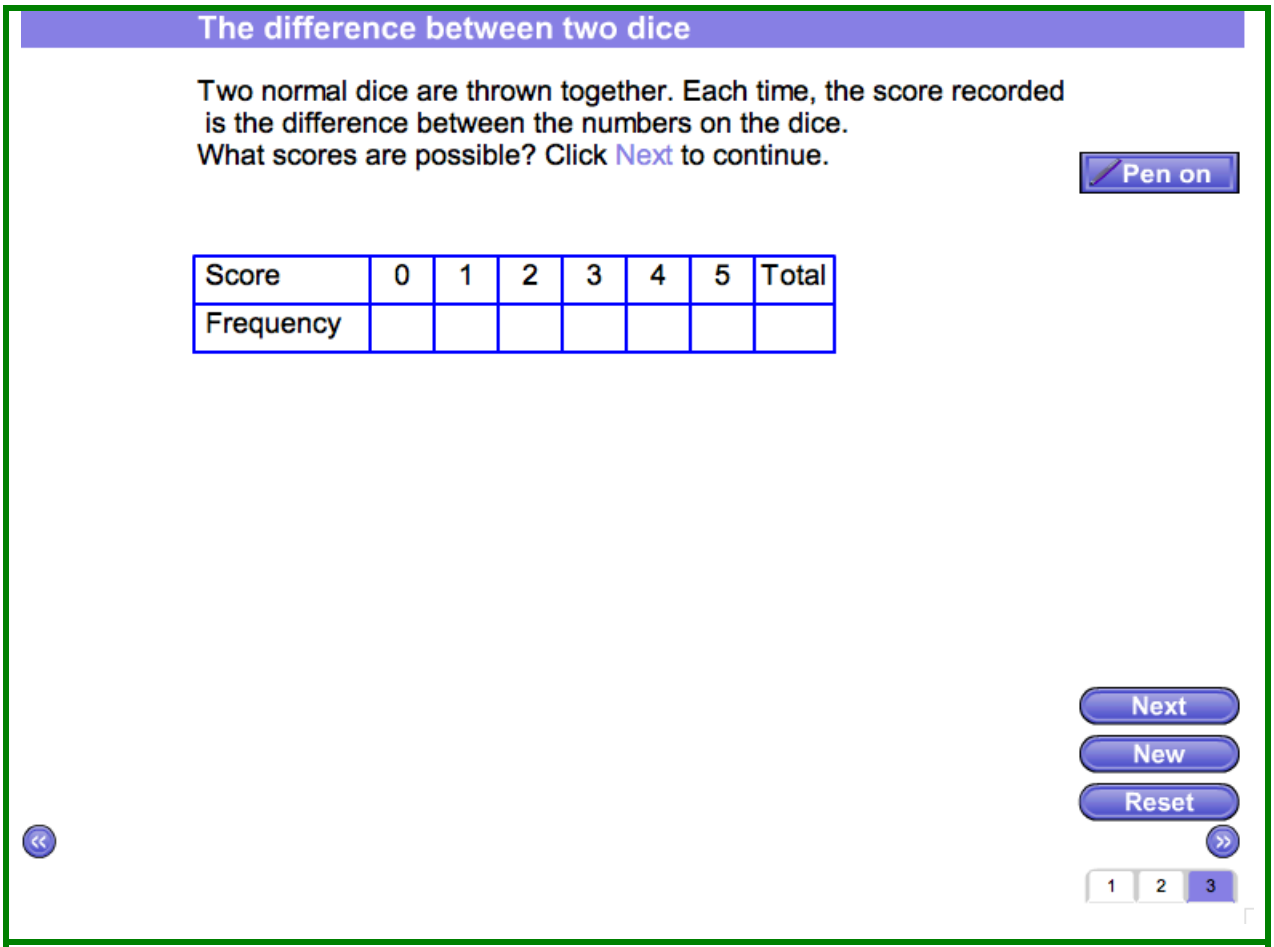
Probability
Next
New
Reset

1 2 3

Screen 2: The dice and coin experiment

A coin and dice are thrown together. If the coin lands heads the score recorded equals the dice number but if it lands tails the score recorded is double the number on the dice. Pupils are asked what scores are possible. A table is then shown with rows of possible scores from 1 to 12 (all numbers included) and frequency. Each time Next is clicked another 100 throws are shown, up to a maximum of 1200. At any point it is possible to show the experimental probability, which may change as the next 100 are added.

Key points: pupils should discuss which scores are likely and then consider an appropriate sample space diagram, converting each outcome to a score; pupils should consider why the 9 possible outcomes are not equally likely; you might wish pupils to keep note of the experimental probabilities after the first 100 throws and compare these with the experimental probabilities after 1200 throws; you may wish to ask pupils to predict the 'final' experimental probability (after 1200 throws) after 100 throws - it may reveal misconceptions (pupils may find it difficult understanding that as the number of throws increases the experimental probabilities should get closer to the theoretical probabilities).



Screen 3: The difference between two dice

Two dice are thrown together. Each time the score to be recorded is the difference between the two numbers on the dice. Pupils are asked what scores are possible. A table is then shown with rows of possible scores from 0 to 5 (all numbers included) and frequency. Each time Next is clicked another 100 throws are shown, up to a maximum of 1200. At any point it is possible to show the experimental probability, which may change as the next 100 are added.

Key points: pupils should discuss which scores are likely and then consider an appropriate sample space diagram, converting each outcome to a score; pupils should consider why the 6 possible outcomes are not equally likely; you might wish pupils to keep note of the experimental probabilities after the first 100 throws and compare these with the experimental probabilities after 1200 throws; you may wish to ask pupils to predict the 'final' experimental probability (after 1200 throws) after 100 throws - it may reveal misconceptions (pupils may find it difficult understanding that as the number of throws increases the experimental probabilities should get closer to the theoretical probabilities).

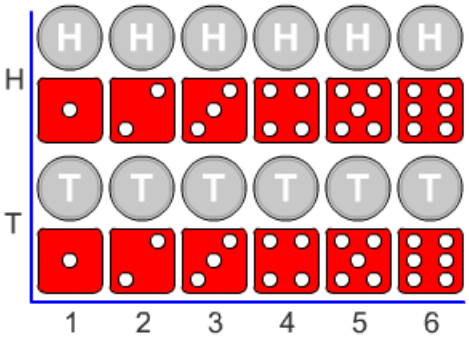
Plenary Whiteboard and Screen information

One coin and one dice: theoretical probability

Click a number between 2 and 12 in the table on the right to see the theoretical probability of that score. The score will be highlighted in the sample space. Click Show to see all of the theoretical probabilities. Click New to start again.

Pen on

Coin and dice sample space



Sample space showing scores

H	1	2	3	4	5	6
T	2	4	6	8	10	12
	1	2	3	4	5	6

Total score	Probability
1	
2	
3	
4	
5	$\frac{1}{12}$
6	
8	
10	
12	

Show

New

Reset

1 2 3 4

Screen 1: One coin and one dice: theoretical probability

A coin and dice are thrown together. If the coin lands heads the score recorded equals the dice number but if it lands tails the score recorded is double the number on the dice. Pupils are shown the sample space for this, first showing the outcomes (for example Head with a 6) and then showing the scores. On the right of the screen is a table that contains the possible 'Total scores' from 1 to 12 and a blank column where the probabilities will be placed. When you click a number in the table a corresponding probability is put in place (in twelfths and simplest form) and the numbers in the sample space are shown selected. Any number of 'Total scores' can be selected or de-selected (by a second click). A Show option puts all the (theoretical) probabilities into the table.

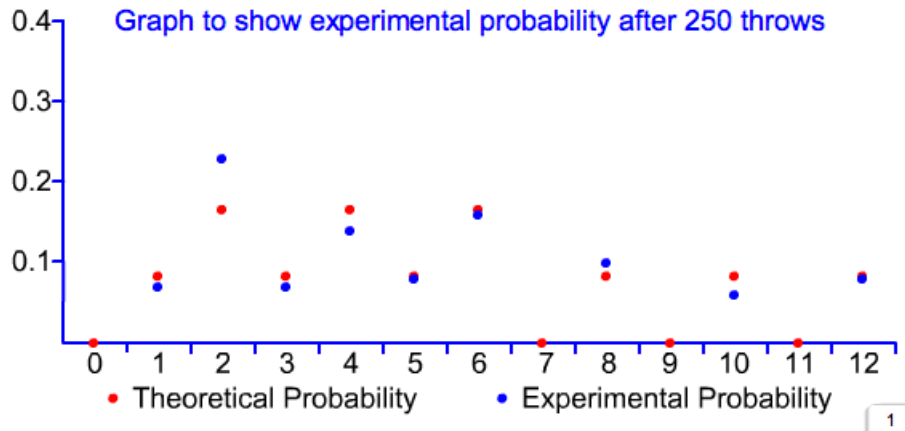
Key points: pupils should discuss the advantage of the sample space diagram and why each outcome shown in it is equally likely; you should consider with your pupils the difference between theoretical and experimental probability and which of them is 'right'.

One coin and one dice: experimental probability

The table shows the recorded scores for 250 throws of the coin and dice, and the graph shows the experimental probabilities for each score. What do you expect to happen if the number of throws is increased? Explain. Click Next for another 250 throws.

Pen on

Score	1	2	3	4	5	6	7	8	9	10	11	12	Total
Frequency	18	58	18	35	21	39	0	25	0	15	0	21	250
Experimental probability	0.07	0.23	0.07	0.14	0.08	0.16	0	0.1	0	0.06	0	0.08	1



Next
New
Reset

1 2 3 4

Screen 2: One coin and one dice: experimental probability

A coin and dice are thrown together. If the coin lands heads the score recorded equals the dice number but if it lands tails the score recorded is double the number on the dice. A coin and dice are thrown together. If the coin lands heads the score recorded equals the dice number but if it lands tails the score recorded is double the number on the dice. A table is then shown with rows of possible scores from 1 to 12 (all numbers included) and frequency together with the experimental probability for that number of throws. A graph of the experimental probability for each of these scores is also shown, linked to the table. Each time Next is clicked another 250 throws are shown, up to a maximum of 3000.

Key points: pupils should consider why the 9 possible outcomes are not equally likely; you may wish pupils to watch how the experimental probabilities change and compare these with the experimental probabilities after 3000 throws, and then project to many more throws; you may wish to ask pupils to predict the 'final' experimental probability; you should consider with your pupils the difference between theoretical and experimental probability and which of them is 'right'.

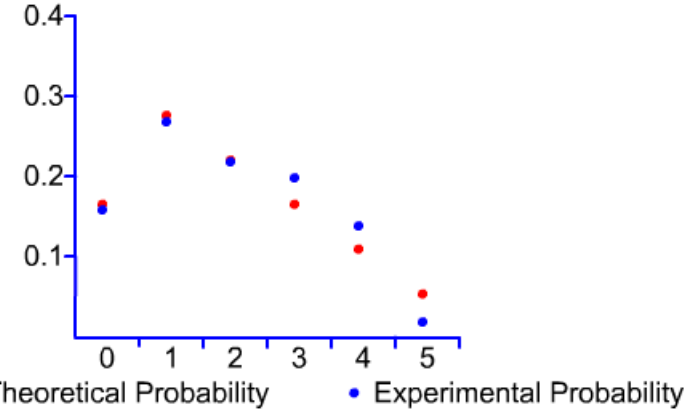
The difference between two dice

The table shows the recorded scores for 250 throws of the dice, and the graph shows the experimental probabilities for each score. What do you expect to happen if the number of throws is increased? Explain. Click Next for another 250 throws.

Pen on

Score	0	1	2	3	4	5	Total
Frequency	39	67	55	49	34	6	250
Experimental probability	0.16	0.27	0.22	0.2	0.14	0.02	1

Graph to show experimental probability after 250 throws



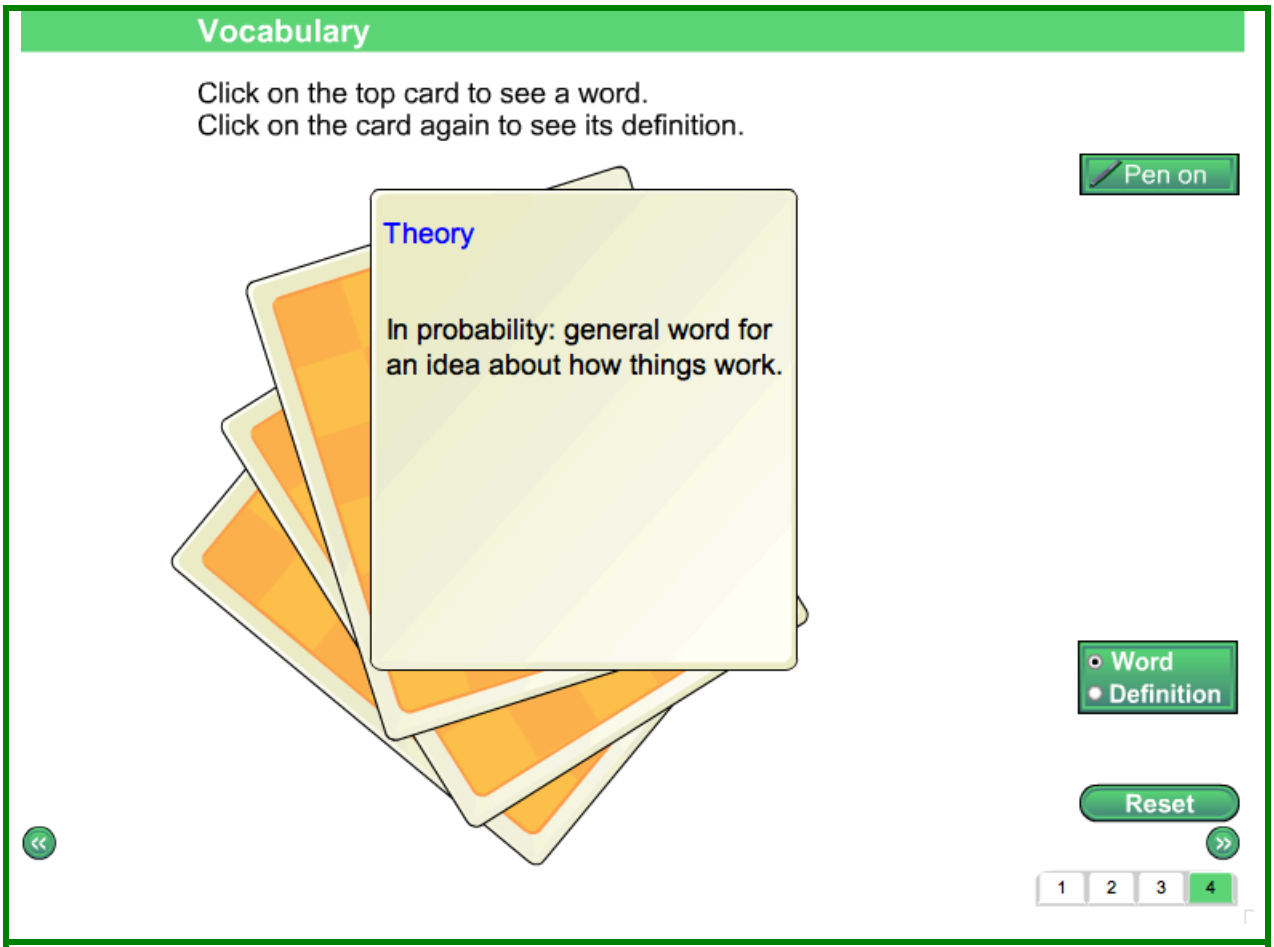
Next
New
Reset

1 2 3 4

Screen 3: The difference between two dice

Two dice are thrown together. Each time the score to be recorded is the difference between the two numbers on the dice. A table is then shown with rows of possible scores from 0 to 5 (all numbers included) and frequency together with the experimental probability for that number of throws. A graph of the experimental probability for each of these scores is also shown, linked to the table. Each time Next is clicked another 250 throws are shown, up to a maximum of 3000.

Key points: pupils should consider why the 6 possible outcomes are not equally likely; you may wish pupils to watch how the experimental probabilities change and compare these with the experimental probabilities after 3000 throws, and then project to many more throws; you may wish to ask pupils to predict the 'final' experimental probability; you should consider with your pupils the difference between theoretical and experimental probability and which of them is 'right'.



Screen 4: Vocabulary

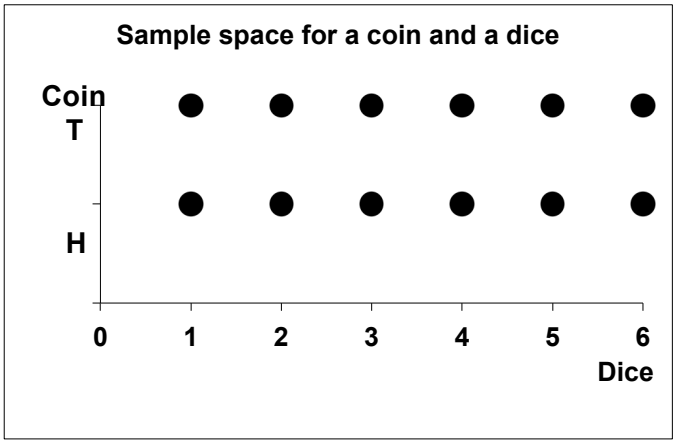
Vocabulary present: Biased, Certain, Chance, Doubt, Equally likely, Even chance, Event, Fair, Fifty-fifty chance, Good chance, Impossible, Interval, Likelihood, Likely, No chance, Outcome, Poor, Possible, Probability, Probable, Random, Risk, Sample, Sample space, Statistic, Theory, Uncertain, Unfair, Unlikely.

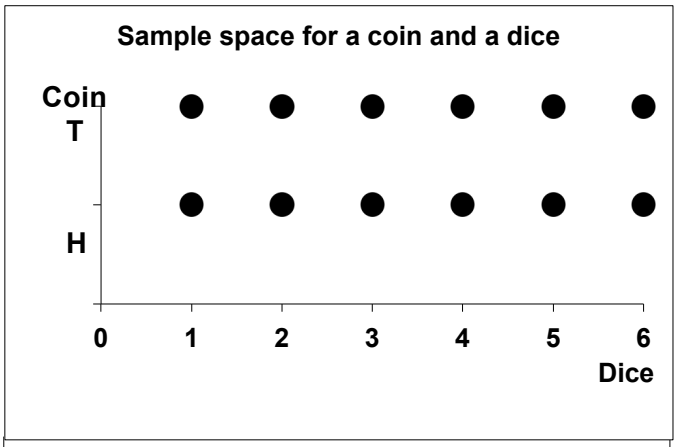
Spire Maths interactive files available in a flash format at: https://spiremaths.co.uk/ia/

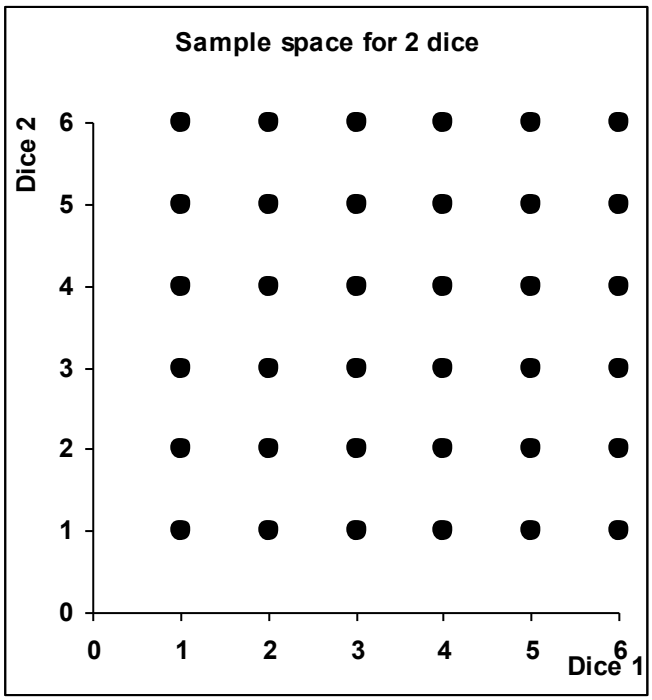
Unfortunately they will not work on iPads or iPhones.

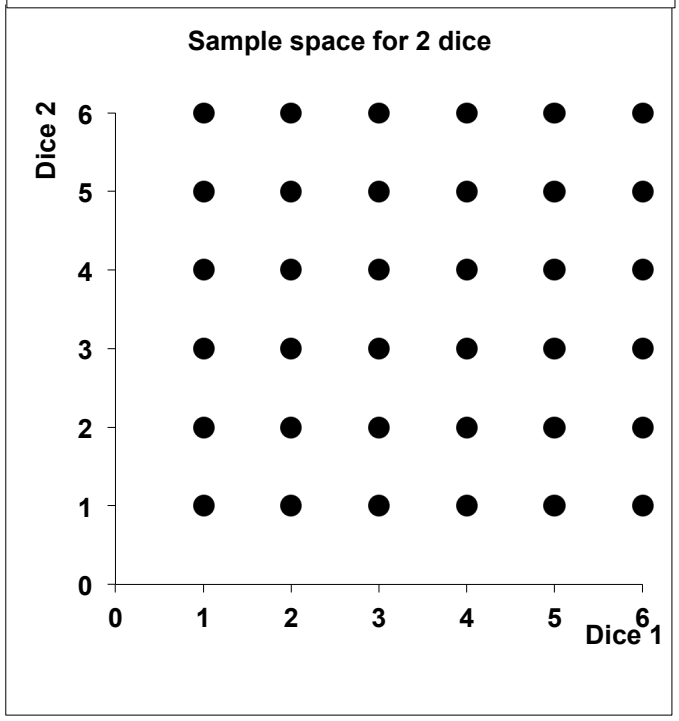
Probability experiments

Sample spaces









Assorted probability

TYPE:	Main
OBJECTIVE(S):	Compare experimental and theoretical probabilities in different contexts.
DESCRIPTION:	1looks at theoretical and experimental probability with dominoes. 2 does the same for an unfair spinner.
OVERVIEW:	Theoretical, experimental probability - dominoes, spinner.
EQUIPMENT:	A set of dominoes or spinners (protractors and cocktail sticks, or blunt, dead matches) and scissors. Two photocopy masters are provided, one contains a set of dominoes, the other a set of spinners.

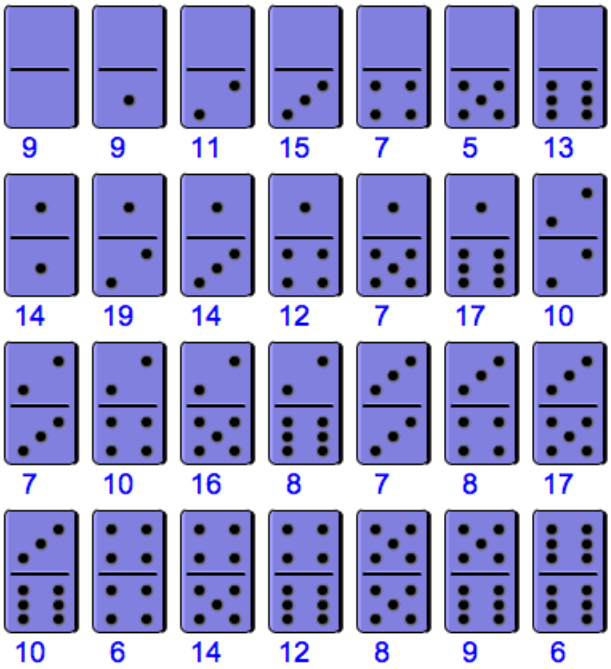
TYPE:	Plenary
OBJECTIVE(S):	Compare experimental and theoretical probabilities in different contexts.
DESCRIPTION:	1 looks at theoretical and experimental probability with dominoes. 2 does the same for an unfair spinner. 3 is vocabulary.
OVERVIEW:	Theoretical, experimental probability - dominoes, spinner.
EQUIPMENT:	None.

Main Whiteboard and Screen information

Dominoes

The table shows the results for 300 selections. Click 100 more to see the results for another 100 selections. Click New to start again.

Pen on



Total	F
0	9
1	9
2	25
3	34
4	31
5	24
6	37
7	41
8	31
9	24
10	20
11	9
12	6
Total	300

100 more
New
Reset

1 2

Screen 1: Dominoes

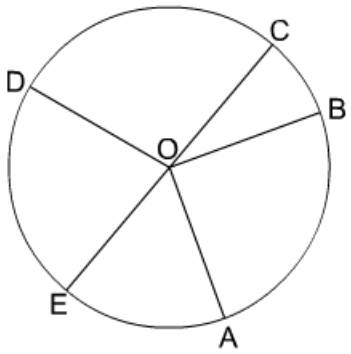
You are told that a set of dominoes is placed in a bag, one is to be drawn out, the spots counted and then it is to be replaced. You are asked what totals can be made. Click Next and you will see the 28 dominoes arranged in a 4 by 7 array (not in any order). Another click of Next and they are put into a systematic order, starting from double blank, then blank one, blank two and so on until double five, five six and double six. On Next again one is drawn out (shown by a red surround and the number one underneath it). At the same time a table with all the totals (0 to 12) is shown with a 1 shown in the appropriate column. You can then click Next to collect 99 more results, one at a time, or you can collect all 100 in one go. After this you can collect another 500 in 100s.

Key points: some pupils may not know what dominoes are, so you may wish to let them find out how many there are and what the totals add up to (if necessary in advance of the lesson); for some pupils it will help to have a set of dominoes that can be physically moved; once pupils have established the set, they should discuss the different possible totals and look to see which are most common; when the table is shown pupils often believe that the probability of each number is one thirteenth - it helps if they discuss what they expect to happen after 1 or 2 throws and then compare this with what happens after 600 throws; the number of 'draws' of the individual dominoes is also shown and these will usually be in the range 12 to 31 (600/28 is about 21.5).

An unfair spinner

You can make a biased spinner and compare the results obtained with those from a computer simulation. Start by drawing a circle and then draw lines from its centre to the circumference at the angles shown. Click Next to continue.

Pen on



$\angle AOB = 90^0$
$\angle BOC = 30^0$
$\angle COD = 100^0$
$\angle DOE = 80^0$
$\angle EOA = 60^0$

Next
Reset

1 2

Screen 2: An unfair spinner

You are shown how to make a 'biased' circular spinner with 5 sectors, where the sector angles are, in order, 90, 30, 100, 80 and 60 degrees. This can be cut out a cocktail stick pushed through and it can then be spun. You are then given a table of results for 50 'spins' of a computer simulation of such a spinner. You can then collect in the table up to 600 spins, going up in 50s. You are then asked to compare the theoretical and experimental probabilities using the table of results.

Key points: it helps if pupils collect their own results for this experiment (blunt, dead matches can be used); pupils will have to decide how the spinner lands and will have to make sure the spinner is level when it starts spinning; the accuracy of their results will need to be considered; many pupils may have difficulty deciding how to 'apportion' the probability and should be given time to discuss the different ways it might be done (five sectors, one fifth each will be a common answer).

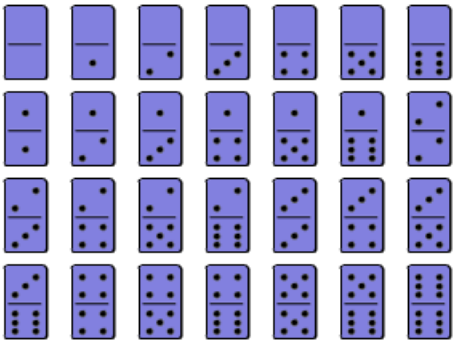
Plenary Whiteboard and Screen information

Dominoes

Click a number between 0 and 12 in the table on the right to see the theoretical probability of that score. The score will be highlighted in the sample space. Click Show to see all of the theoretical probabilities. Click New to start again.

Pen on

Domino sample space



Sample space showing totals

0	1	2	3	4	5	6
2	3	4	5	6	7	4
5	6	7	8	6	7	8
9	8	9	10	10	11	12

Total	Probability
0	
1	
2	
3	
4	
5	$\frac{3}{28}$
6	
7	
8	
9	
10	
11	
12	
Total	

Show

New

Reset

1 2 3

Screen 1: Dominoes

You are told that a set of dominoes is placed in a bag, one is to be drawn out, the spots counted and then it is to be replaced. You are asked what totals can be made. Click Next and you will see the 28 dominoes arranged in a 4 by 7 array in a systematic order, starting from double blank, then blank one, blank two and so on until double five, five six and double six. Another Next and you see a sample space where all the dominoes are replaced with a total (the sum of the dots on the domino). A further click of Next lets you see all the possible totals (0 to 12) in a table. When you click a number in the table a corresponding probability is put in place (in 28ths and simplest form) and the numbers in the sample space are ringed. Any number of 'Total scores' can be selected or de-selected (by a second click). A Show option puts all the probabilities into the table.

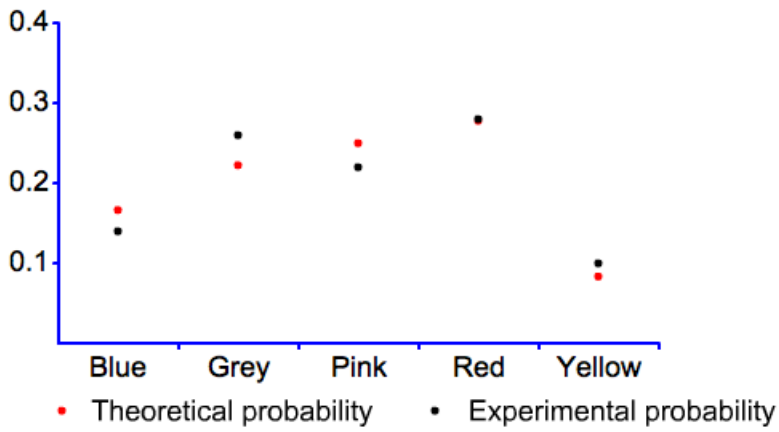
Key points: some pupils may think that probabilities will be twelfths or thirteenths and this should be discussed; some pupils might still think that it is harder to get a double six than any other double; some pupils may still not be familiar with dominoes.

An unfair spinner

The table shows the results of a computer simulation of 50 spins of the spinner. EP stands for experimental probability. Click Next to simulate another 50 spins.

Pen on

Colour	Blue	Grey	Pink	Red	Yellow	Total
TP	$\frac{60}{360} = \frac{1}{6}$	$\frac{80}{360} = \frac{2}{9}$	$\frac{90}{360} = \frac{1}{4}$	$\frac{100}{360} = \frac{5}{18}$	$\frac{30}{360} = \frac{1}{12}$	1
F	7	13	11	14	5	50
EP	0.14	0.26	0.22	0.28	0.1	1



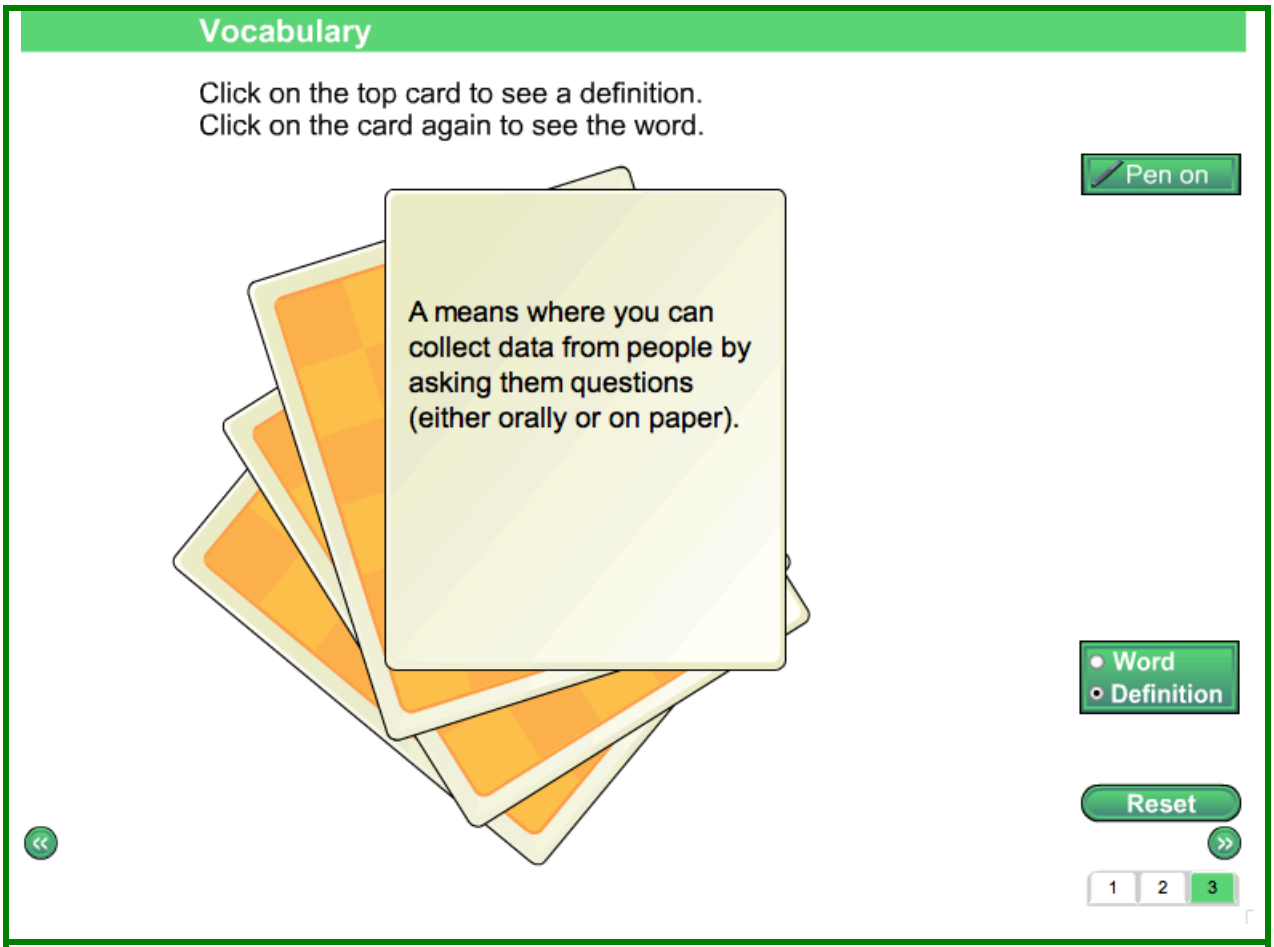
Next
Reset

1 2 3

Screen 2: An unfair spinner

You are shown how to make a 'biased' circular spinner with 5 sectors, where the sector angles are, in order, 90, 30, 100, 80 and 60 degrees. This can be cut out a cocktail stick pushed through and it can then be spun. You are asked for the theoretical probability and this is then shown in a table and on a graph. You are then given a table of results for 50 'spins' of a computer simulation of such a spinner together with the corresponding experimental probabilities, and these probabilities are also shown on the graph. You can keep collecting results, in fifties, until you reach 600 spins.

Key points: pupils should consider why the 5 possible outcomes are not equally likely; you may wish pupils to watch how the experimental probabilities change and compare these with the experimental probabilities after each set of throws; you may wish to ask pupils to predict the 'final' experimental probability; you should consider with your pupils the difference between theoretical and experimental probability and which of them is 'right'.



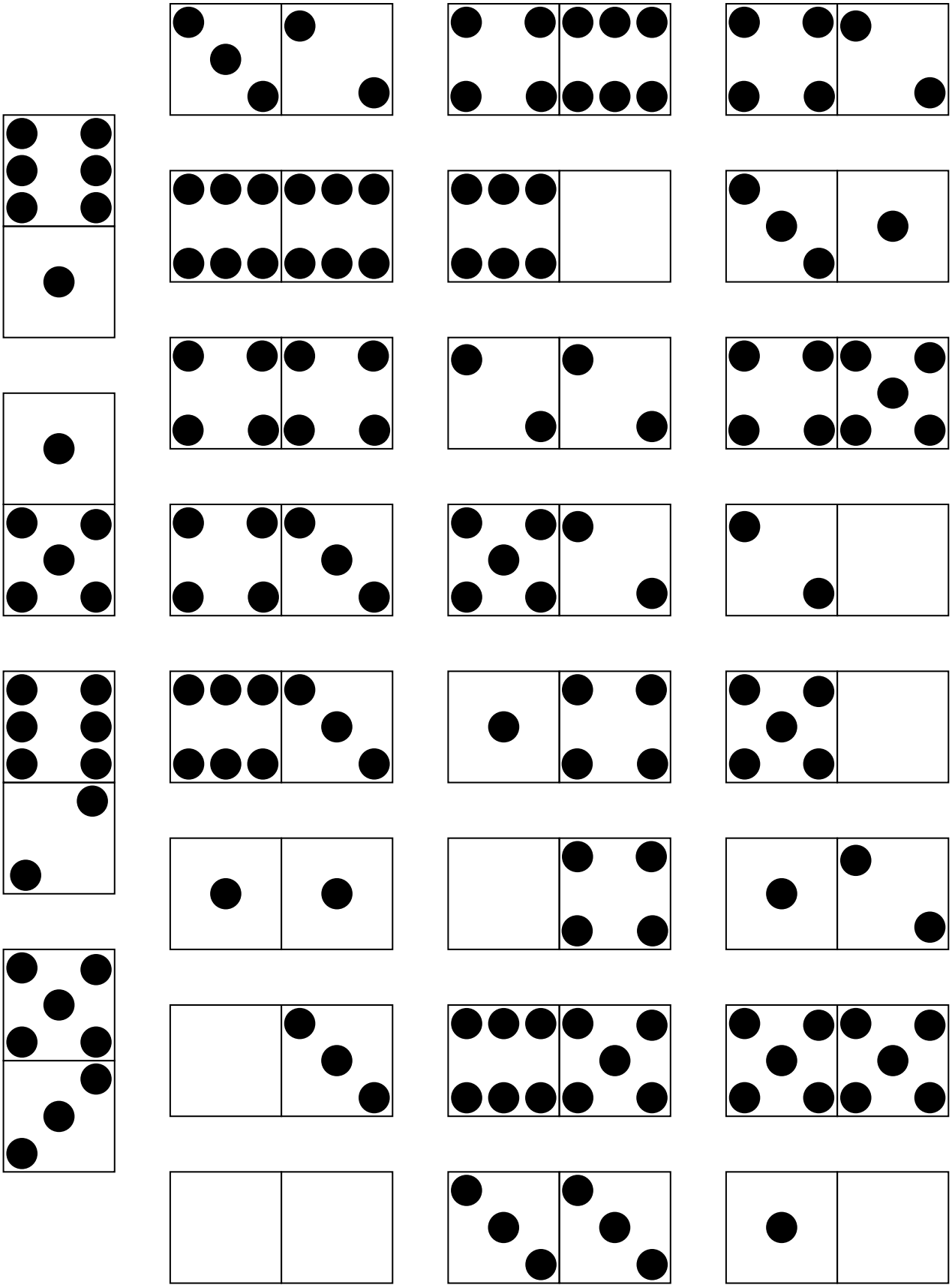
Screen 3: Vocabulary

Vocabulary present: Average, Class interval, Continuous, Data, Data collection sheet, Data log, Database, Discrete, Experiment, Frequency, Grouped data, Mean, Median, Modal class/group, Mode, Primary source, Questionnaire, Range, Sample, Secondary source, Statistics, Stem-and-leaf diagram, Survey, Table, Tally, Two-way table.

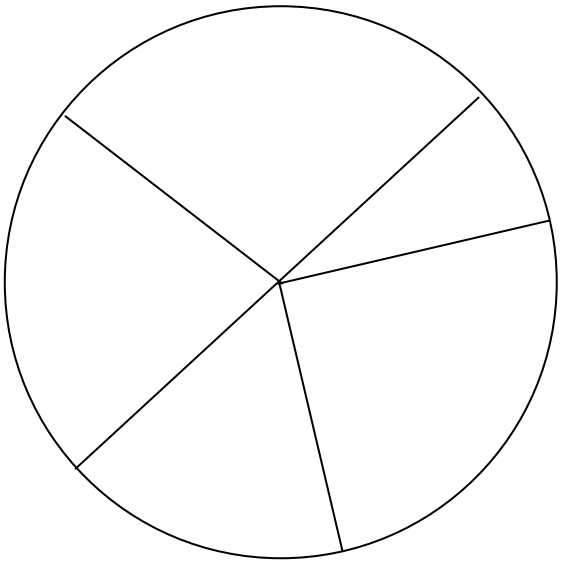
Spire Maths interactive files available in a flash format at: https://spiremaths.co.uk/ia/

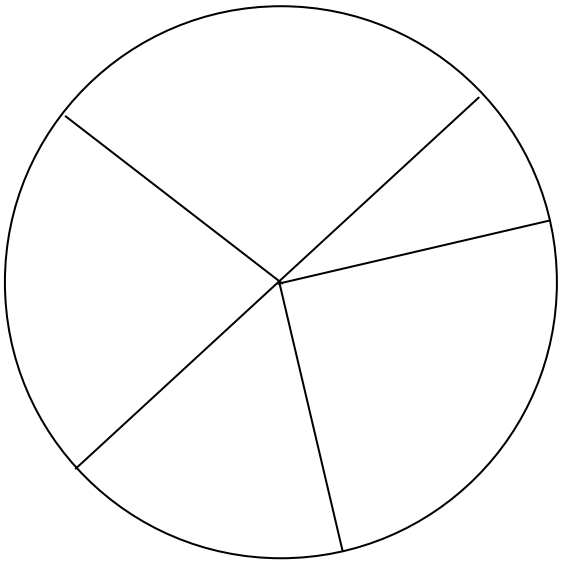
Unfortunately they will not work on iPads or iPhones.

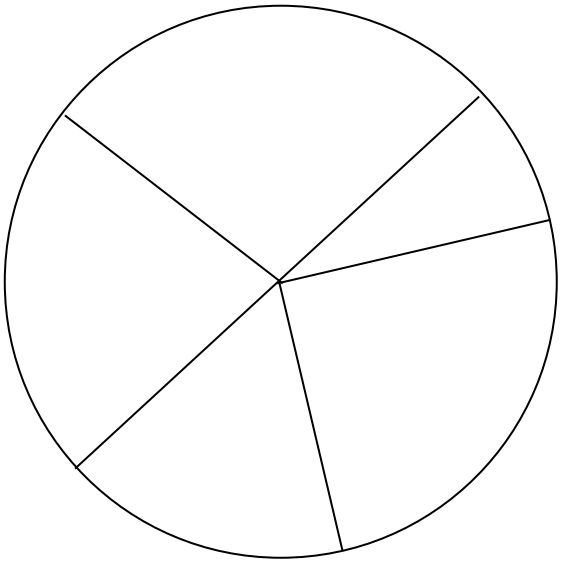
Dominoes

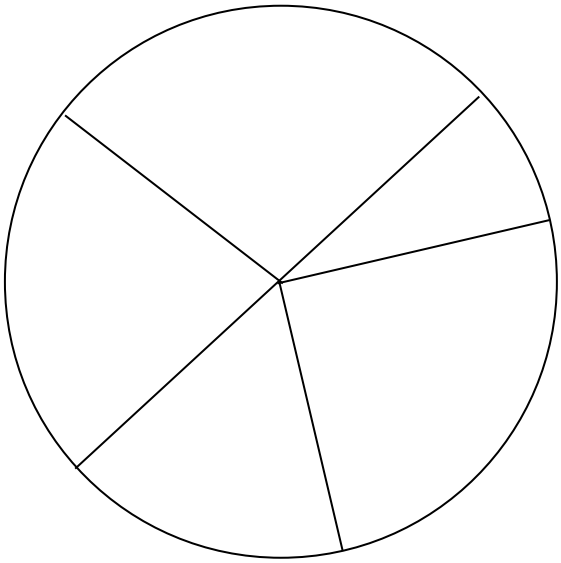


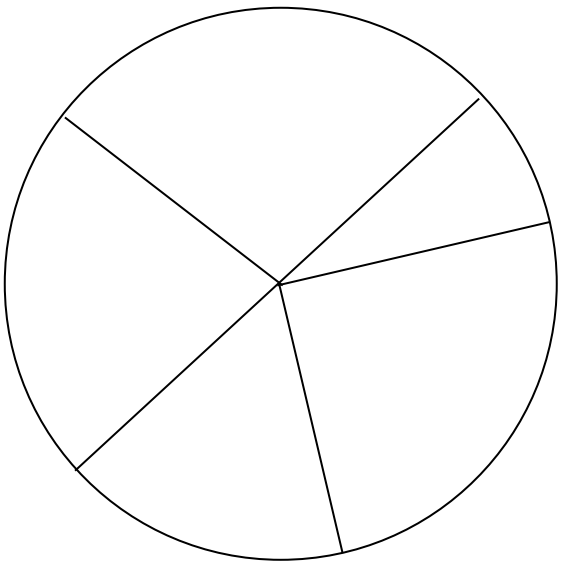
Spinners

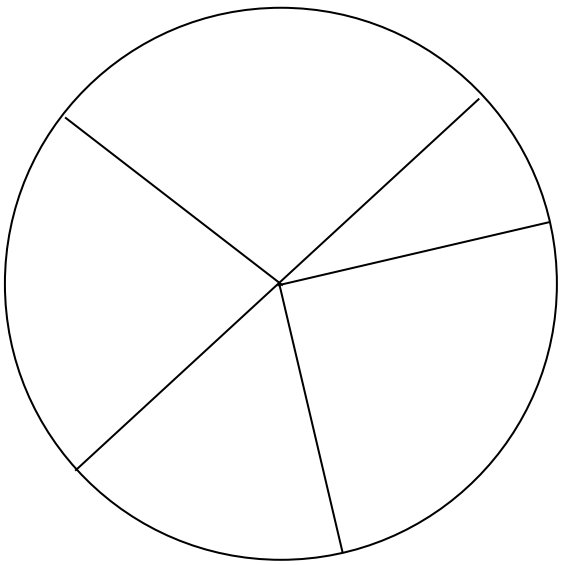












Probability statements and fruit

TYPE:	Main
OBJECTIVE(S):	Use the vocabulary of probability in interpreting results involving uncertainty and prediction.
DESCRIPTION:	1 looks at probability statements. 2 compares probabilities.
OVERVIEW:	Discuss probability statements
EQUIPMENT:	None.

TYPE:	Plenary
OBJECTIVE(S):	Use the vocabulary of probability in interpreting results involving uncertainty and prediction.
DESCRIPTION:	1 is probability statements. 2 make sample to fit probability. 3 is SATs page. 4 is vocabulary.
OVERVIEW:	Discuss probability statements
EQUIPMENT:	None.

Main Whiteboard and Screen information

Probability statements

Look at the statement below. Discuss this statement. Do you agree or disagree with it? Give reasons for your answer. Click New for another statement.

Pen on

The probability of picking a red counter from a bag is 0.6.
If I add more blue counters this probability will go down.

New

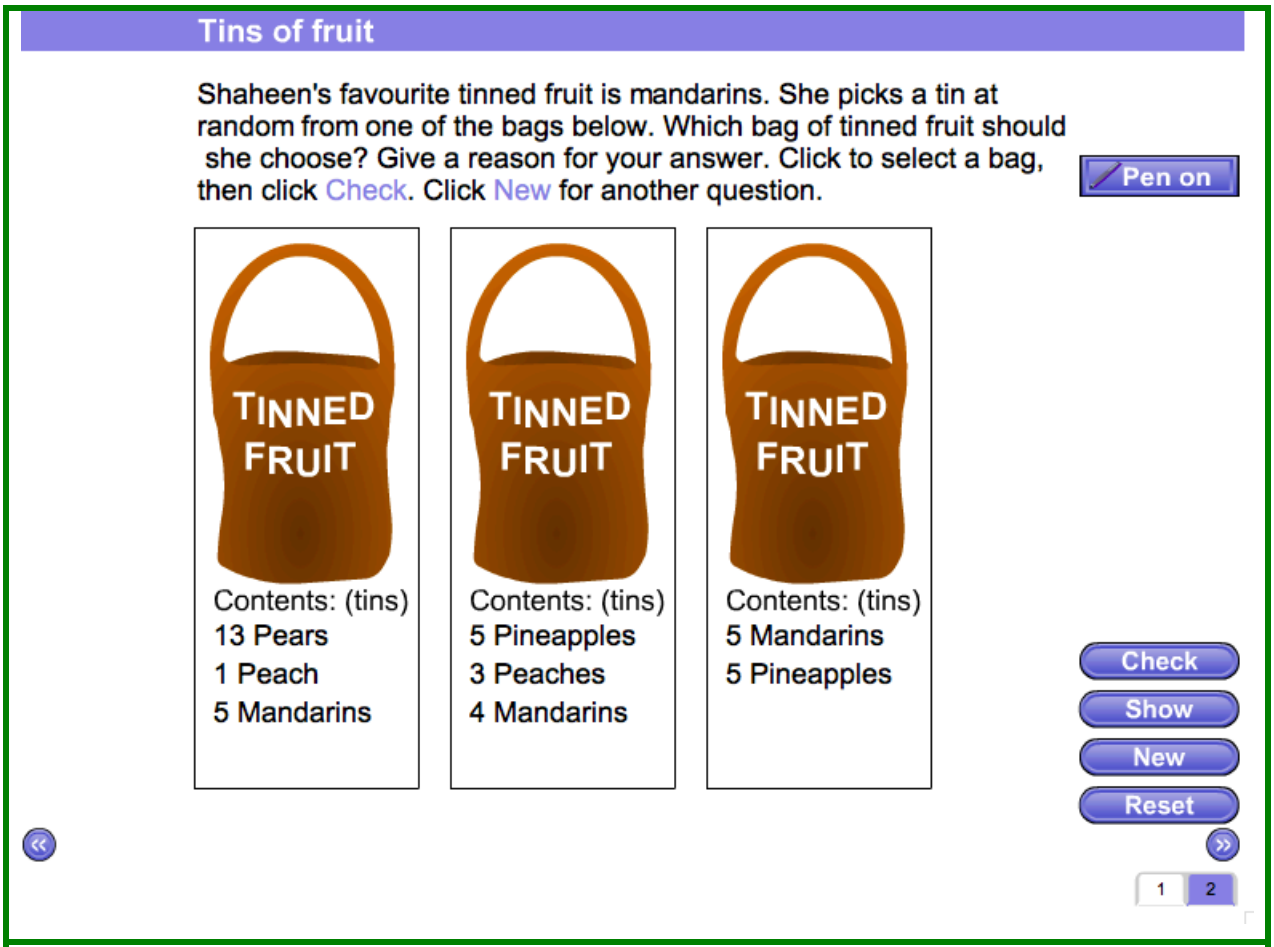
Reset

1 2

Screen 1: Probability statements

A statement about probability or chance is shown and you are asked to discuss the statement and state whether you agree or disagree with it, giving reasons for your answer.

Key points: pupils should give reasons rather than just agree or disagree; many of these include standard misconceptions and so should provoke discussion.

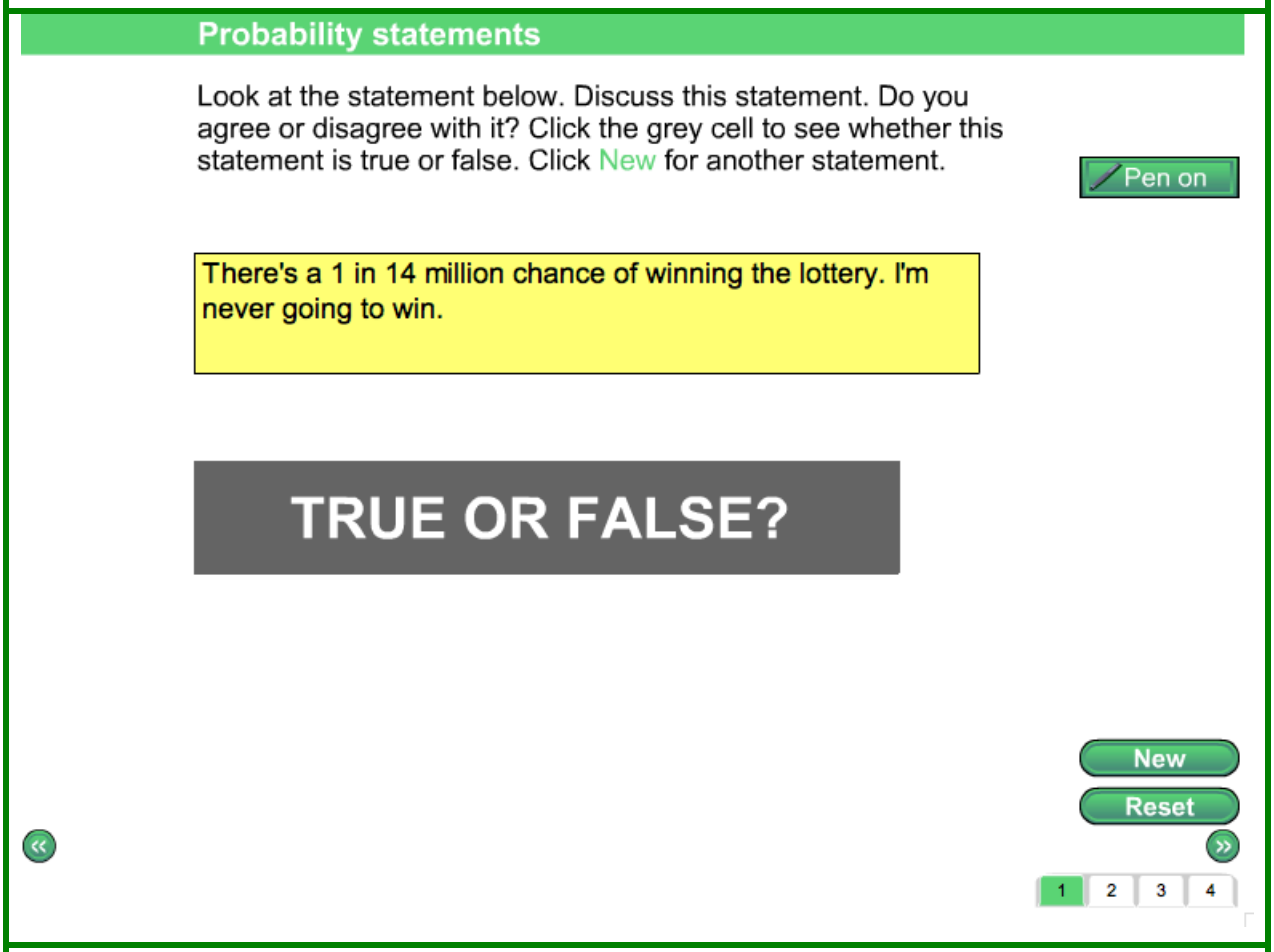


Screen 2: Tins of fruit

You are told that a person's favourite tinned fruit is, for example, pears and that the person can pick at random one tin of fruit from a bag containing tins of fruit. You are told the contents of three bags, shown on screen, in terms of the types of tins of fruit it contains and asked to decide which bag the person should choose from. You click the bag and are told if you are correct. Finally you are asked to explain the probabilities for picking a particular tin of fruit.

Key points: pupils need to discuss the probabilities of the event of picking a particular tin of fruit and then compare the fractions or percentages; pupils may need to be reminded about comparing fractions, but in some cases the answer can be found without 'directly using' equivalent fractions; encourage pupils to be precise in their reasoning.

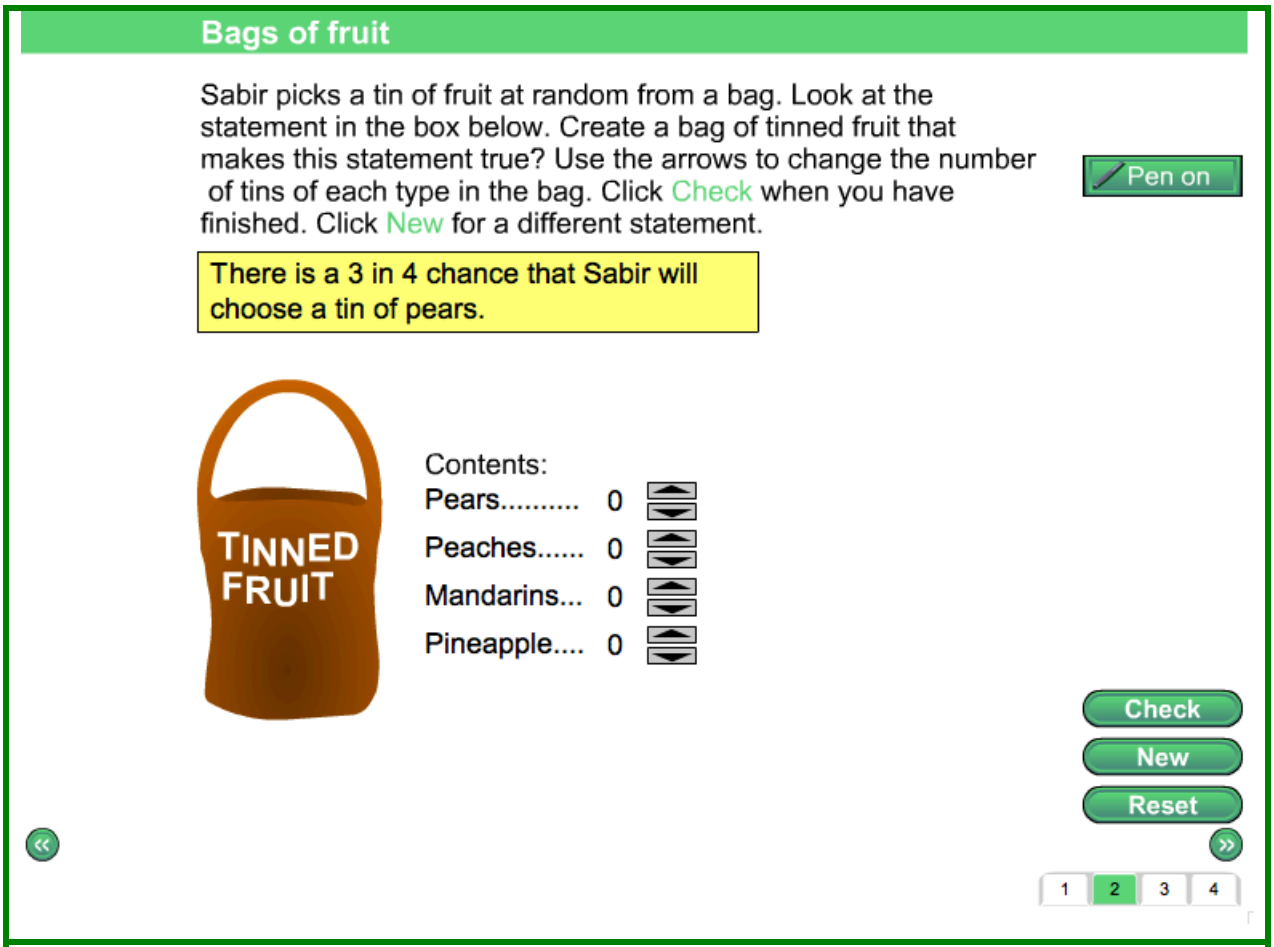
Plenary Whiteboard and Screen information



Screen 1: Probability statements

A statement about probability or chance is shown and you are asked to discuss the statement and state whether you agree or disagree with it, giving reasons for your answer. You can then click a true/false cell to see if it is true or false – a reason is given with this.

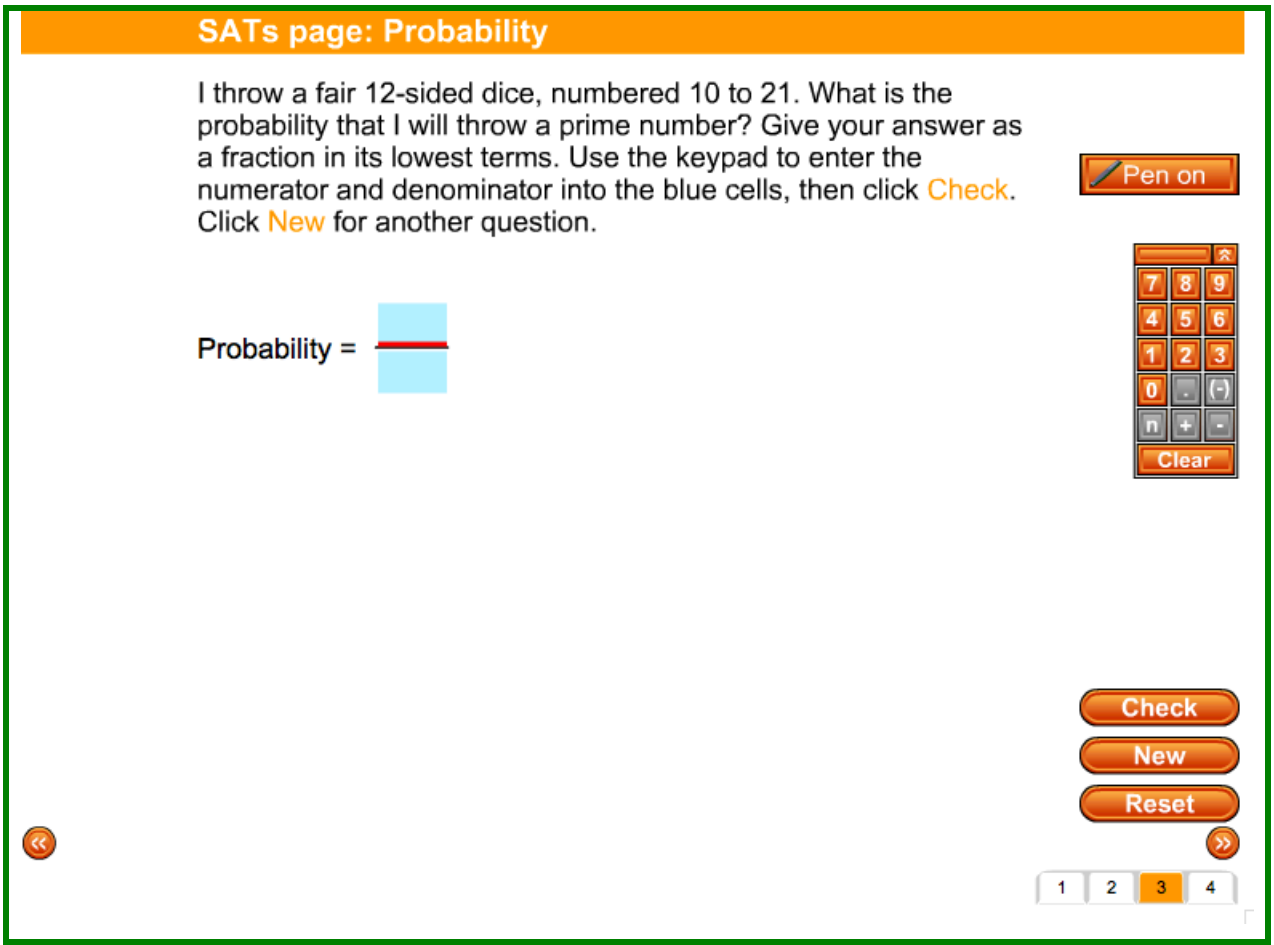
Key points: it is important that pupils discuss reasons and compare them with the reason given.



Screen 2: Tins of fruit

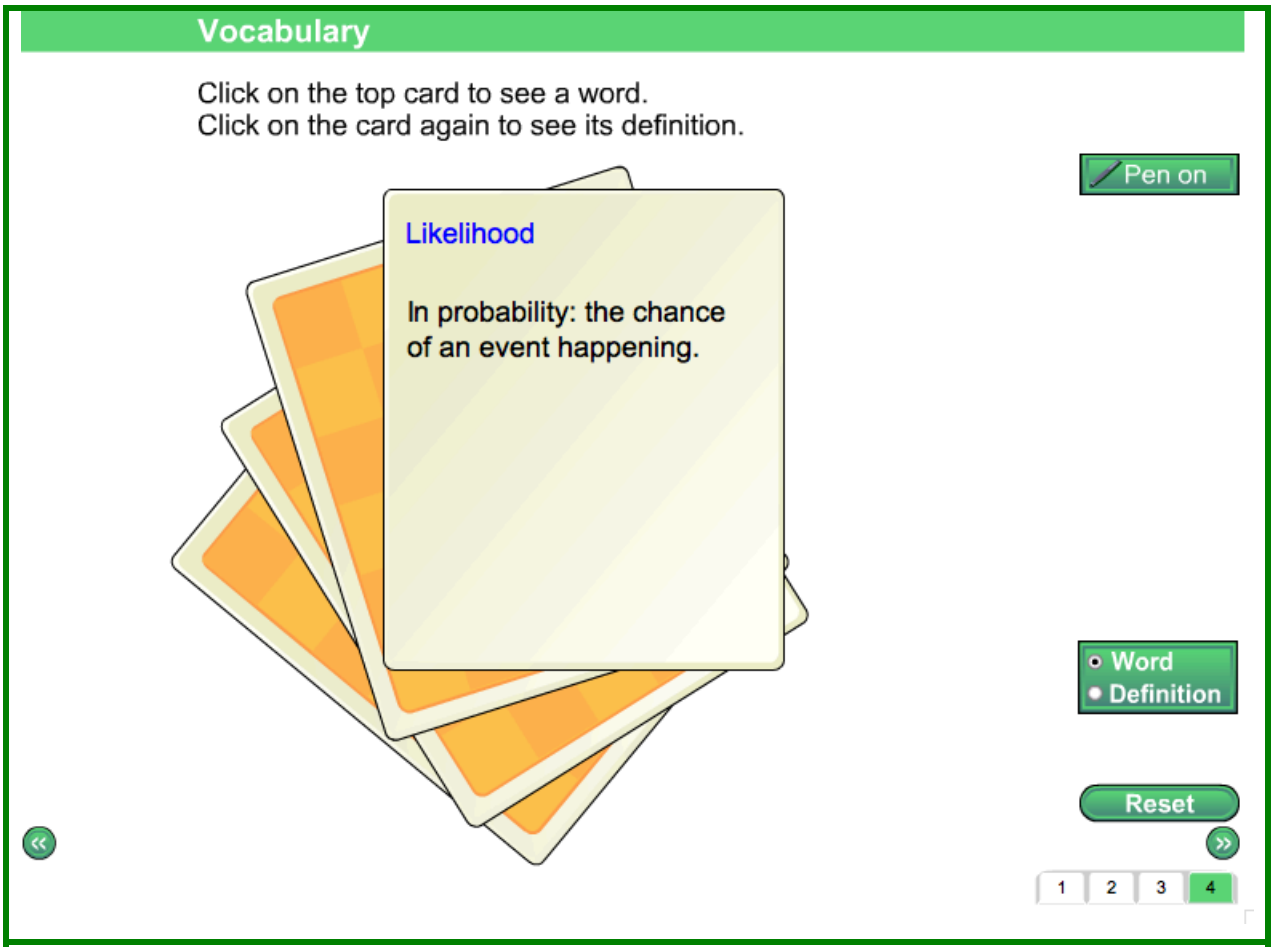
You are told that a person picks a tin of fruit at from a bag containing tins of fruit. A statement of likelihood or chance is shown in a yellow box about what the person will choose (for example "Liam is more likely to choose a tin of peaches than a tin of pears"). You are asked to create a bag for which the statement in the yellow box will be true by using spinners to add tins of fruit to the bag. Once correct you are asked to explain why your answer is correct.

Key points: pupils will fix the spinners by considering numbers of tins of fruit of each sort rather than use probabilities; on questions that for example involve, for example, a two in three chance pupils are likely to pick two of one type of tinned fruit and three of another, let them do this – since this common problem needs to be considered; encourage pupils to explain why they choose particular values and to be precise in their reasoning.



Screen 3: SATs page: Probability

A fair dice that has n sides is thrown (n is 6, 8, 10, 12 or 20). It is numbered with n consecutive numbers. You are asked for the probability that a prime number will be thrown. You have to give the result as a fraction in its lowest terms.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Chance, Doubt, Equally likely, Even chance, Event, Exhaustive, Fair, Fifty-fifty chance, Good chance, Impossible, Independent, Likelihood, Likely, Limit, Mutually exclusive, No chance, Outcome, p(n), Poor, Possible, Probability, Probable, Random, Relative frequency, Risk, Sample, Sample space, Theory, Tree diagram, Uncertain, Unfair, Unlikely.

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Probability and octagonal spinners

TYPE:	Main
OBJECTIVE(S):	Use the vocabulary of probability in interpreting results. Use estimates of probability. Consider outcomes and sample spaces.
DESCRIPTION:	1 is choosing a rule. 2 is choosing a spinner. 3 is your own spinner.
OVERVIEW:	Game like probability work with octagonal spinners
EQUIPMENT:	Spinners - pupils could make these from regular octagons (on card) with a cocktail stick through the middle (or used matches) - though you would need to take an appropriate risk assessment - these work for a small number of spins. You could also use octahedral dice. Three photocopiable masters are available: one is a set of octagons; the other two comprise data collection sheets for the spinner activities.

TYPE:	Plenary
OBJECTIVE(S):	Use the vocabulary of probability in interpreting results. Use estimates of probability. Consider outcomes and sample spaces.
DESCRIPTION:	1 looks at reasons for choosing rules for octagonal spinners. 2 is SATs page. 3 is vocabulary.
OVERVIEW:	Game like probability work with octagonal spinners
EQUIPMENT:	None.

Main Whiteboard and Screen information

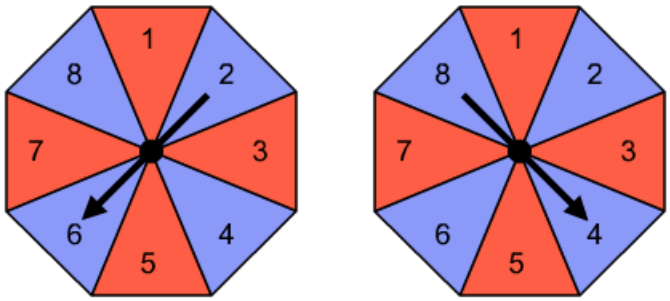
Choosing a rule

Clicking Spin will simulate a single spin of both spinners. The results are recorded in the table below. Click x 10 to simulate ten spins. Which player do you think has chosen the better rule? Give a reason for your answer. Click Reset to start again.

Pen on

	Player 1	Player 2
Rule	Same colour	The total is less than 10
Score	6	9

Number of spins = 15



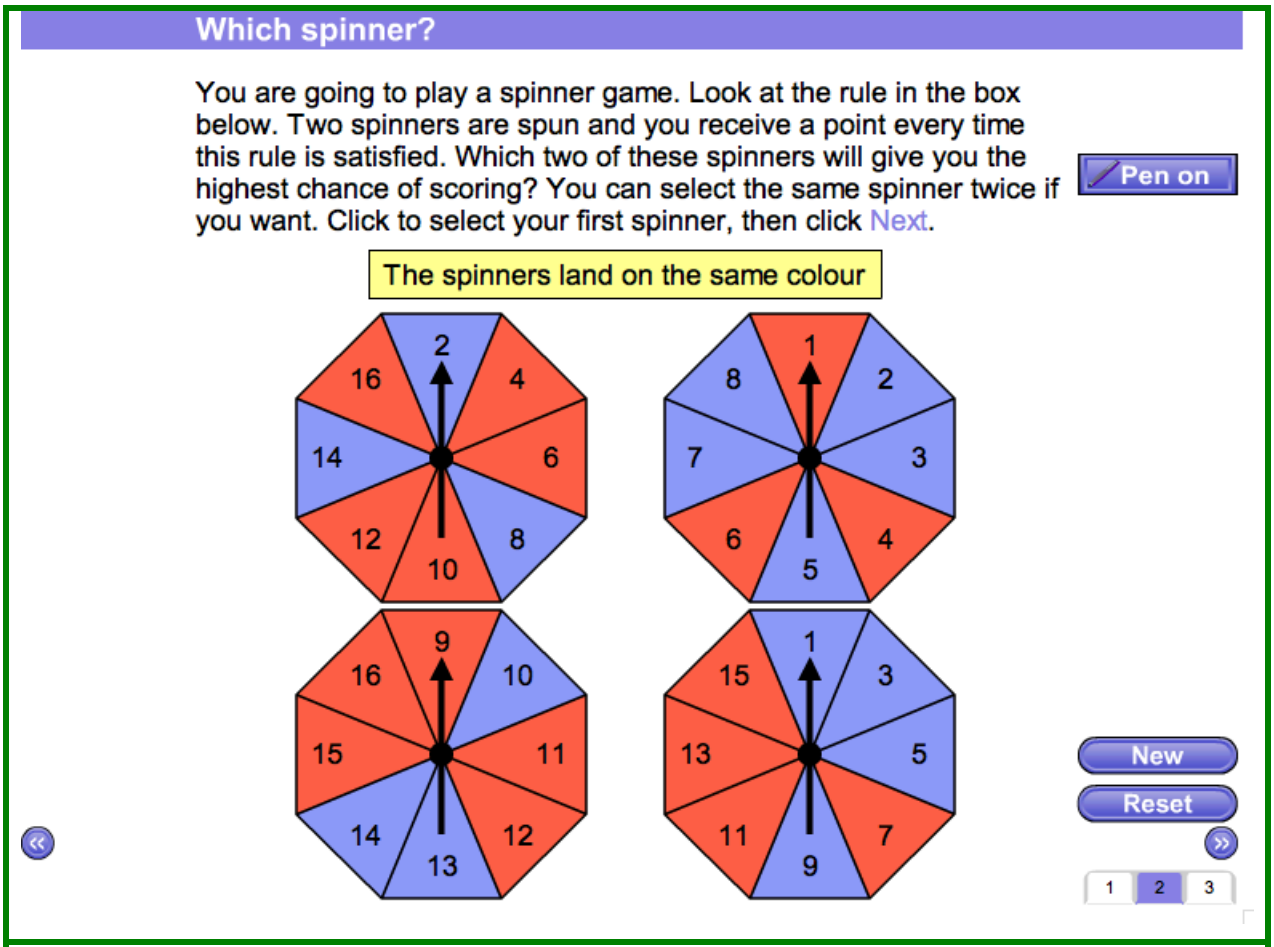
Spin
x 10
Reset

1 2 3

Screen 1: Choosing a rule

Two eight-sided spinners, numbered one to eight are shown together with eight rules for how to score points when they are spun. Two players click a rule in turn with the intention of scoring the most points. The game is then played. A table is then shown that shows the number of points (i.e. successes) for each rule and the number of spins. You can 'collect' spins in ones or in tens. You select when the game will finish. You are asked which player has the better rule and how you might show this by using probabilities.

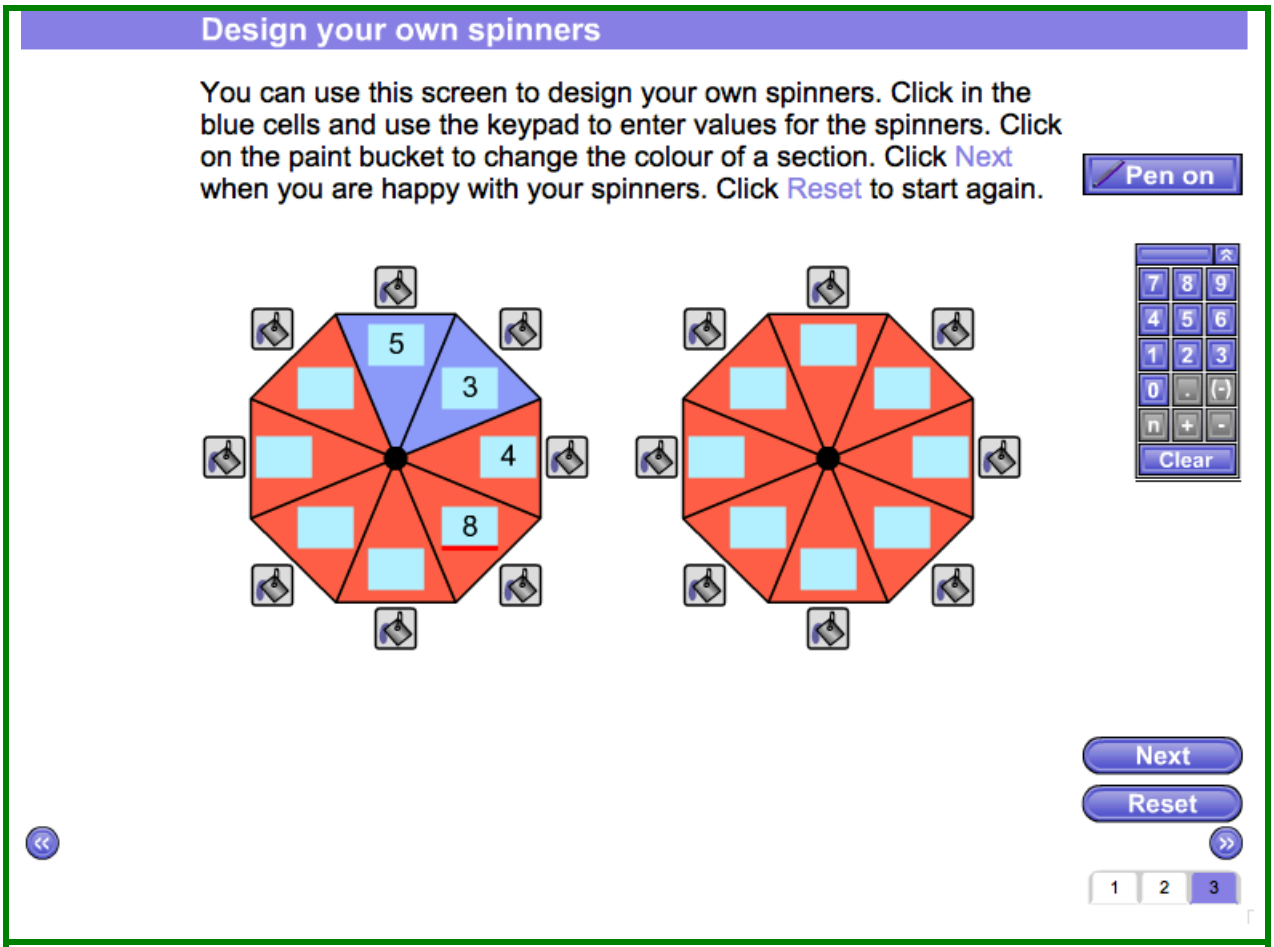
Key points: do not let pupils spend too long choosing rules first time as they will get more idea after one example; it will help if pupils try the game for themselves and you could 'share' out the rules amongst them; you should ask pupils whether it makes any differences how the spinners are numbered; when considering outcomes some pupils may think that there are sixteen outcomes; at some point pupils should consider the complete set of outcomes using a sample space, experimental probability or some other method; for some pupils you may wish to extend this work for spinners with different shapes (hexagonal, pentagonal or even two different ones).



Screen 2: Which spinner?

Four eight-sided spinners, numbered and coloured in different ways are shown together with a rule for how to score points when they are spun (there are many rules). You are asked to click the two spinners that give you the best chance of scoring points (and you can choose the same one twice). The game is then played. A table is shown that shows the number of points (i.e. successes) for the rule and the number of spins. You can 'collect' spins in ones or in tens. You select when the game will finish. You can then try different spinners with the same rule.

Key points: do not let pupils spend too long choosing spinners first time as they will get more idea after one example; it will help if pupils try the game for themselves and you could 'share' out the rules amongst them; you should ask pupils whether it makes any differences how the spinners are numbered or coloured; when considering outcomes some pupils may think that there are sixteen outcomes; at some point pupils should consider the complete set of outcomes using a sample space, experimental probability or some other method; for some pupils you may wish to extend this work for spinners with different shapes (hexagonal, pentagonal or even two different ones).



Screen 3: Design you own spinners

Here you can number (1 - 99) and colour (two choices) your own two spinners which you can then spin.

Key points: you could ask pupils to design a pair of spinners that would score points, for example 25% of the time - this could be used to test their theories.

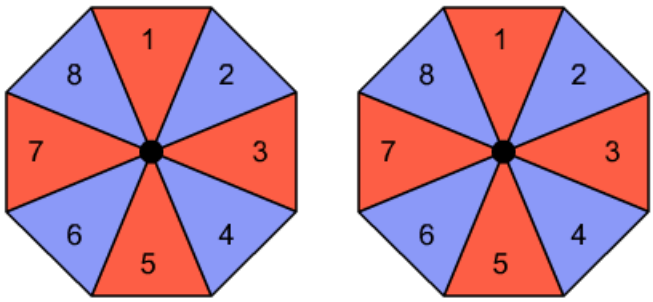
Plenary Whiteboard and Screen information

Which rule?

In a game two identical eight-sided spinners are spun. Each player chooses a rule. On each spin, players receive a point if their rule is satisfied. Which of the rules below should you choose to give yourself the best possible chance of scoring? Click to select a rule then click Check.

Pen on

The total is even	The total is a multiple of three	The total is a multiple of four



Show

Check

New

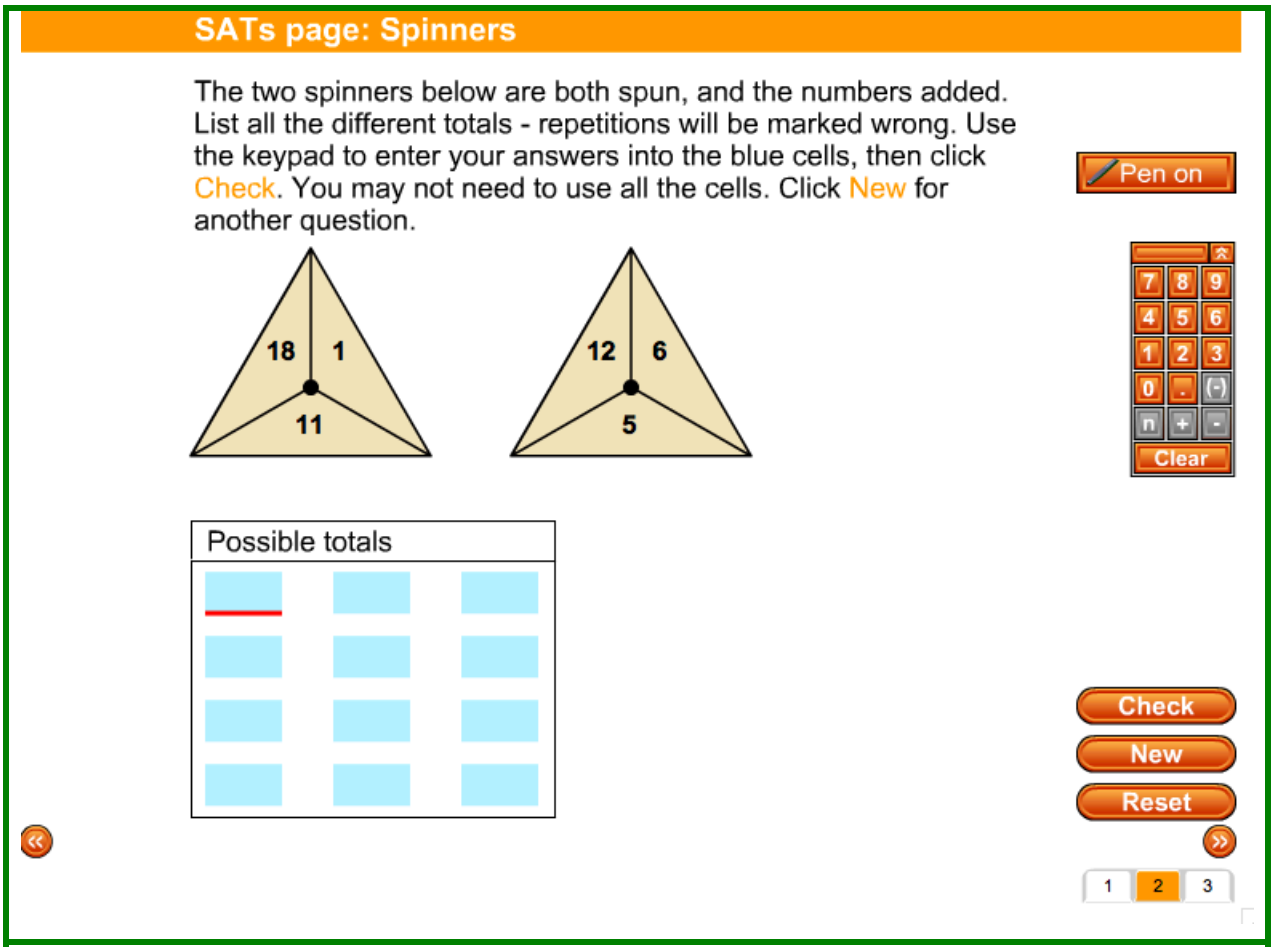
Reset

1 2 3

Screen 1: Which rule?

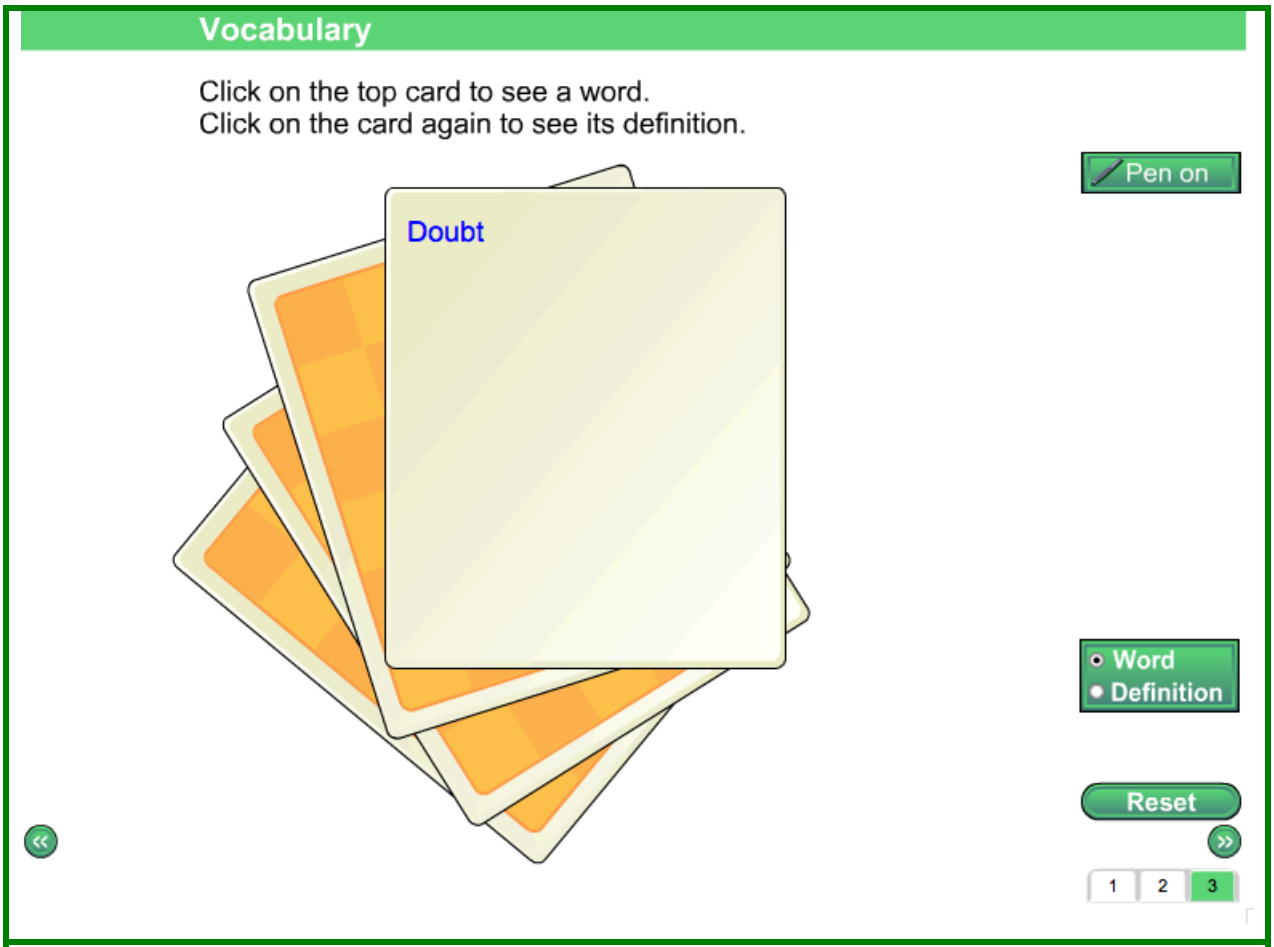
Two eight-sided spinners, numbered one to eight are shown together with three rules for how to score points when they are spun. You are asked to decide which rule to choose to give yourself the best chance of scoring. You click the rule and can check to see if it is correct. Once you have the correct answer you can click Show to see the probability sample space for the two spinners to confirm the correct answer.

Key points: pupils may consider the complete set of outcomes using the sample space or may use a 'logical' approach that we initially show.



Screen 2: SATs page: Spinners

Three questions types are given. The first asks you to 'paint' an eight-sided spinner so that the probability of an arrow landing on a painted part is a given percentage. The second shows two numbered triangular spinners where the numbers are added and you have to find all the possible different totals made by adding the numbers. The third shows an eight-sided spinner numbered one to eight and asks a probability question based on this looking for the answer as a percentage.



Screen 3: Vocabulary

Vocabulary present: Biased, Certain, Chance, Chance, Doubt, Equally likely, Even chance, Event, Exhaustive, Fair, Fifty-fifty chance, Good chance, Impossible, Independent, Likelihood, Likely, Limit, Mutually exclusive, No chance, Outcome, p(n), Poor, Possible, Probability, Probable, Random, Relative frequency, Risk, Sample, Sample space, Theory, Tree diagram, Uncertain, Unfair, Unlikely.

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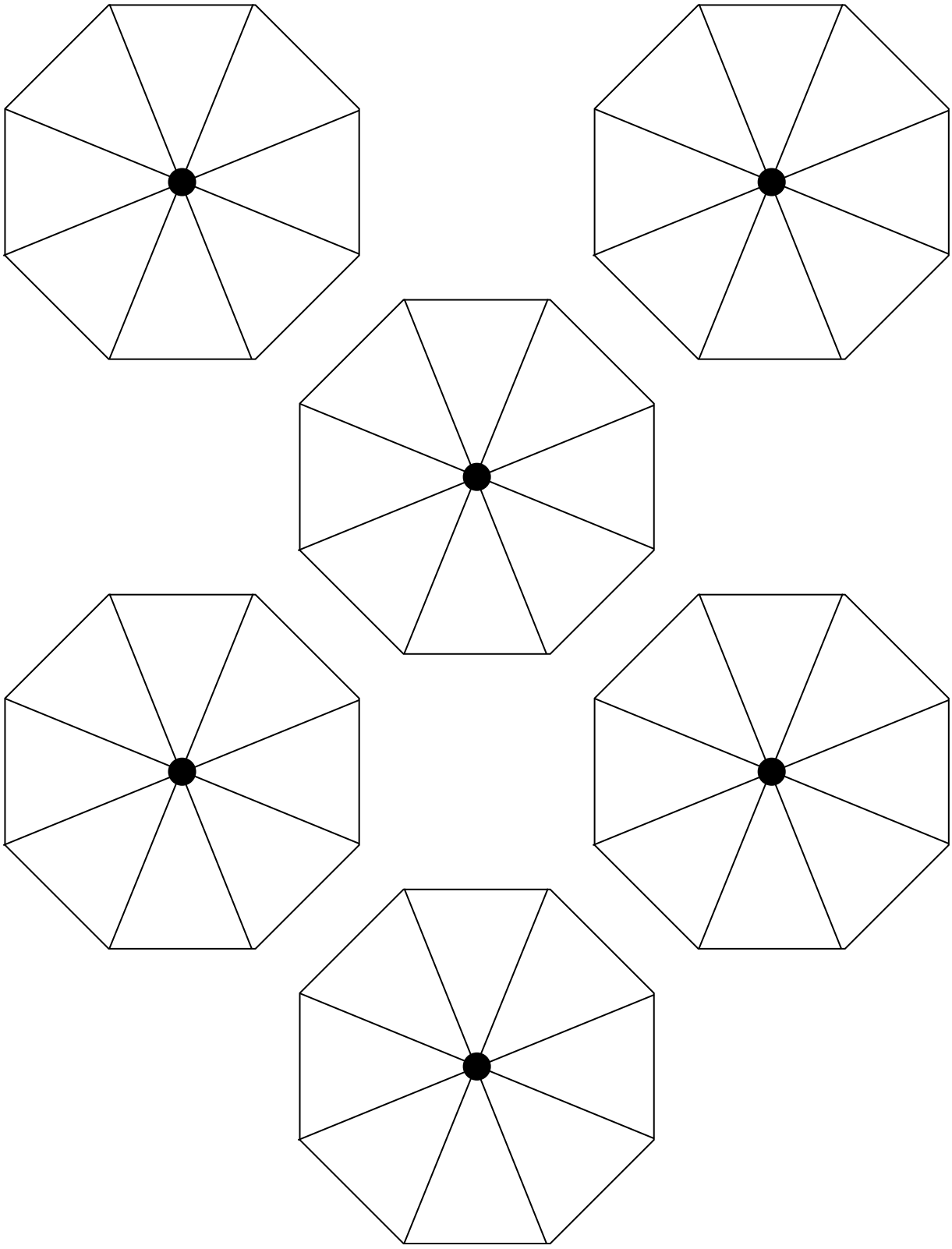
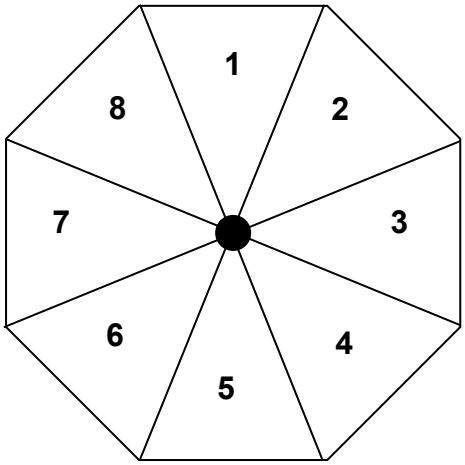
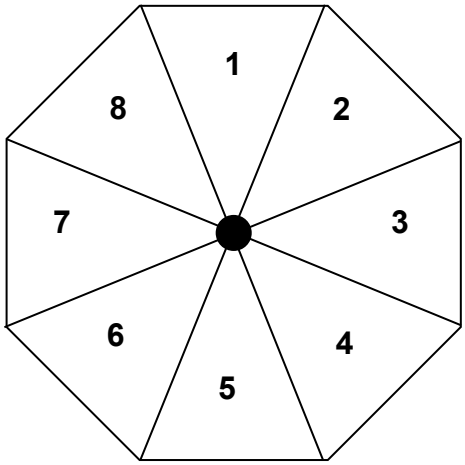
Octagonal spinners



Data collection sheet for octagonal spinners testing two rules





	Spinner 1	Spinner 2
Rules:		
Points:		
Total number of spins:		

Sample space for rule 1

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Sample space for rule 2

	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

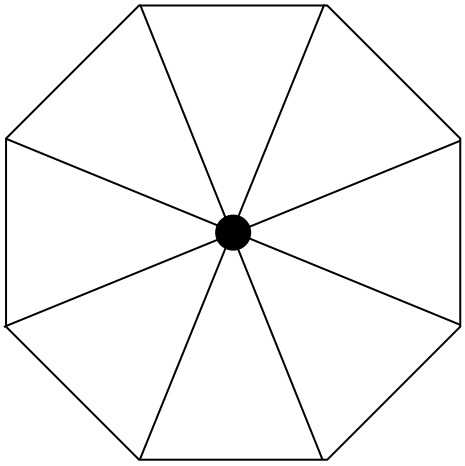
Conclusion: _____

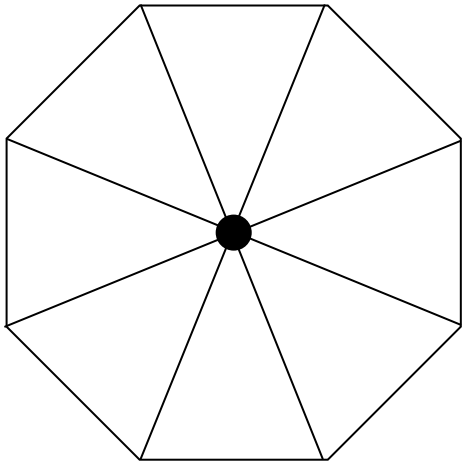
_____

_____

_____

Data collection sheet for testing two octagonal spinners





Rule:

Points:	
Total number of spins:	

Sample space for the rule

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1								
2								
3								
4								
5								
6								
7								
8								

Conclusion: _____

_____

_____

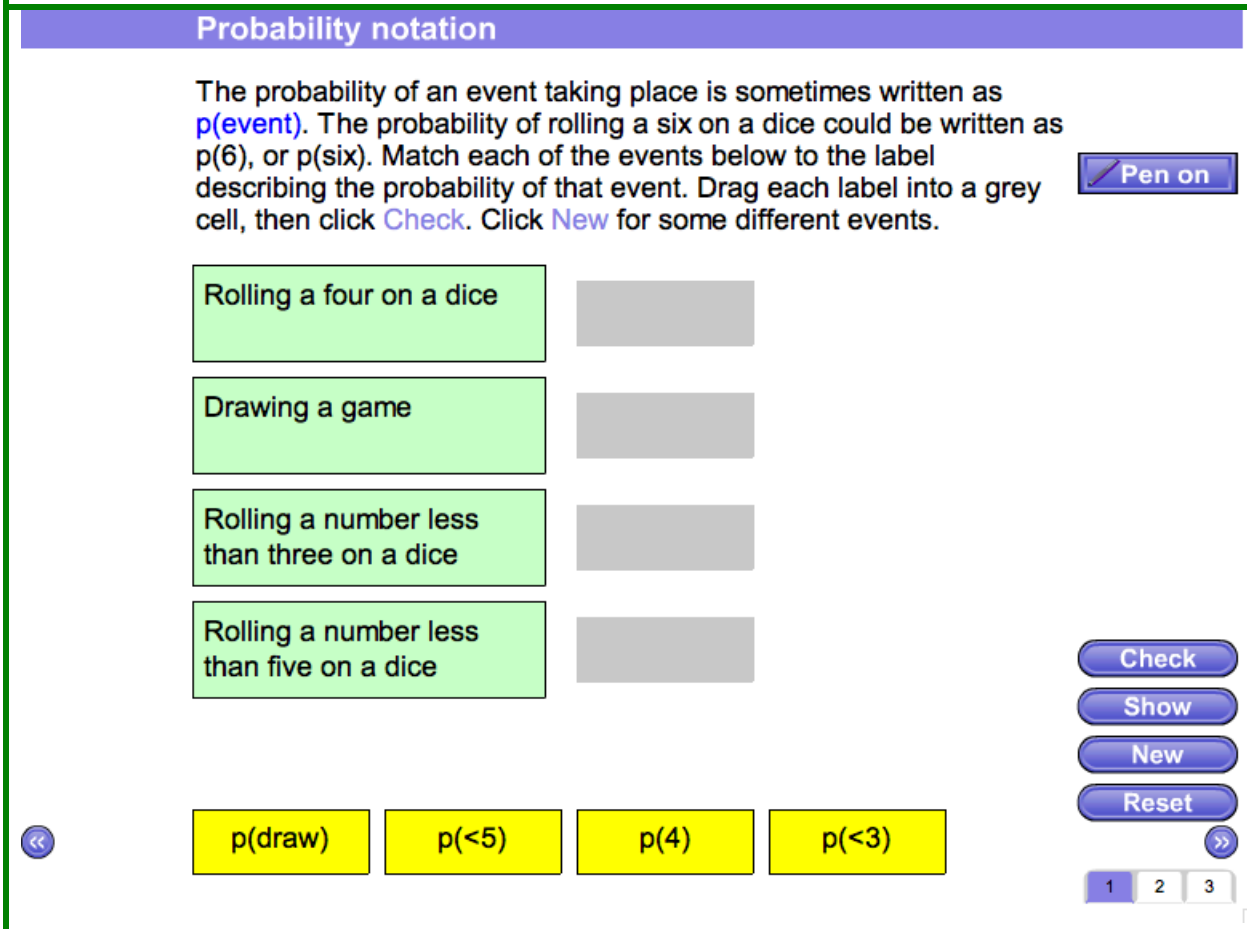
_____

Probability events with counters

TYPE:	Main
OBJECTIVE(S):	Identify all mutually exclusive outcomes and know that the sum of probabilities of all mutually exclusive outcomes is 1.
DESCRIPTION:	1 considers p(event). 2 looks at counters and probabilities of events. 3 is design a bag of counters.
OVERVIEW:	Probability with counters: mutually exclusive events.
EQUIPMENT:	A bag of counters or equivalent could help with this work.

TYPE:	Plenary
OBJECTIVE(S):	Identify all mutually exclusive outcomes and know that the sum of probabilities of all mutually exclusive outcomes is 1.
DESCRIPTION:	1 is probability events. 2 is probability calculation from mutually exclusive events. 3 is SATs page. 4 is vocabulary.
OVERVIEW:	Probability with counters: mutually exclusive events.
EQUIPMENT:	None.

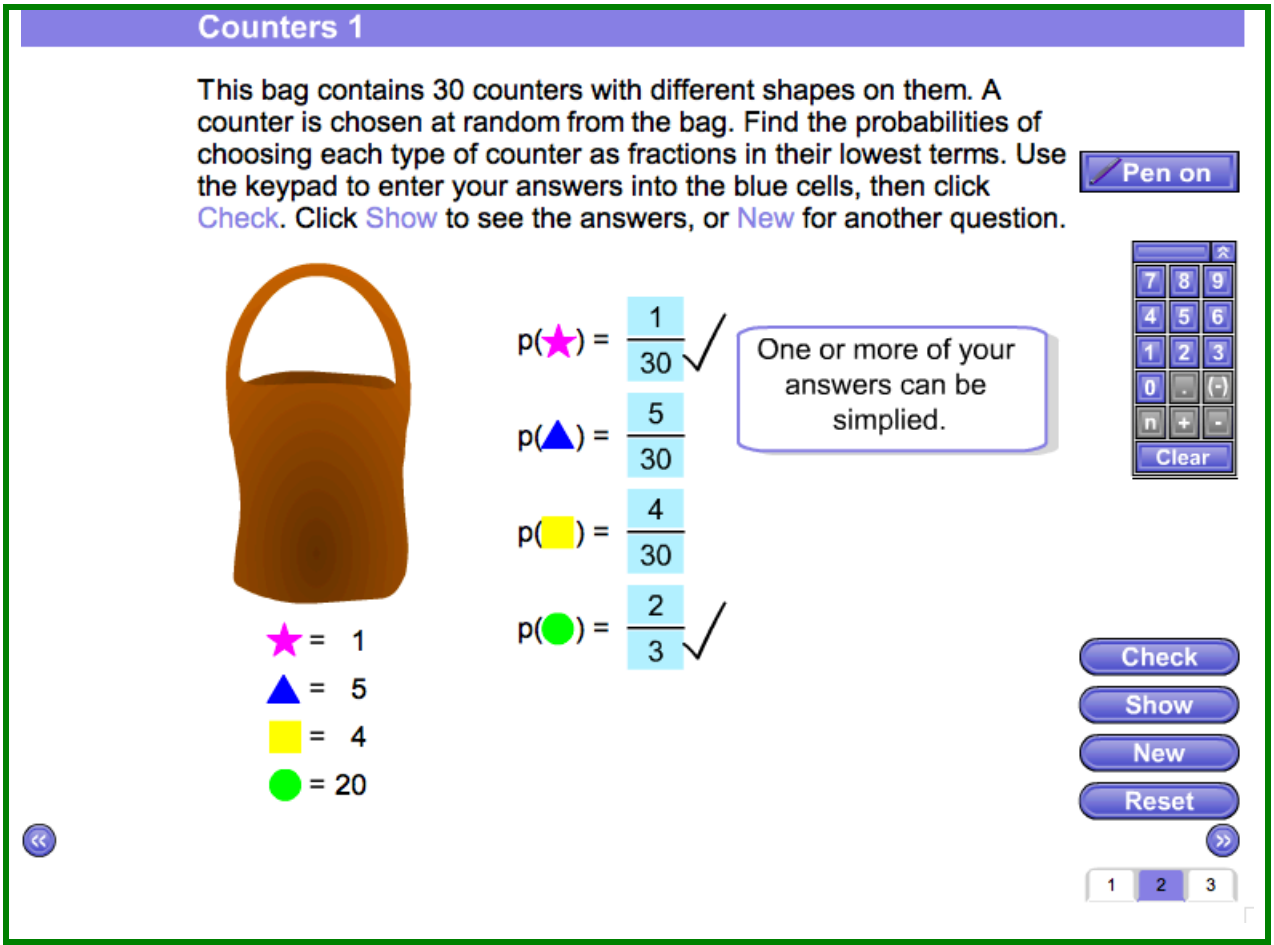
Main Whiteboard and Screen information



Screen 1: Probability notation

Four events are shown and you are asked to drag four 'labels' to match the events; for example, you would match p(one) to 'rolling one on a dice'.

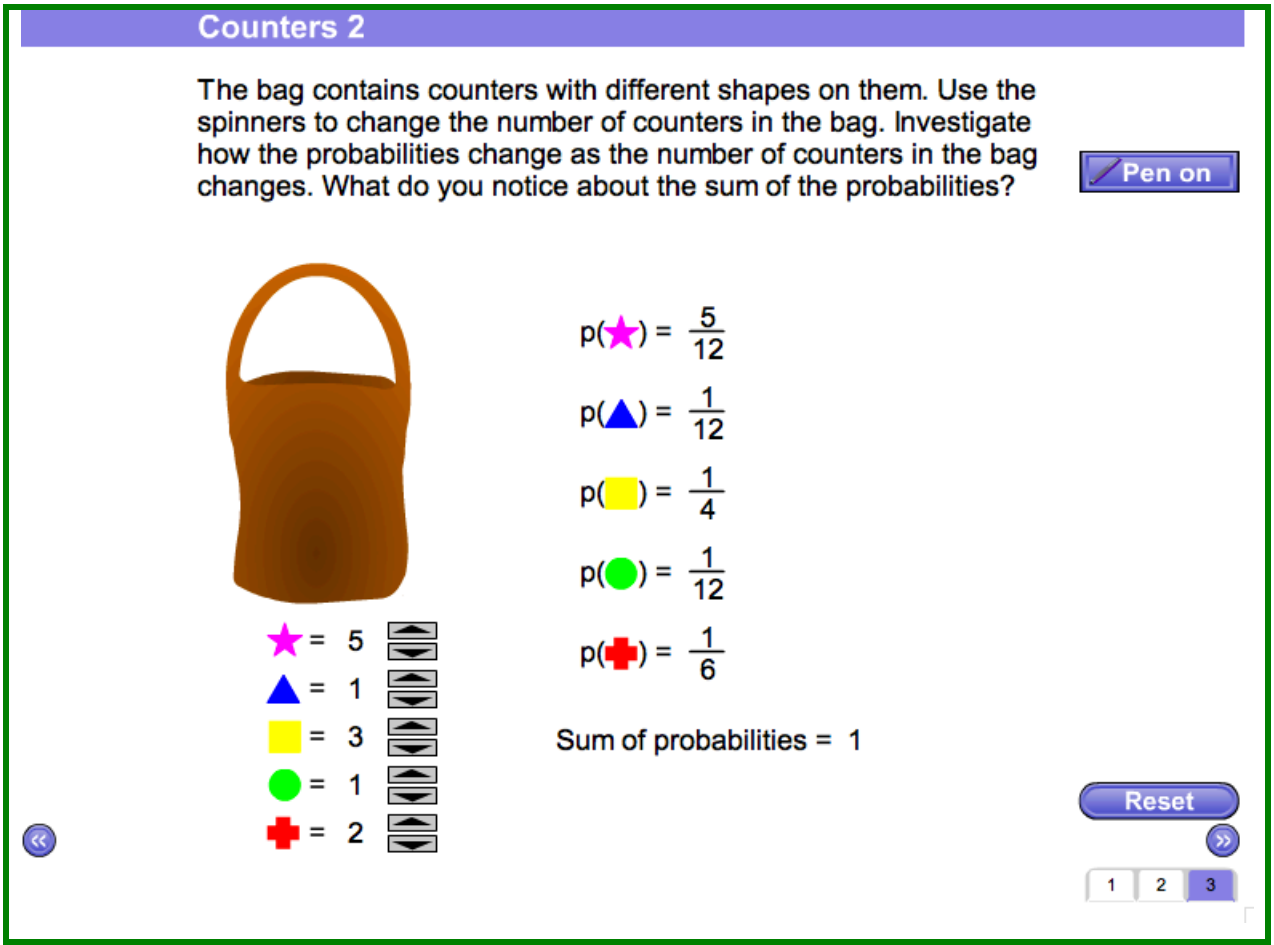
Key points: have pupils discuss these and extend it to similar things - at times there may be reasonable alternatives to these.



Screen 2: Counters 1

A bag is shown and you are told that it contains a given number of counters that have different shapes on them. You are told how many of each shape appear on the counters and asked to find the probability associated with each of the events p(shape 1) as a fraction in its lowest terms. When you have them all correct you are asked to find the sum of the probabilities.

Key points: pupils need to make sure they have counted all the options; fractions and lowest terms will create problems for some pupils, though we have kept to small numbers; all the answers are required before they are checked; encourage use of 'mutually exclusive'.

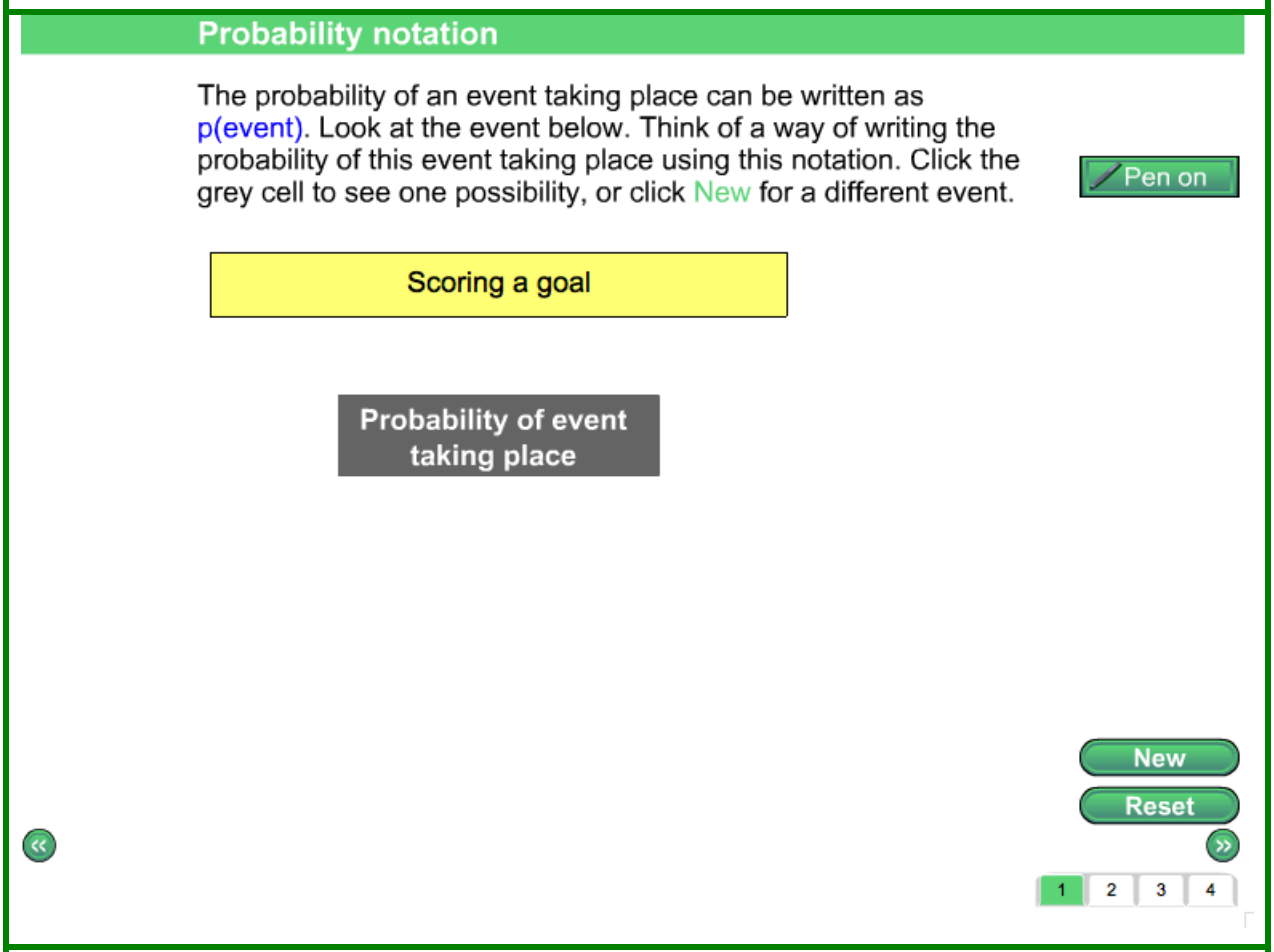


Screen 3: Counters 2

A bag is shown and you are told that it can contain a given number of counters that have different shapes on them. You can now experiment and add up to twenty of five different shaped counters. As you add counters the probabilities, which are shown all the time, change. The sum of the probabilities is also shown.

Key points: this will let you (or your pupils) set conditions and then 'make' a bag of counters to conform to these conditions; pupils should be aware that the sum of probabilities is always one and that you can find p(not blue) by subtracting p(blue) from 1; encourage use of 'mutually exclusive'.

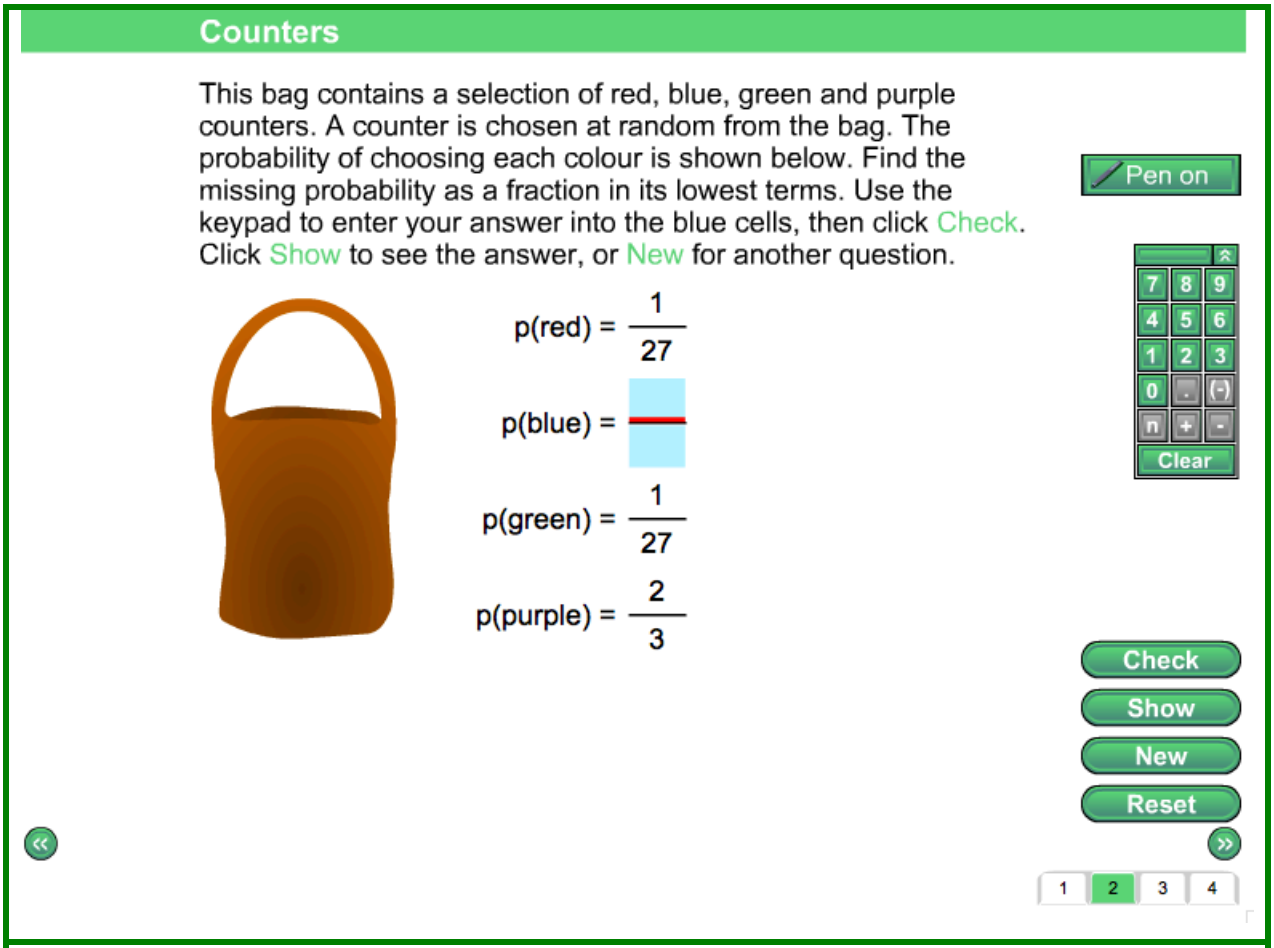
Plenary Whiteboard and Screen information



Screen 1: Probability notation

An event is shown and you are asked to write it in a form p(event). You can then click on a grey cell to see one way to do this.

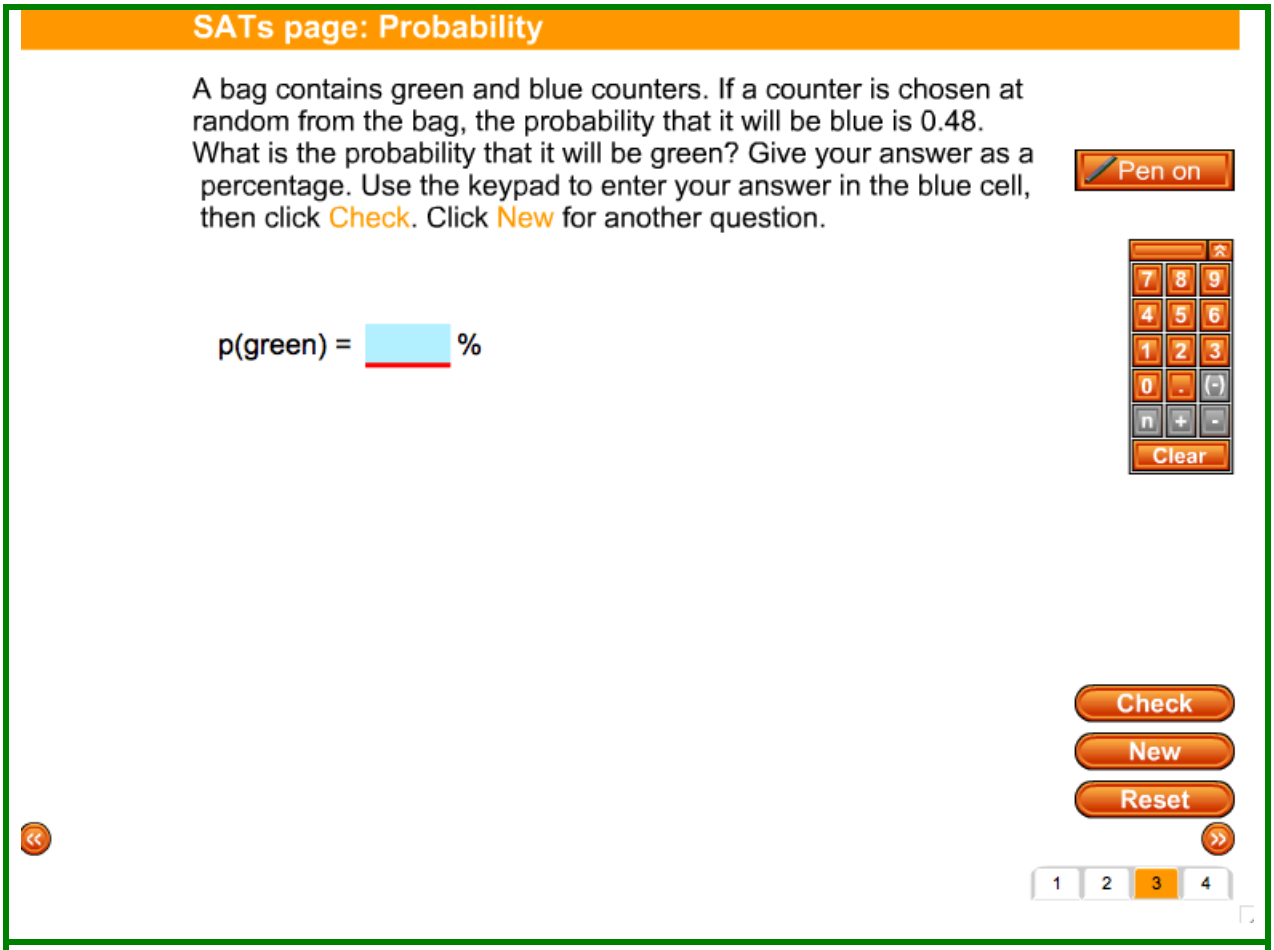
Key points: have pupils discuss how to do it and compare this with the given suggestion; pupils should understand that there will be reasonable alternatives, but it does help to have meaningful event names.



Screen 2: Counters

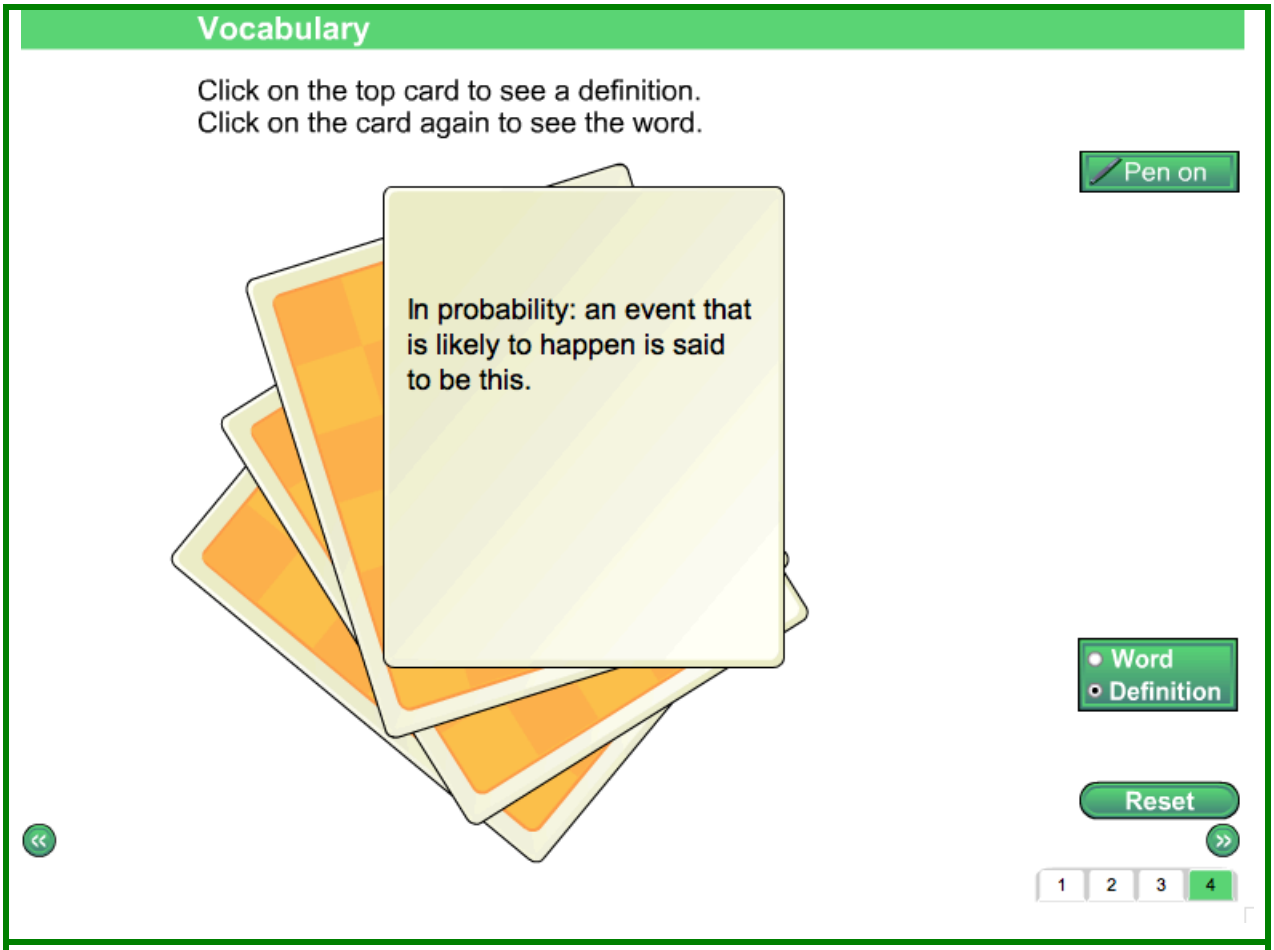
A bag is shown and you are told that it contains red, blue, green and purple counters. You are given the probabilities associated with three of the four events p(red), p(blue), p(green) and p(purple) and asked to find the missing probability. All are given or have to be found as fractions in their lowest terms. Once you have the correct answer you are reminded that the sum of the probabilities is one.

Key points: pupils may need to discuss how to add the fractions, especially when not all the denominators are the same; encourage use of 'mutually exclusive.



Screen 3: SATs page: Probability

You are given one of two situations. In the first you are told the probability of choosing a blue counter from a bag containing green and blue counters (as a decimal) and asked to find the probability of choosing a green counter (as a percentage). In the second you are told that you have a biased dice and your are given a table of probabilities for throwing five of the six numbers and asked to find the missing probability - all are given as decimals.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Chance, Doubt, Equally likely, Even chance, Event, Exhaustive, Fair, Fifty-fifty chance, Good chance, Impossible, Independent, Likelihood, Likely, Limit, Mutually exclusive, No chance, Outcome, p(n), Poor, Possible, Probability, Probable, Random, Relative frequency, Risk, Sample, Sample space, Theory, Tree diagram, Uncertain, Unfair, Unlikely.

Spire Maths interactive files available in a flash format at: https://spiremaths.co.uk/ia/

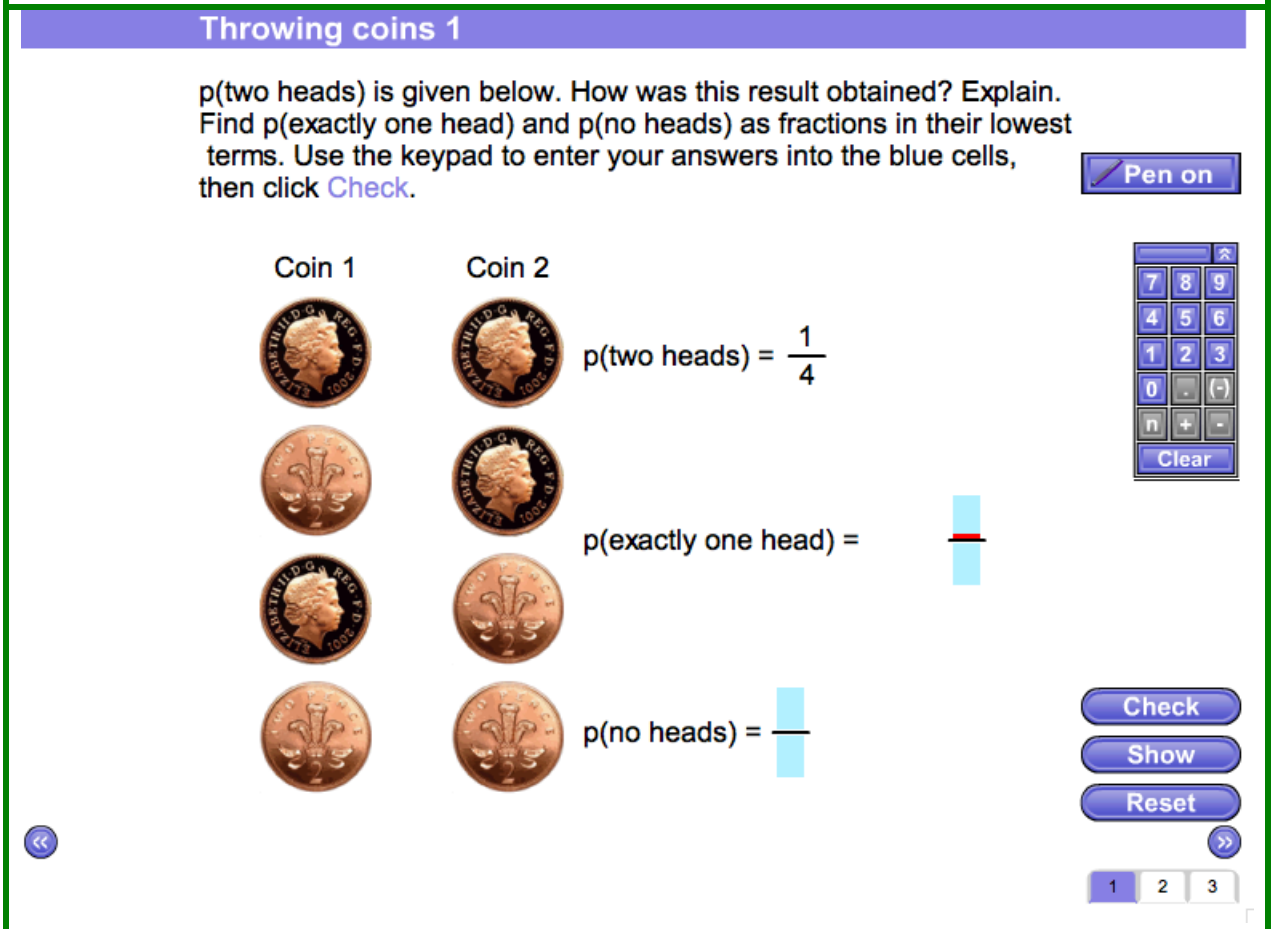
Unfortunately they will not work on iPads or iPhones.

Probability of two events

TYPE:	Main
OBJECTIVE(S):	Identify all mutually exclusive outcomes of an experiment; know sum of probabilities of all mutually exclusive outcomes is 1. Use this.
DESCRIPTION:	1 is throwing three coins. 2 is throwing three coins. 3 is using two octagonal spinners.
OVERVIEW:	Two and three coins and spinner probabilities.
EQUIPMENT:	Coins for pupils to use (preferably their own). Coins for pupils to use (preferably their own). Spinners - pupils could make these from regular octagons (on card) with a cocktail stick through the middle (or used matches) - though you would need to take an appropriate risk assessment - these work for a small number of spins. You could also use octahedral dice. Four photocopiable masters are available: one is a set of octagons; the other three comprise one for the sum, one for the product and one for the difference of the numbers on the octagonal spinners - they have the table of outcomes ready made.

TYPE:	Plenary
OBJECTIVE(S):	Identify all mutually exclusive outcomes of an experiment; know sum of probabilities of all mutually exclusive outcomes is 1. Use this.
DESCRIPTION:	1 is throwing coins. 2 is spinners. 3 is SATs page. 4 is vocabulary.
OVERVIEW:	Two and three coins and spinner probabilities.
EQUIPMENT:	None.

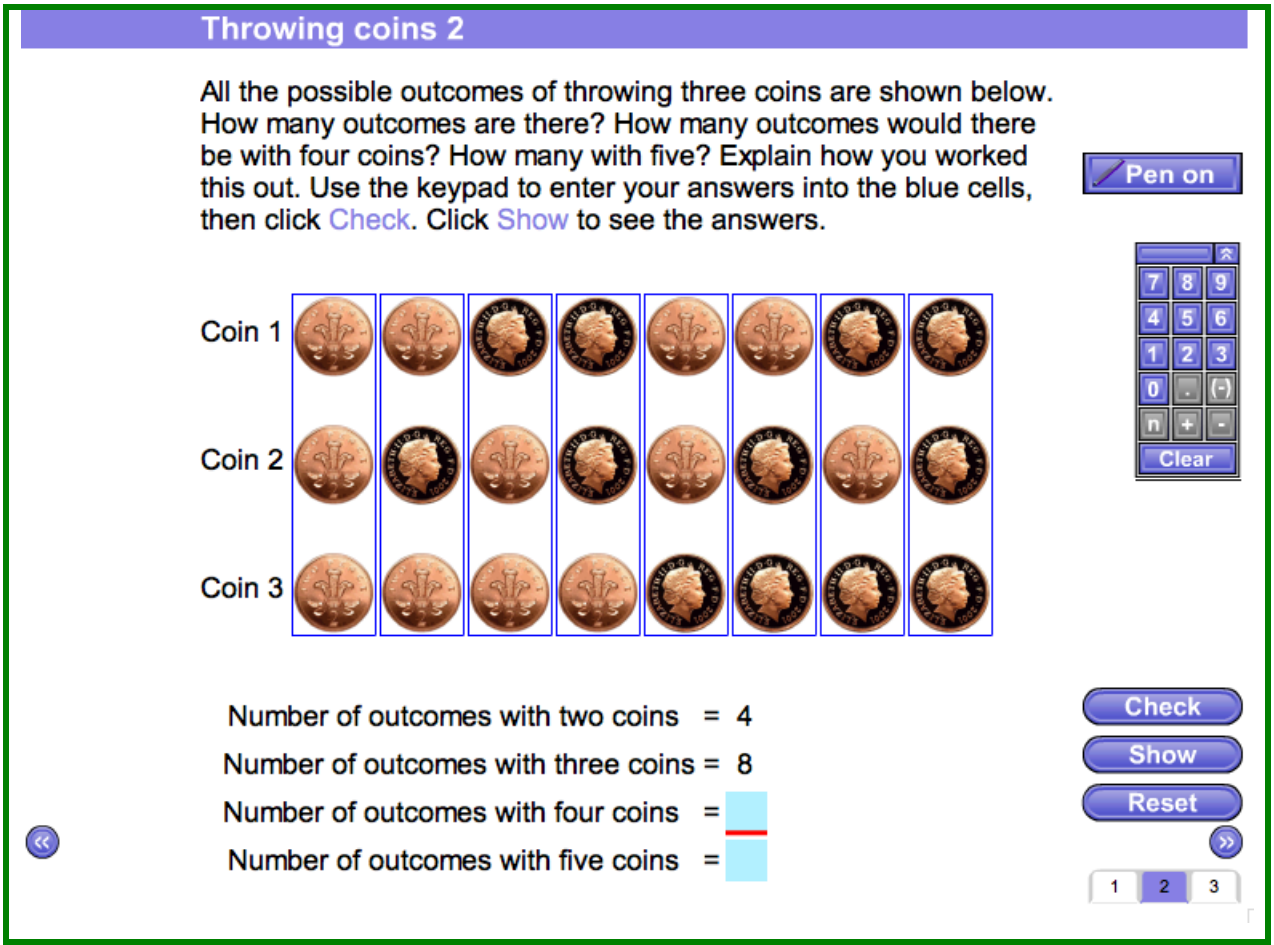
Main Whiteboard and Screen information



Screen 1: Throwing coins 1

Two coins are shown and you are asked how many possible outcomes there are when two coins are thrown. You can click Throw to simulate throwing the coins. On Next you are shown the four possible outcomes of throwing the two coins and a definition of the probability of an event is given. You are then asked for the probability of throwing two heads. On Next you are shown the probability of throwing two tails and asked to work out the probabilities of exactly one head and no heads (or they can be shown). The next screen follows a similar thing for three coins.

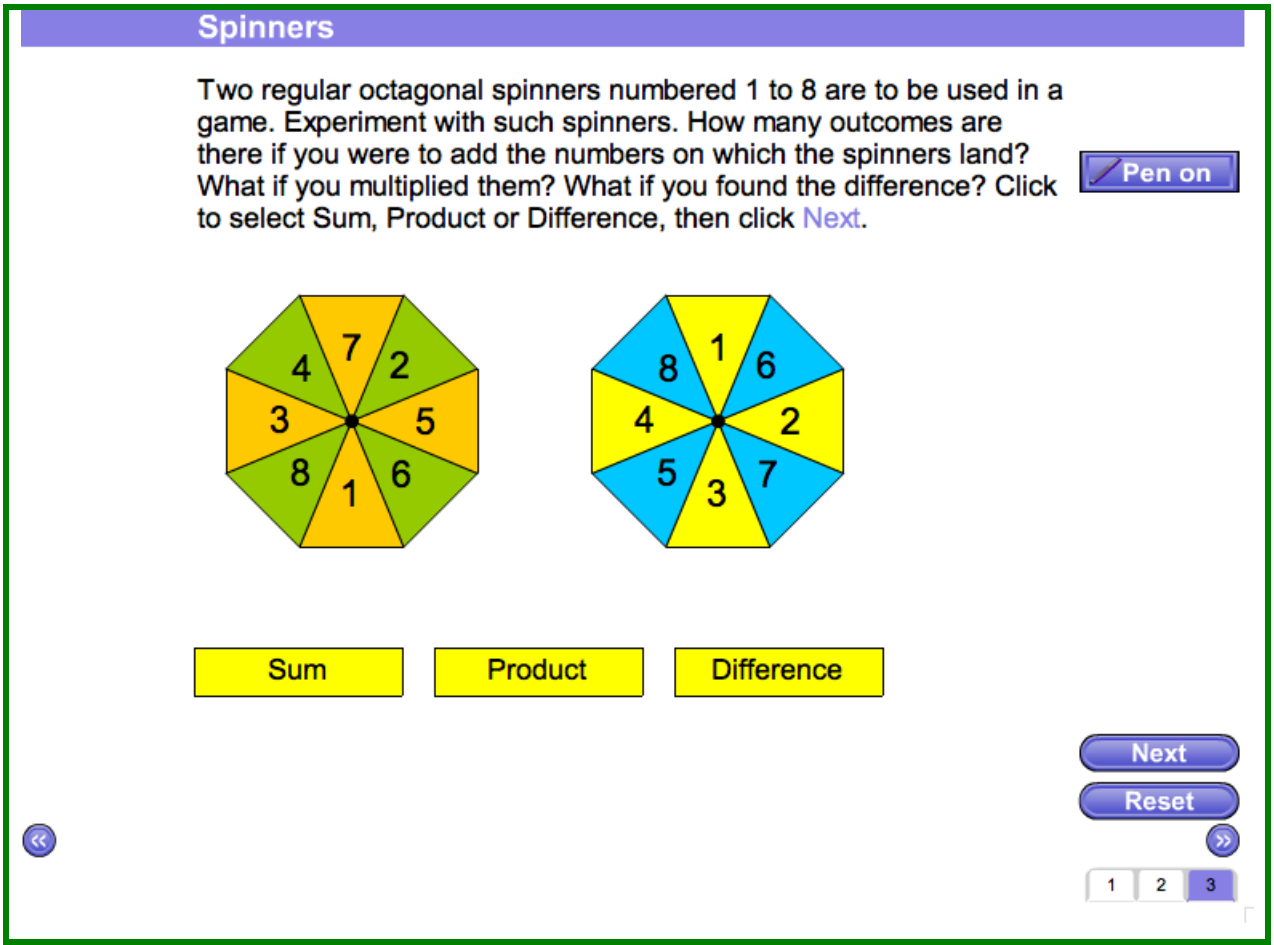
Key points: you should ask pupils whether it makes any differences if the coins are the same or different, how they are thrown, whether one lands first, whether anything else is likely to happen on a flat surface - each of these may expose misconceptions; when considering probabilities, you should ask pupils why probability is not one third for two heads; some pupils may benefit from the class throwing two coins several times each to see that the experimental probability is not one third (it should be one head that looks as thought it is about a half that is evident first); you may wish to have pupils consider sample space diagrams - we do this for spinners later, but not here.



Screen 2: Throwing coins 2

Two coins are shown and you are asked how many possible outcomes there are when three coins are thrown. You can click Throw to simulate throwing the coins. On Next you are shown the eight possible outcomes of throwing the three coins and asked to predict the total number of outcomes with four and five coins. You are then asked to discuss the probability of throwing exactly three, two, one or no heads when you have three coins The previous screen follows a similar idea for two coins.

Key points: you should ask pupils whether it makes any differences if the coins are the same or different, how they are thrown, whether one lands first, whether anything else is likely to happen on a flat surface - each of these may expose misconceptions; when considering probabilities, you should ask pupils why probability is not in thirds, sixths or even quarters; some pupils may benefit from the class throwing three coins several times each to see what the experimental probabilities are.

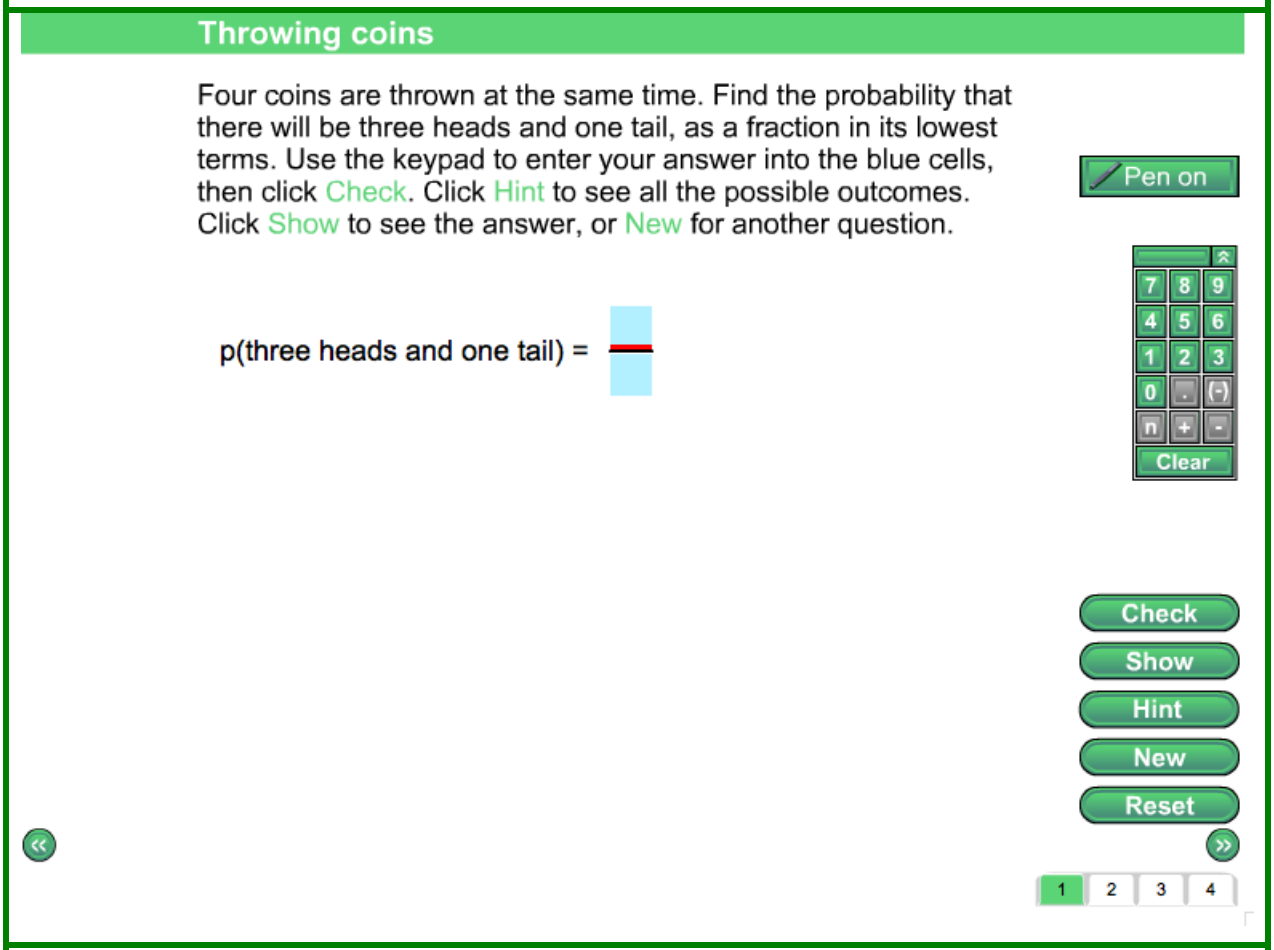


Screen 3: Spinners

Two regular octagonal spinners each containing the numbers 1 to 8 are shown and you are asked: to discuss the probabilities that might arise from using one or both of them; to experiment with such spinners; how many outcomes are there if you were to add (multiply or find the difference between) the numbers on which the spinners land. You can then select Sum, Product or Difference, and click Next. On doing this you are shown the relevant sample space and asked to find the total number of outcomes. When you click Next again four possible rules (such as 'the sum is even') are shown and you can click on any of them to see how many outcomes there are in the table - they are shaded. Finally you are asked about the probability of each of the rules being satisfied.

Key points: you should ask pupils whether it makes any differences how the spinners are numbered, how they are thrown, whether one lands first, whether anything else is likely to happen on a flat surface - each of these may expose misconceptions; when considering outcomes some pupils may think that there are sixteen outcomes; you may have to explain 'product'; try not to rush to get to the next stages; have pupils discuss the probabilities for the four rules before you show the outcomes; pupils may need to be reminded about lowest terms for fractions; for some pupils you may wish to extend this work for spinners with different shapes (hexagonal, pentagonal or even two different ones).

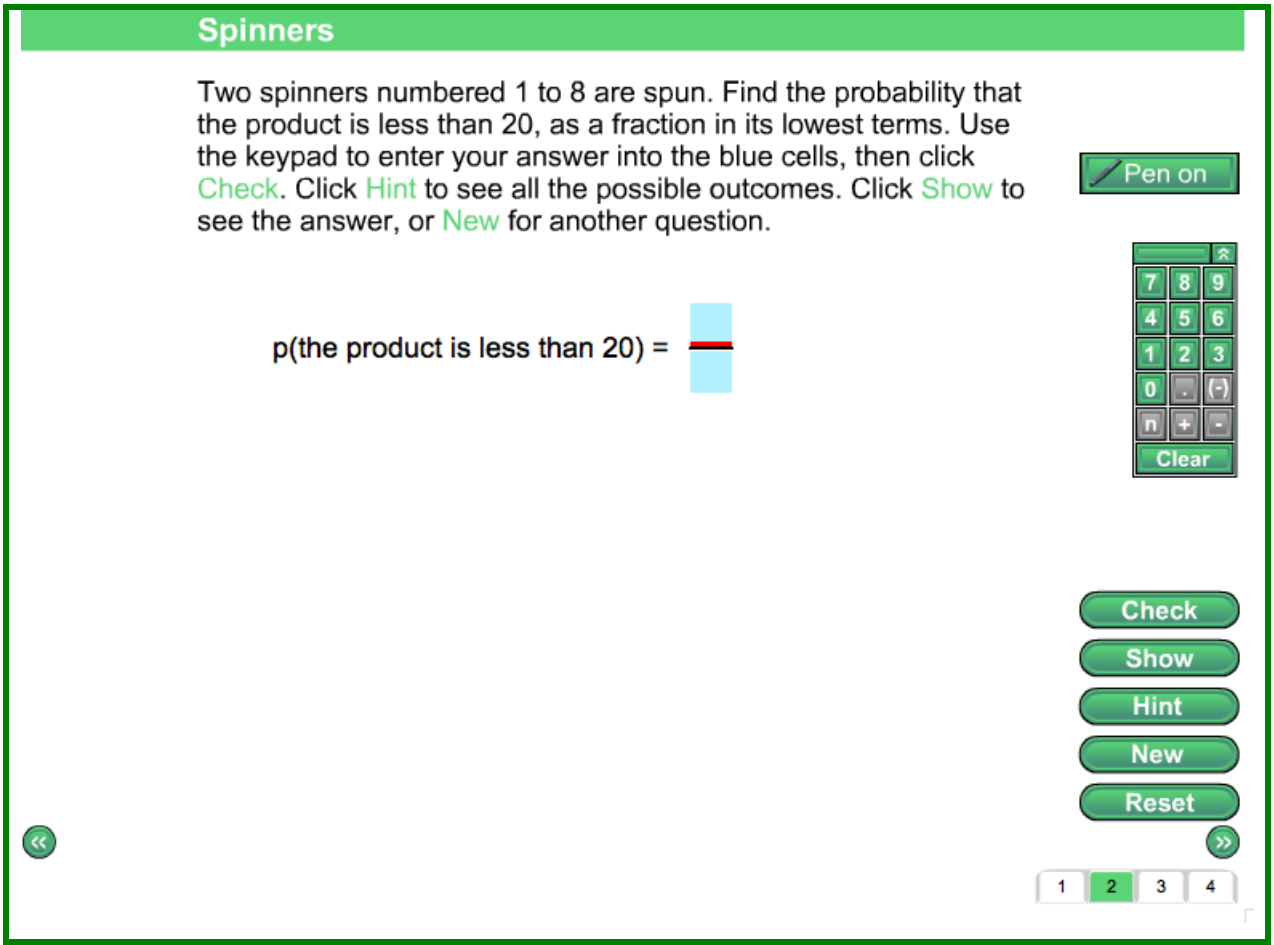
Plenary Whiteboard and Screen information



Screen 1: Throwing coins

You are told that three or four coins are thrown at the same time and asked for the probability of a given specific event related to the number of coins thrown. A hint is available (it shows the individual outcomes in a table) across the page and when you click Show it shows where the outcomes are in the table (by surrounding relevant columns in red). Note that the table is constructed systematically, but not necessarily in the best way to solve this problem. Key points are coloured in blue in the text at the top.

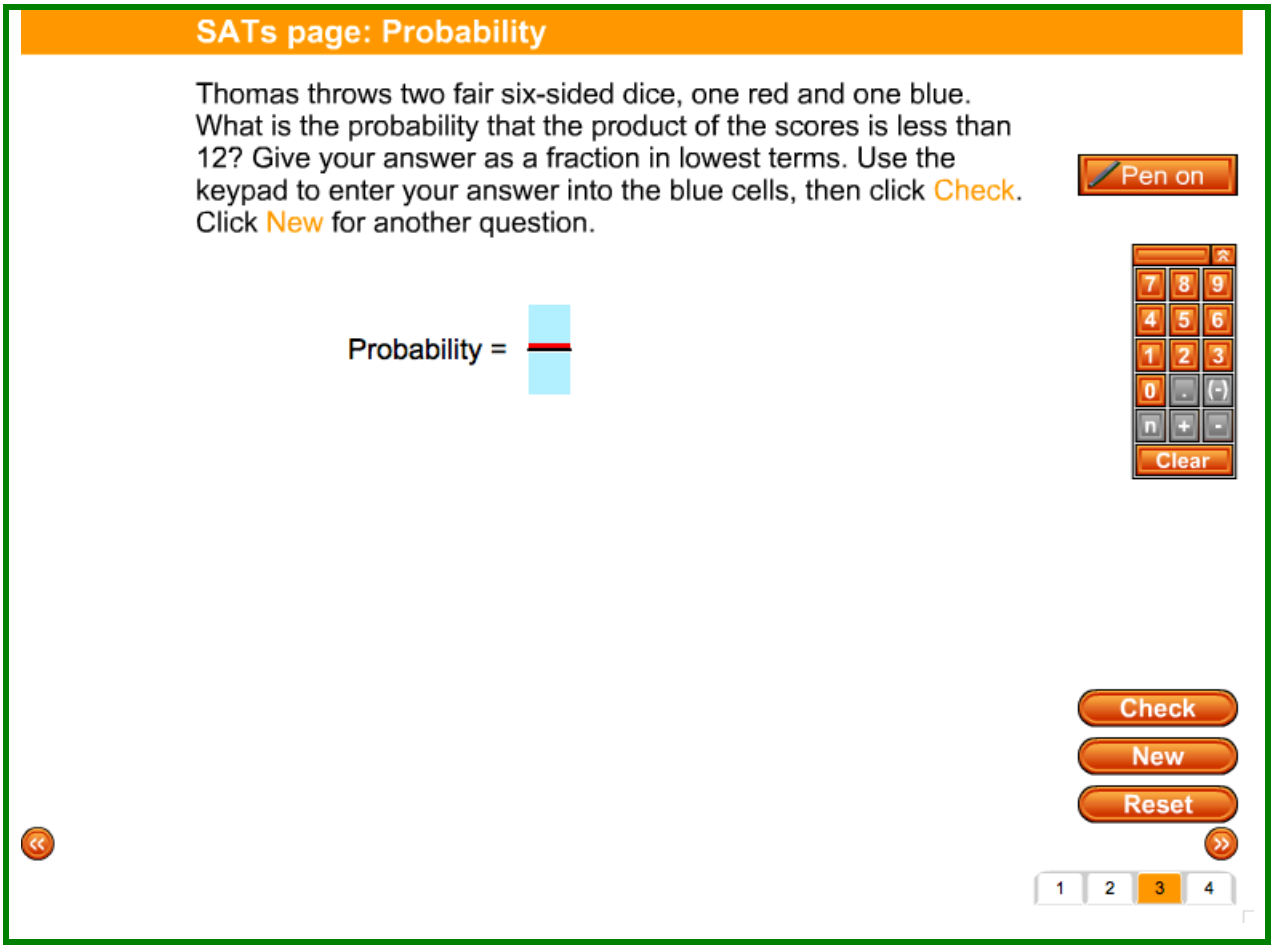
Key points: pupils should consider the total number of events for the outcome and then decide how many of the outcomes are required; you may wish pupils to consider the system used in the hints table and how a better table might be designed for this activity.



Screen 2: Spinners

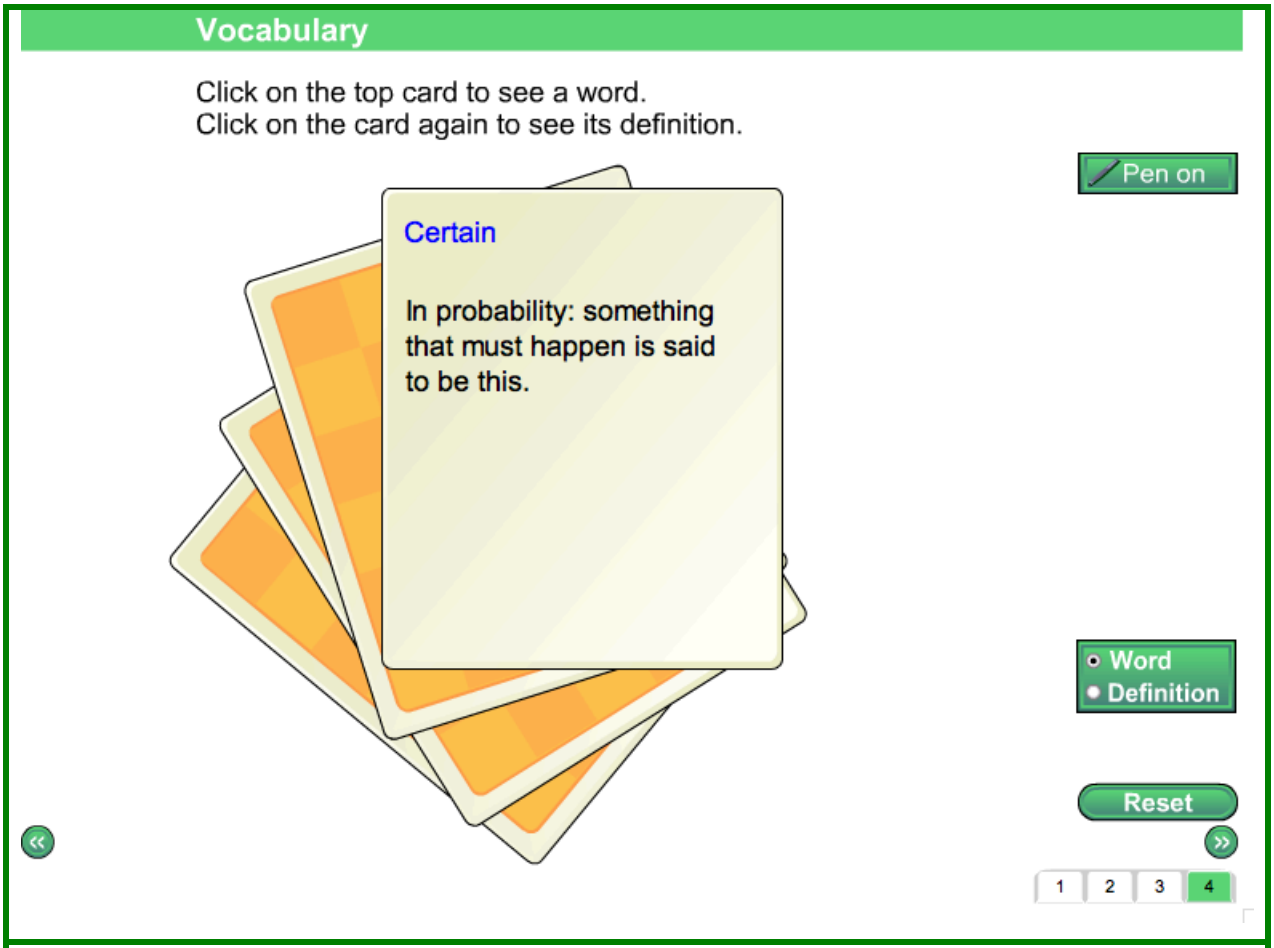
You are asked to click Sum, Difference or Product. On doing this you are told that two regular octagonal spinners each containing the numbers 1 to 8 are spun and you are asked to find the probability of a specific outcome. A hint is available (it shows the individual outcomes in a table) and when you click Show it colours in the relevant outcomes.

Key points: pupils should consider the total number of events for the outcome and then decide how many of the outcomes are required.



Screen 3: SATs page: Probability

You are given information about two dice being thrown and asked to answer a probability question.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Chance, Doubt, Equally likely, Even chance, Event, Exhaustive, Fair, Fifty-fifty chance, Good chance, Impossible, Independent, Likelihood, Likely, Limit, Mutually exclusive, No chance, Outcome, P(n), Poor, Possible, Probability, Probable, Random, Relative frequency, Risk, Sample, Sample space, Theory, Tree diagram, Uncertain, Unfair, Unlikely.

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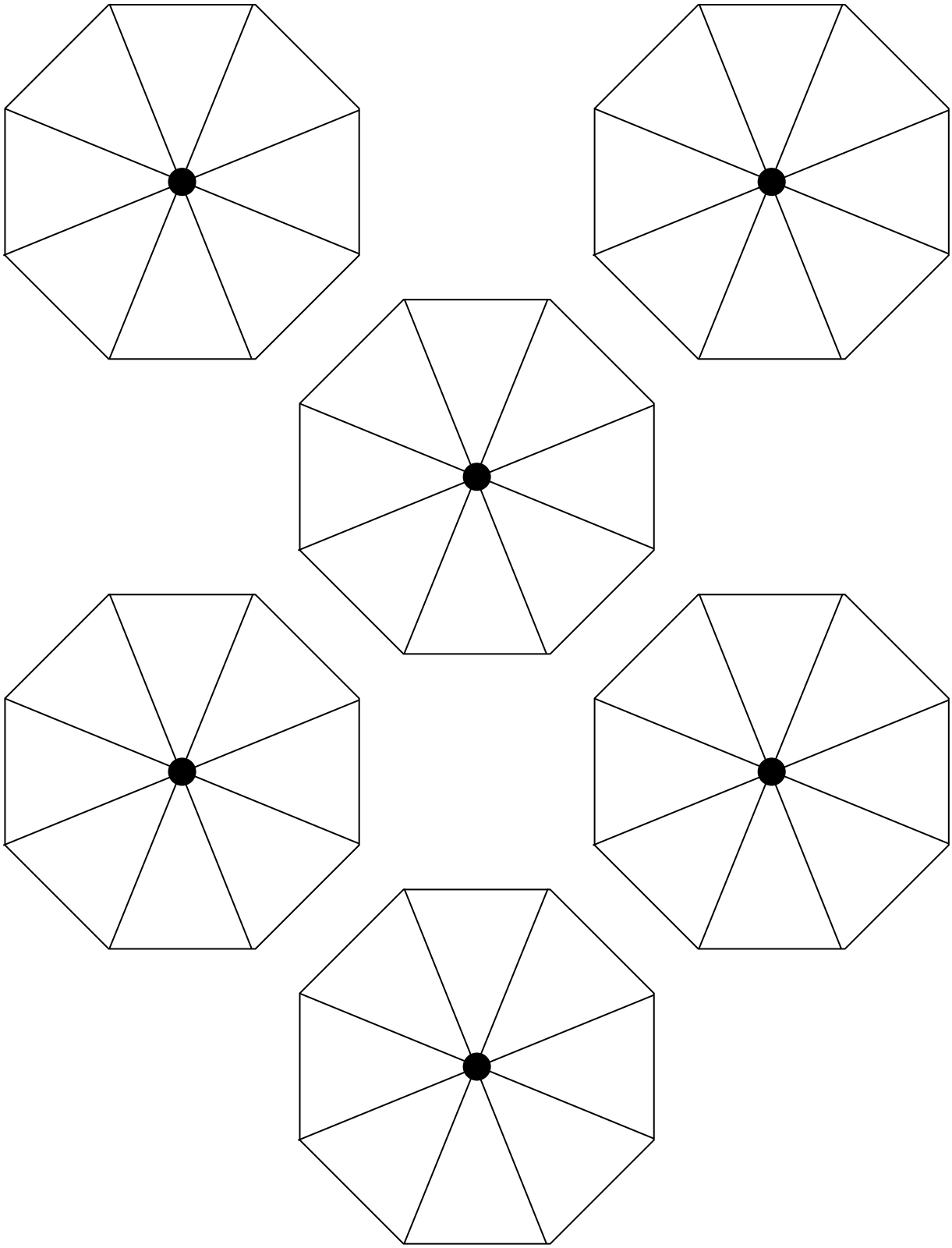
Octagonal spinners



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Octagonal spinners: probability with sum of numbers

+	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

+	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

+	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

+	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

Octagonal spinners: probability with product of numbers

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

Rule: ______________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

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Octagonal spinners: probability with difference of numbers

D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0	1	2	3	4	5	6	7
2	1	0	1	2	3	4	5	6
3	2	1	0	1	2	3	4	5
4	3	2	1	0	1	2	3	4
5	4	3	2	1	0	1	2	3
6	5	4	3	2	1	0	1	2
7	6	5	4	3	2	1	0	1
8	7	6	5	4	3	2	1	0

Rule: ________________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0	1	2	3	4	5	6	7
2	1	0	1	2	3	4	5	6
3	2	1	0	1	2	3	4	5
4	3	2	1	0	1	2	3	4
5	4	3	2	1	0	1	2	3
6	5	4	3	2	1	0	1	2
7	6	5	4	3	2	1	0	1
8	7	6	5	4	3	2	1	0

Rule: ________________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0	1	2	3	4	5	6	7
2	1	0	1	2	3	4	5	6
3	2	1	0	1	2	3	4	5
4	3	2	1	0	1	2	3	4
5	4	3	2	1	0	1	2	3
6	5	4	3	2	1	0	1	2
7	6	5	4	3	2	1	0	1
8	7	6	5	4	3	2	1	0

Rule: ________________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

D	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0	1	2	3	4	5	6	7
2	1	0	1	2	3	4	5	6
3	2	1	0	1	2	3	4	5
4	3	2	1	0	1	2	3	4
5	4	3	2	1	0	1	2	3
6	5	4	3	2	1	0	1	2
7	6	5	4	3	2	1	0	1
8	7	6	5	4	3	2	1	0

Rule: ________________________________

____

Total number of outcomes: ______________

Number of successful outcomes: __________

Probability of rule:

____

Probability: Buffon's needle

TYPE:	Main
OBJECTIVE(S):	Consider experimental probability; appreciate the difference between mathematical explanation and experimental evidence.
DESCRIPTION:	1 is designing an experiment. 2 is simulating the experiment. 3 is considering outcomes.
OVERVIEW:	Buffon's needle. You drop 'needles' onto lined paper.
EQUIPMENT:	Needles' and A3 paper on which you can draw parallel lines so pupils can do this experiment. You can then compare the results for different distances between the lines. You can also do this with a square grid and consider how many times not touching a line.

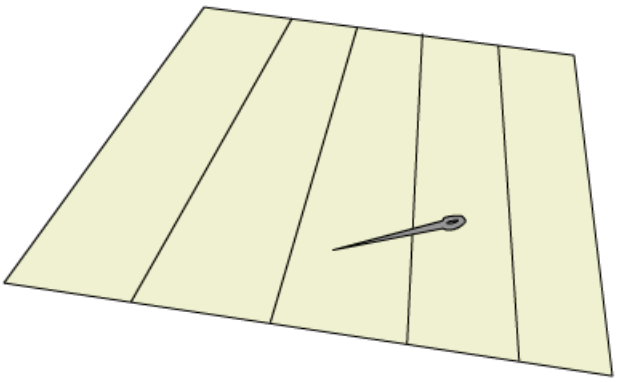
TYPE:	Plenary
OBJECTIVE(S):	Consider experimental probability; appreciate the difference between mathematical explanation and experimental evidence.
DESCRIPTION:	1 is factors affecting experiments. 2 is factors affecting experimental probabilities. 3 is SATs page. 4 is vocabulary.
OVERVIEW:	Buffon's needle. You drop 'needles' onto lined paper.
EQUIPMENT:	None.

Main Whiteboard and Screen information

Designing an experiment

You are dropping a needle onto a piece of paper with lines drawn on it. What are the possible outcomes? What do you think will happen? How often do you think that the needle will land overlapping one of the lines? What factors do you think will affect the outcome? Click Next to continue.

Pen on



Next

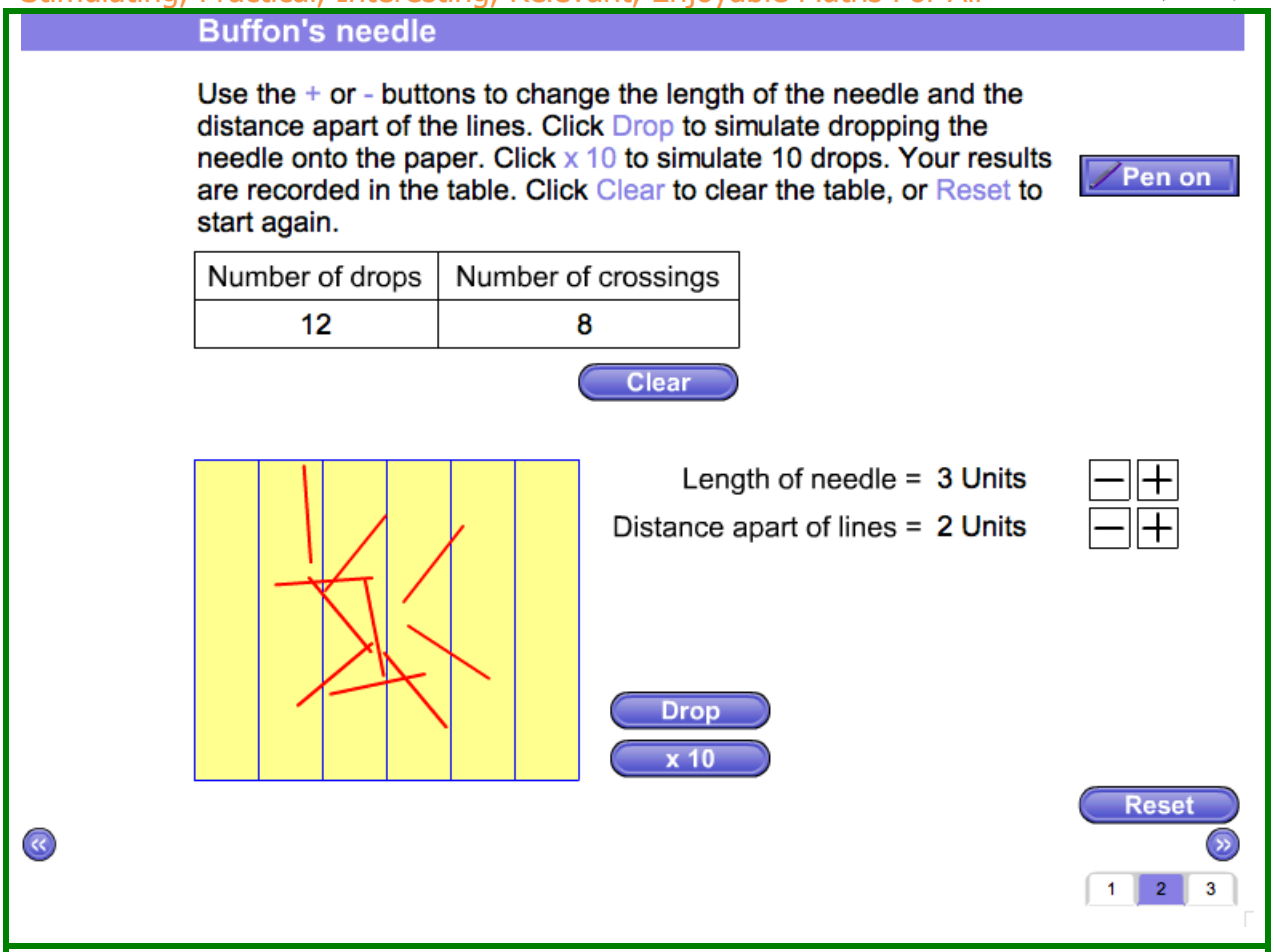
Reset

1 2 3

Screen 1: Designing an experiment

You are told that you are dropping a needle onto a piece of paper with lines drawn on it and are asked for the possible outcomes. You are also asked how often you thing that the needle will land overlapping one of the lines and what factors will influence this. You are then given four factors in yellow boxes and asked to click on those that are relevant.

Key points: pupils should consider possible outcomes and may have other things; note that some pupils might say that the temperature could have an impact – but any increase in length is negligible.



Screen 2: Buffon's needle

The activity is known as Buffon's needle after the eighteenth century French mathematician who carried out this experiment. It consists of dropping a needle onto a piece of paper with a set of parallel lines on it. You can vary the length of the needle and the distance between the lines. You can 'drop' the needle once or ten times and the results are recorder in a table.

Key points: pupils should carry out this experiment and see what happens; they will need to consider how to eliminate other potential variables (for example, the height of the drop) and apply principles from science to make sure it is a fair experiment; pupils should make a conjecture before undertaking this experiment.

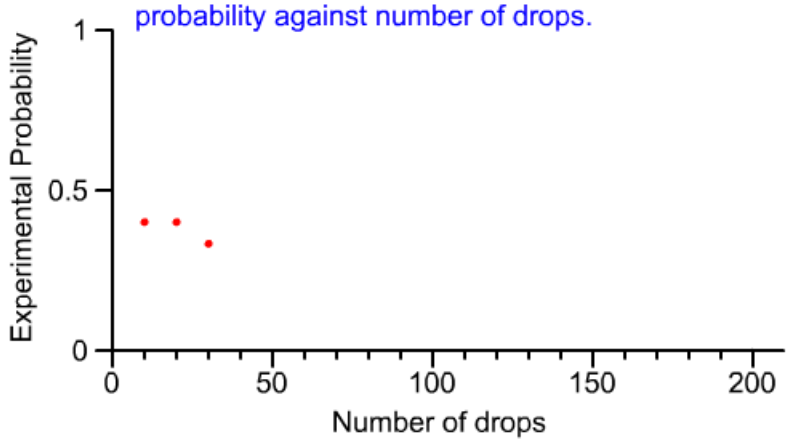
Experimental probability

The results of the experiment after 30 drops are shown below. Click Graph off to hide the graph. Click Next to see the results of more trials.

Pen on

Number drops	10	20	30
Number of crossings	4	8	10
Experimental probability	0.4	0.4	0.333

Scatter graph showing experimental probability against number of drops.



Graph off
Next
Reset

1 2 3

Screen 3: Experimental probability

You are given the results of the experiment in a table after 10, 20, 30, 50, 100, 150 and 200 throws (table is added to after each Next). The experimental probability is given as well. A graph of the experimental probability against the number of drops can be shown (and then turned off). You are asked to comment on the complete graph and the actual probability of this needle crossing this line.

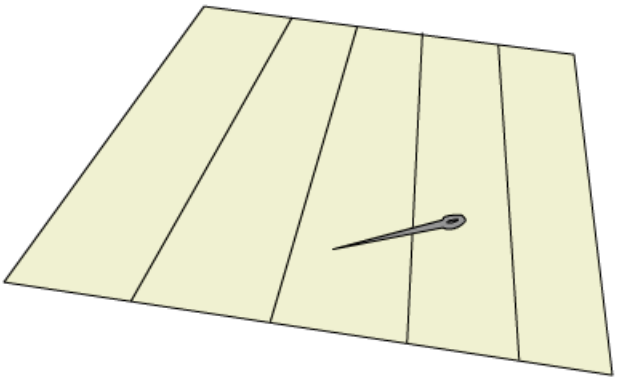
Key points: pupils should discuss what they see and make comments about the relative lengths of the needle and the distances between the lines – this is where it will help if pupils have experimental results themselves (and have shared them).

Plenary Whiteboard and Screen information

Extending the experiment

You are conducting an experiment dropping a needle onto a piece of paper with lines draw on it. How would you alter your experiment to make it more or less likely for the needle to land crossing a line? What factors would you change? What factors would you keep the same? Click Next to continue.

Pen on



Next

Reset

1 2 3 4

Screen 1: Extending the experiment

You are told that you are dropping a needle onto a piece of paper with lines drawn on it and are asked what things you could change to make it more or less likely that the needle will cross a line. You are then given four factors and asked if they will make it more or less likely for the needle to cross the line or if they are not relevant. You have to drag the factors into one of two columns. You are then asked how the likelihood would change if you were given a square grid rather than a grid of parallel lines.

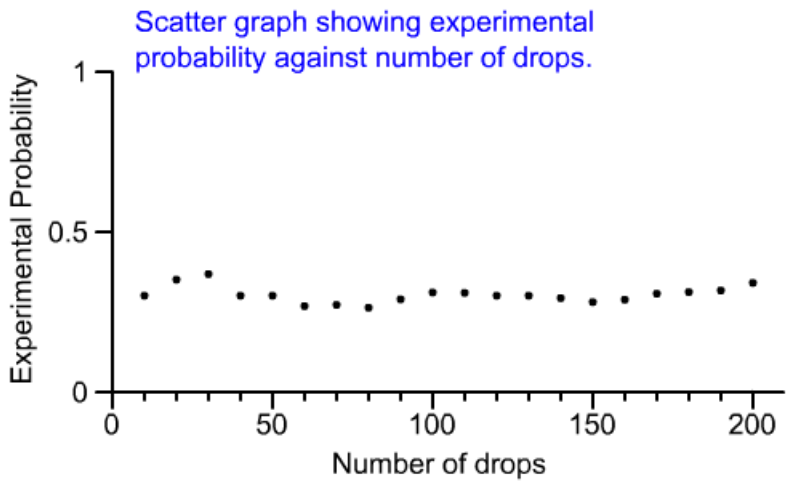
Key points: pupils should discuss how the factors are likely to alter things; if necessary you could have practical evidence to fall back on.

Grids

A needle was dropped 200 times onto a piece of paper with lines drawn on it. The experimental probability of the needle crossing a line as the number of drops increased is shown. How will changes in the length of the needle affect the experimental probability? Explain. Click Next to continue.

Pen on

Number drops	10	20	30	50	100	150	200
Experimental probability	0.3	0.35	0.367	0.3	0.31	0.28	0.34



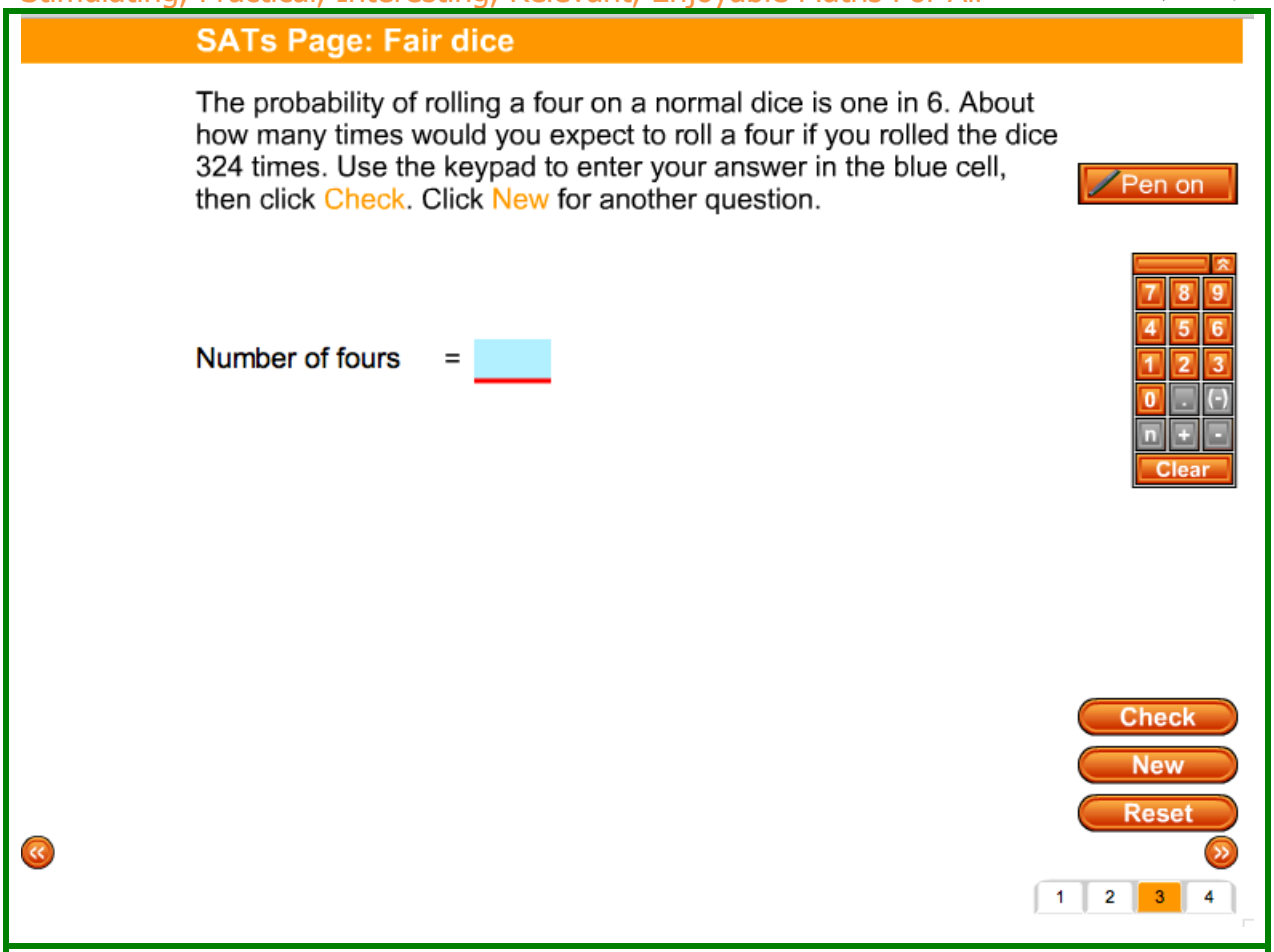
Next
New
Reset

1 2 3 4

Screen 2: Changing the experiment

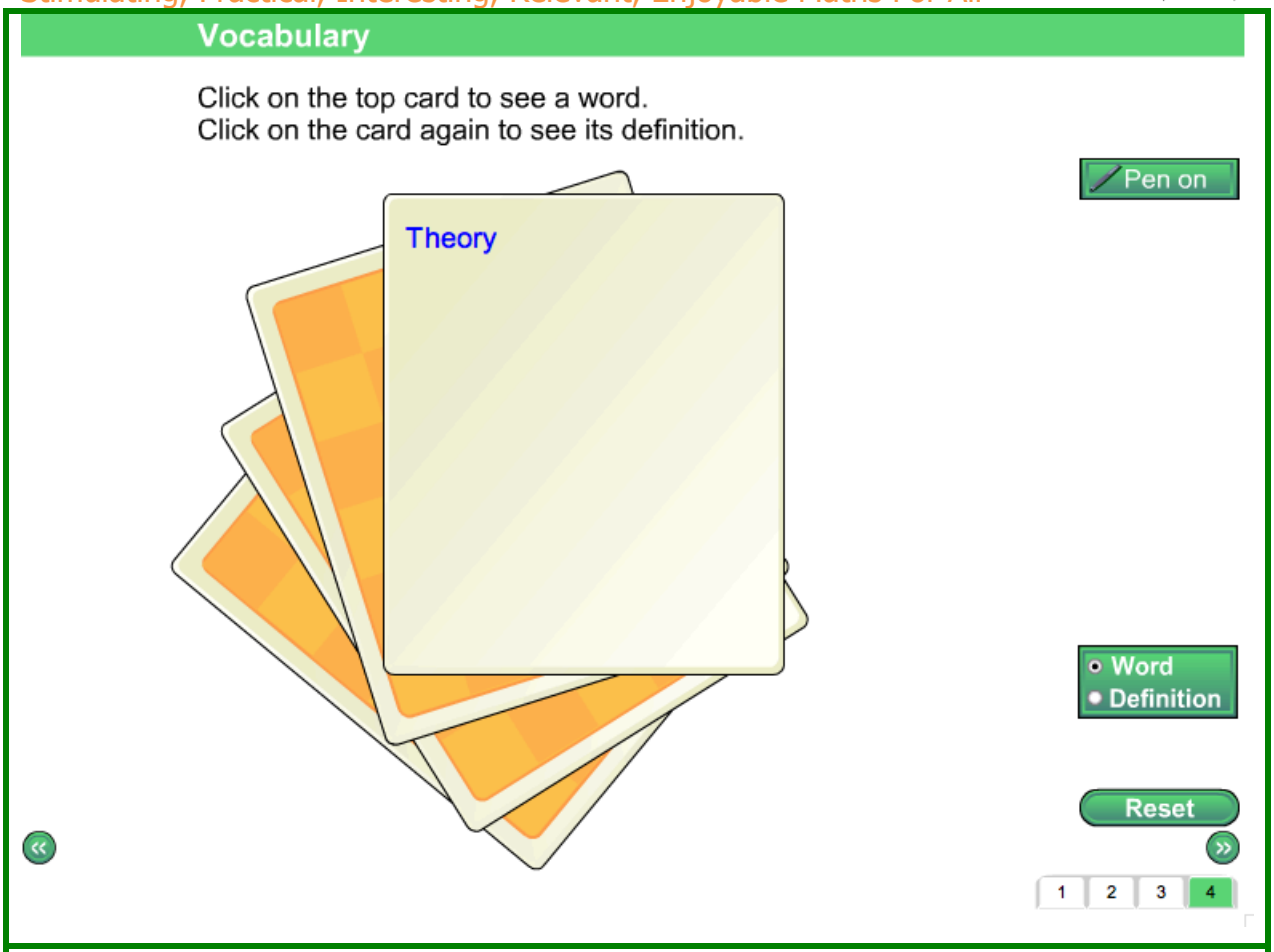
You are shown the data collected from 200 drops of a needle onto a piece of lined paper (the number of drops and the experimental probability linked to them). The graph of experimental probability is shown plotted in red against the number of drops. You can then choose to change the length of the needle or the distance between the lines. You are then given the corresponding experimental probabilities for the same number of drops for the new equipment. These experimental probabilities are plotted in blue on the same graph.

Key points: pupils should discuss what, if anything might change if they were to undertake the new experiment; they should try and justify their comments, which ideally should be based on having tried out similar experiments for themselves; you might want them to consider what happens if you double the length.



Screen 3: SATs page: Fair dice

You are asked one of three question types: the number of times you would expect to see a given number appear on a dice when you are given the total number of rolls; the probability of getting a given number on a biased dice when you have a table of frequencies; the number of a particular coloured counters in a bag you would expect when you know how many are there in total and how many times this colour has been drawn out so far.



Screen 4: Vocabulary

Vocabulary present: Biased, Certain, Chance, Chance, Doubt, Equally likely, Even chance, Event, Exhaustive, Fair, Fifty-fifty chance, Good chance, Impossible, Independent, Likelihood, Likely, Limit, Mutually exclusive, No chance, Outcome, P(n), Poor, Possible, Probability, Probable, Random, Relative frequency, Risk, Sample, Sample space, Theory, Tree diagram, Uncertain, Unfair, Unlikely.

Spire Maths interactive files available in a flash format at: https://spiremaths.co.uk/ia/

Unfortunately they will not work on iPads or iPhones.