TSST 2015 Day 1, 10 November 2015

Resources

Will be found at: https://spiremaths.co.uk/ for activities completed in sessions and at: http://www.cimt.plymouth.ac.uk/ske/ for the Assessment tests and materials to support your mathematical development.

National Curriculum, Aims and Themes of Course

Background to mathematics NC, it's purpose and aims and the themes we would be following on the course – which is intended to support you for the next 5 – 10 years. These themes are:

- Concrete, Pictorial, Abstract (CPA)
- Multiple Representations
- Mastery and Mastery with greater depth
- Conceptual Understanding with Procedural Fluency
- Regularly Reasoning Mathematically
- Solving Problems: a natural part of the Mathematics Programme

Links are given to KS3 overview and the Progression Maps, that look like this:

Year 7	Year 8	Year 9
Structure		
understand and use place value for decimals, measures and integers of any size	state the multiplicative relationship between the numbers represented by any two digits in any number	state in the form A × 10^n (n any positive or negative integer) the multiplicative relationship between the numbers represented by any two digits in any number
order positive and negative integers, decimals and fractions	order positive and negative integers, decimals, fractions and numbers given in the form $\sqrt{n}$	order positive and negative integers, decimals, fractions and numbers given in the standard form A x 10^n $1 \leq A < 10$, where n is a positive or negative integer or zero

Example shown is taken from that for Number.

Place Value: Chairs activity

We then looked at place value chairs game: making numbers 10 times bigger and 10 times smaller (which could be extended to higher powers of 10 and then by e.g. tenths – misconceptions about multiplying making numbers bigger). Further worked on writing numbers in digits and words and re-transcribing.

Powers of 10

Leading from the chairs activity we looked at Headings of Th, H, T and 1s (or Units) and extended to Billions and to millionths (B and m). Noticing how e.g. 60,000 is 1000 times bigger than 60. Billions now in UK meaning 1000 Million (after Harold Wilson changed it in 1974 – see Statistical literacy guide1, 'What is a billion?' And other units http://researchbriefings.files.parliament.uk/documents/SN04440/SN04440.pdf.

The use of headings like 10^3 is also included, because it helps understanding, though it is rarely made explicit.

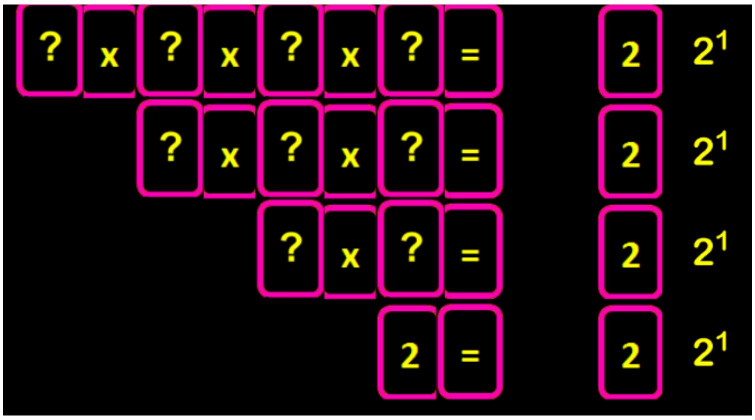
Powers of 2

Using patterns we explored the powers of 2, showing how conventions determine that $2^0 = 1$ since it makes for a consistent system of powers. We also extended the work to negative powers showing that e.g. 2^{-3} is the reciprocal of 2^3 (1 over).

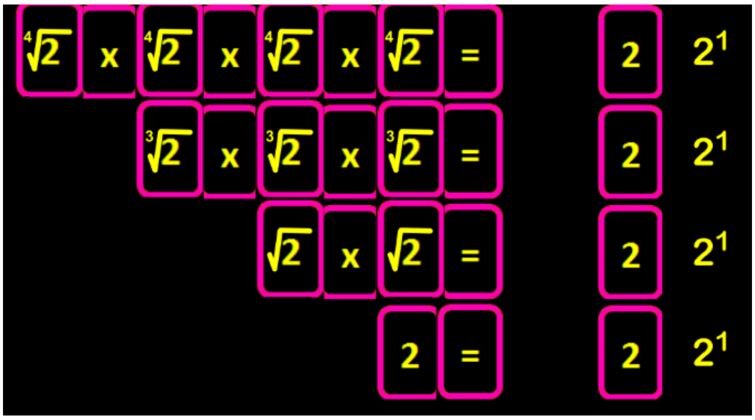
Fractional powers of numbers

I did not cover fractional powers but it is in the flipchart and smart notebook file. Looking at the diagram below the ? in the top line represents the same value multiplied by itself 4 times that equals 2 which is 2^1. This value is the fourth root of 2.

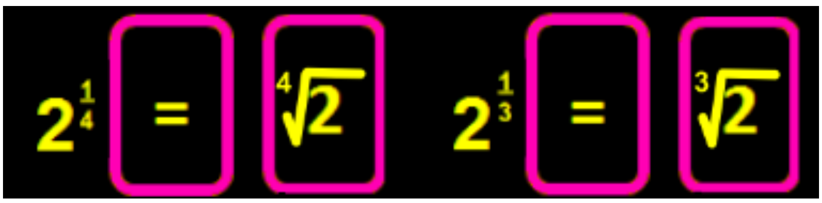
The second and third lines follow in a similar way for the cube root and the square root.

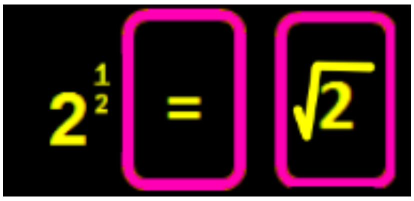


In the diagram that follows the ?s are replaced by the usual symbols for the fourth, cube and square roots.



In terms of powers of indices it means that the fourth root in the top line as a number is one quarter (since 4 quarters add up to one). So we have:





Summarising for Indices and Powers

If we have for example 64 raised to the power of minus two thirds then the answer is one sixteenth: i.e.

$$64^{\frac{-2}{3}} = \frac{1}{16}$$

You can do the powers in any order, so you could square the 64 (giving 4096), then cube root this (16); the final step is to do the minus which turns it into 1 over 16. Alternatively it is usually easier to do the root aspect first, because this keeps the numbers smaller, so cube root of 64 is 4; then this squared is 16 and the final step is the same.

The Clever, Lazy Mathematician

This is what we should be and encourage our pupils to be the same. Making sure that we use the notation and conventions of mathematics correctly, looking for solutions that are quick and short, but fully detailed.

Examples:

- 2^4 is better than 2 x 2 x 2 x 2 x 2
- 2 is better than 2^1
- 4y is better than 4 x y or y4 or y x 4
- a is better than 1a or 1 x a

Improving Leaning in Mathematics Resources

Over 50 activities with lesson plans etc. and IWB versions for Smart and Inspire: these are at: https://spiremaths.co.uk/ilim/. Booklet at: https://spiremaths.co.uk/ilimcs/. The resources are built on some underlying principles:

(i) Build on the knowledge learners bring to sessions
(ii) Expose and discuss common misconceptions
(iii) Develop effective questioning
(iv) Use cooperative small group work
(v) Emphasise methods rather than answers
(vi) Use rich collaborative tasks
(vii) Create connections between mathematical topics
(viii) Use technology in appropriate ways

We also compared a ‘Transmission’ view with a ‘Connected, challenging’ view, as follows:

A 'Transmission' view		'Connected', 'challenging' view
A given body of knowledge and standard procedures that has to be 'covered'.	**Mathematics is …** ⟷	An interconnected body of ideas and reasoning processes.
An individual activity based on watching, listening and imitating until fluency is attained.	**Learning is …** ⟷	A collaborative activity in which learners are challenged and arrive at understanding through discussion.
Structuring a linear curriculum for learners. Giving explanations before problems. Checking that these have been understood through practice exercises. Correcting misunderstandings.	**Teaching is …** ⟷	Exploring meaning and connections through non-linear dialogue between teacher and learners. Presenting problems before offering explanations. Making misunderstandings explicit and learning from them.

Then we started SS1: Classifying Shapes, where we sorted shapes into different groups according to their properties. I shared out examples of Number, Algebra and Statistics work.

At the end of the day we looked at the card sort activity for N12: Using Indices which linked back to earlier work in the day. You worked them out and tried to match them.

$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$	x^{-3}	x^3	x^2	$\sqrt{x}$
$x^{-\frac{1}{3}}$	x^{-2}	$\frac{1}{2x^2}$	$x^{\frac{2}{3}}$	$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^3}$	$x\sqrt{x}$
x^{-4}	$\frac{1}{2\sqrt{x}}$	x^0	$x^{-\frac{3}{2}}$	$x^{-\frac{1}{2}}$	$2x^{-2}$	$\frac{\sqrt{x}}{x}$	x^4
$\frac{2}{\sqrt{x}}$	$\frac{1}{x}$	$\frac{1}{2x}$	$\sqrt[3]{x^2}$	$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{2}x^{-2}$	$x^{\frac{1}{3}}$	x

The lesson plan suggests that for some pupils they can be used in a different way, turning one over and looking for matches as more are added and also seeing which pair might be the equivalent to a single one. There were number versions as well.

CIMT website

http://www.cimt.plymouth.ac.uk/ske/

We looked together at the Introductory Problem for Strand B in groups and managed to find the solution, though each table did something different. We found that the problem was special to the numbers in the 50s. There was some conjecturing and reasoning and together we established that the results follow because e.g.:

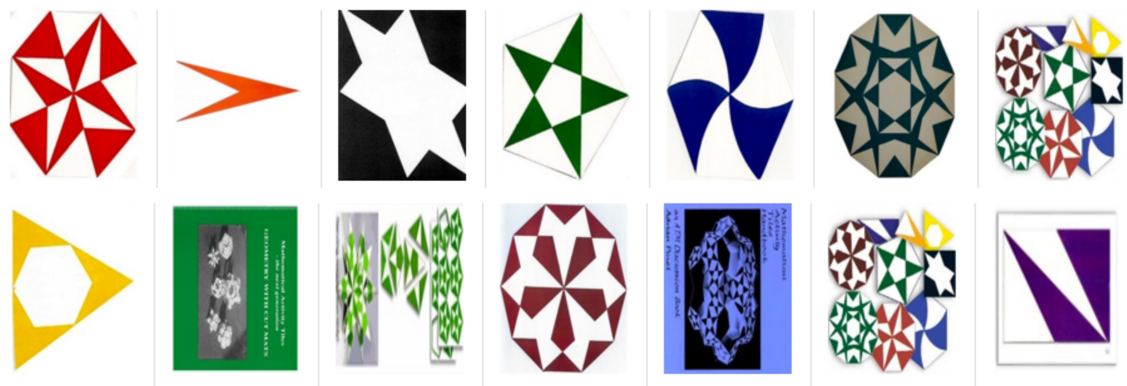
$$(50 + 2)(50 + 2) = 2500 + 200 + 4$$

and the 50 is crucial. It was quite a step to get to this from noticing this from $52^2 = 2704$ and $54^2 = 2916$ suggests that 57^2 will have first two digits are $5^2 + 7 = 32$ and second 2 digits 49 meaning that $57^2 = 3249$. A familiar algebraic technique (multiplying brackets) used with partitioned numbers to prove why something is true. Something to add to your problem solving 'armoury'.

Association of Teachers (ATM) Mats

The Mats and booklets related to their use are found at:

http://www.atm.org.uk/corecode/search/SearchResults.aspx?q=mats (product tab)



We hardly touched the surface of how they might be used, but I did suggest that you look at the Platonic solids which are made by using only one shape with the same number of the shape meeting at each vertex (I mistakenly referred to them as the Archimedean solids.)

Really good site for seeing solids is http://www.korthalsaltes.com/ though annoyingly it now has a lot of adverts.

For solid animations http://www.mathsisfun.com/geometry/polyhedron-models.html?m=Dodecahedron is well worth a visit.

If any of you have good 3D models then it would be good if you could bring an example along next time.

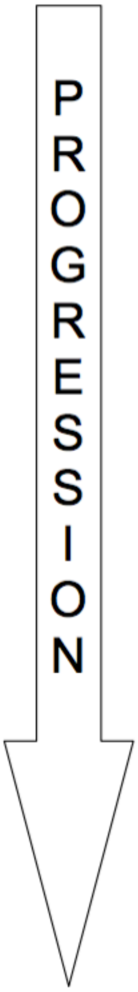
I mentioned Euler's relationship $V + F = E + 2$: I just consider the cube, I don't need to remember the rule other than noting that 2 is important.

The ATM are currently offering a free journal for a limited time (you just have to register your details - which is free). http://www.atm.org.uk/Special-Edition-MT249

Bowland Assessment Materials

Info can be found via my blog at: https://spiremaths.co.uk/bowlanda/. You tried the 110 years on and came up with a number of different answers that offered a range of solutions. I then gave each of you, at random, an example of another of the 35 activities. Each activity also includes a Progression in Key Processes grid (see below) with accompanying pupil responses.

Progression in Key Processes



Representing	Analysing	Communicating and reflecting
Simplification of the information and assumptions to complete the model. A method to represent multiplication and/or time	Accurate calculations and logical working	Clear communication throughout
Draws a simple diagram or lists some key events sequentially. Uses given facts, eg uses 4 (or 3) for the number of children born to the girl but 2 for their final generation Pupil A	Uses a counting method to find the number of great and great, great grandchildren.	Shows which generation is which, and the number of children born in each generation Pupil A
Draws a diagram showing at least 3 generations or a timeline with some key events shown Pupil A Even if not explicit, the number of years between generations is clearly implied Pupils B+C	Uses a counting method to find the total number of descendants Pupil A	Show their methods such that someone else can follow their reasoning reasonably well Pupils B+C
Chooses a method to represent both multiplication and time Pupils B and C. Even if not explicit, the years between generations and at what age people died are clearly implied Pupil D	Uses a multiplicative method to find the number of great or great, great grandchildren Pupils B+C	Uses methods that are explicit and 'flow'
Chooses an effective method to represent both multiplication and time. Assumptions are explicit and detailed, eg as previously and all descendants have children, and premature deaths are to be ignored Pupil D	Uses a multiplicative method to find the total number of descendants Pupil D	Communicates clearly, effectively and concisely throughout, using a method that could extend to further generations Pupil D

1 million Cubic Centimetres

Using Dienes blocks we 'made' a volume of 1 cubic metre = 1,000,000 cm^3 = 100^3 cm^3. Pupils don't always realise how easy it can be to represent a million.

Homework – between session tasks

Research suggests that you get more from a course if you use some of the learning in the very near future, so you were asked to use, if you can fit it in:

- any Bowland activity
- any Improving Learning in Mathematics activity
- anything else related to the day

Have fun!