Fractions & Multiple Representations
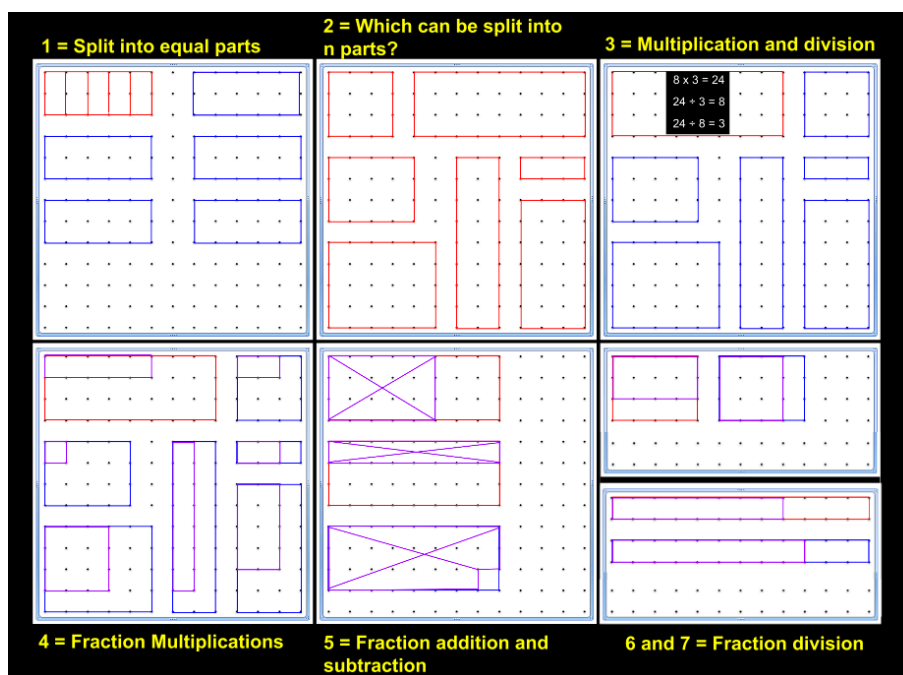

8 x 3 = 24
24 ÷ 3 = 8
24 ÷ 8 = 3

A Spire Maths Activity

Table of Contents

Summary of Activities

This overview shows an approach to using multiple representations for fraction calculations. It makes use of square geoboard grids (e.g. through iPad app GeoDraw – click link below) or online grids such as that found on nRich or through use of equivalent paper or geoboards (bought ones are usually 11 by 11). It provides:

- pictorial representations for the four operations
- relates multiplication of fractions to multiplication of numbers
- allows a common representation for the other three operations
- offers a number of different representations for division
- explains the standard algorithm for division of fractions

GeoDraw the iPad app

nRich Geoboard (uses flash) http://nrich.maths.org/2883

for printable paper see: https://spiremaths.co.uk/printable-paper/

Two helpful articles are *One third...of what? The interpretation of fractions in KS2 and KS3* which, amongst other things, comments on the importance of a fraction being considered as a number, and *Relational Understanding and Instrumental Understanding* (Skemp 1976) which summarises these two 'understandings' and why we need to attach importance to conceptual understanding as well as procedural fluency. They can be found, respectively, at the following web addresses:

https://www.ncetm.org.uk/resources/45492
http://mathed.net/wiki/Skemp_(1976)

ActivInspire file: http://bit.ly/fractionrepsflip
PDF file: http://bit.ly/fractionrepspdf

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Rectangles: Equal parts 1 and 2

Rectangles: Equal Parts 1

Here is a 13 by 13 square dotted grid with a red 5 by 2 rectangle on it.

The red rectangle is split into 5 equal parts, using the horizontal and vertical grid lines.

1. Still using the horizontal and vertical grid lines split each blue rectangle into a different number of equal parts.

2. What do you notice?

3. Start with a different rectangle and repeat 1 and 2 for it.

4. What do you notice?

1. The first activity involves making rectangles and splitting them into equal parts. So the 5 by 2 rectangle (across then up/down) can be split, using the vertical and horizontal lines of the grid only, into 10 equal parts of area 1; 5 of area 2; 2 of area 5 and 1 of area 10. Also covering work of factors.

Rectangles: Equal Parts 2

Here is a 13 by 13 square dotted grid with 7 red rectangles on it.

1. Using the horizontal and vertical grid lines which, if any, of them can be split into 2 equal parts.

2. Which into 3 equal parts?

3. What do you notice?

2. The next activity looks directly at the connection of parts to the shape of the rectangle.

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Rectangles: Multiplication 1 and 2

Rectangles: Multiplication 1

Here is a 13 by 13 square dotted grid with some rectangles on it. The red rectangle shows

$$8 \times 3 = 24$$

$$24 \div 3 = 8$$

$$24 \div 8 = 3$$

1. What calculations are shown by each of the blue rectangles?

3. Here rectangles are linked to multiplication tables and the equivalent divisions. Note that the consistent approach here is that the row of 8 is repeated 3 times as the interpretation of 8 x 3 – but this need not be the case, as it is not made explicit here.

Rectangles: Multiplication 2

Here is a 13 by 13 square dotted grid with some rectangles on it. The red rectangle shows

$$\frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$$

1. What calculations are shown in each of the blue rectangles?

4. In the diagram above the red rectangle represents the multiplication of 1 by 1 and the pink part within it shows $\frac{5}{8}$ across and $\frac{1}{3}$ down. This leads to the

multiplicatior $\frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$ resulting in the answer being given as the area of

the resulting pink rectangle (since each small square in the red rectangle represents one twenty-fourth of the 1 unit rectangle. Here the fraction is considered just like a number – and it is important that this is noted.

Rectangles: Addition and Subtraction of Fractions

5. Here the pictorial representations are of $\frac{5}{8}$ and $\frac{1}{3}$ since they hare to be added,

they require the same Lowest Common Denominator ((LCD), here 24, hence the first two diagrams shown above. The diagram that results just places all the parts in the diagrams above into the same shaped rectangle (if there are more then 24 then a second rectangle of the same size is needed and gives a result of over 1.

Subtraction can be considered at the same time since the diagrams are the same and the process is virtually the same (it does help to make sure that the larger one is put first).

The result here is:

$$\frac{5}{8} + \frac{1}{3} =$$

$$\frac{15}{24} + \frac{8}{24} = \frac{23}{24}$$

Rectangles: Division of Fractions

Rectangles: Division of Fractions

Here is a 13 by 13 square dotted grid with two rectangles on it. The shaded part of the red rectangle shows

$$\frac{2}{3}$$

The shaded part of the blue rectangle shows
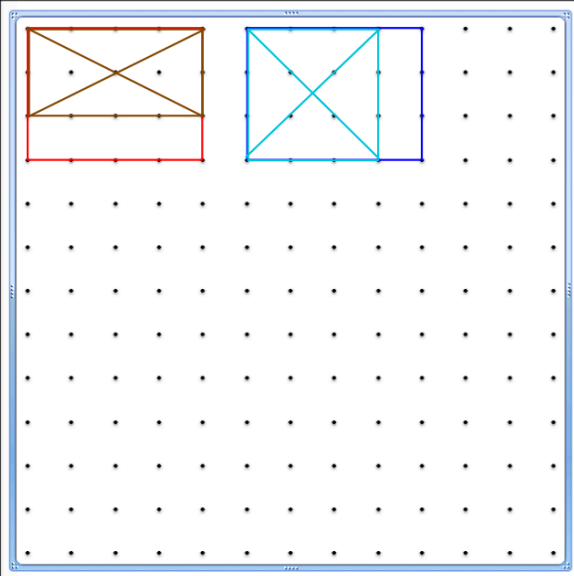
$$\frac{3}{4}$$

1. Using this explain why

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$$

2. And why

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$$

3. Do some more fraction divisions.

6.   Here division of fractions is through a pictorial representation where the LCD is again used. So for thirds and quarters the LCD is 12. The diagrams show that

$$\frac{2}{3} = \frac{8}{12} \text{ and } \frac{3}{4} = \frac{9}{12}$$

so when the division is $\frac{2}{3} \div \frac{3}{4}$ then the solution is $\frac{8}{9}$ since the first fraction has 8 squares shaded and the second has 9 shaded, so the division is equivalent to 8 ÷ 9. Giving $\frac{8}{9}$ as the answer.

When the division is the other way round then the equivalent division is 9 ÷ 8, giving $1\frac{1}{8}$ as the answer.

Fraction Wall: Division of Fractions

7.   An alternative, but similar pictorial representation uses the fraction wall idea. Here the division requires that both fractions can be represented on the same row of the wall (i.e. the LCD of the fractions is needed) and then the first problem is seen as the equivalent, in this case, of 8 ÷ 9. So it is virtually identical to the case above.

Fraction Wall: Division of Fractions

Here is a 13 by 13 square dotted grid with two rectangles on it. The shaded part of the red rectangle shows

$$\frac{2}{3}$$

The shaded part of the blue rectangle shows
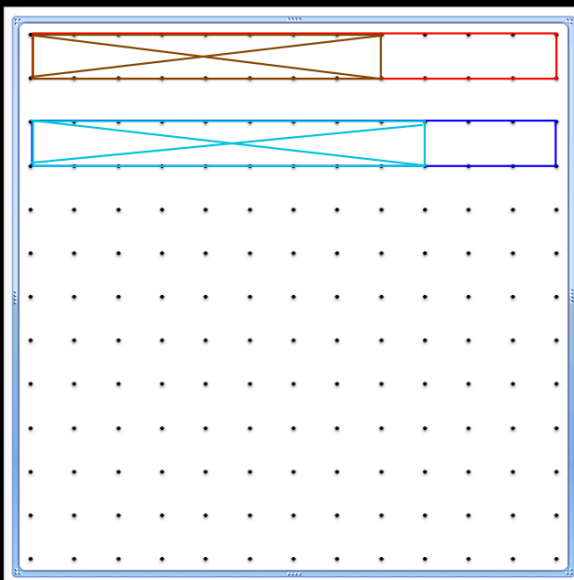
$$\frac{3}{4}$$

1. Using this explain why

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$$

2. And why

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$$

3. Do some more fraction divisions.

Equivalent Fractions: Division of Fractions

Equivalent Fractions: Division of Fractions

Here is a calculation to show that

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$$

1. Explain what is happening.

2. Do a similar calculation for

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$$

3. Do some more fraction divisions like this.

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12}$$

$$= \frac{\frac{8}{12}}{\frac{9}{12}}$$

$$= \frac{8}{9}$$

8. This method of dividing fractions converts each of the fractions into the division into fractions with the LCD (i.e. as the two earlier methods do) and then uses the fact that division can be considered as the creation of a fraction where the dividend (the number to be divided) is the numerator and the divisor is the denominator. Then the two fractions (twelfths above) can be multiplied by the LCD leading to the answer as shown.

Make Division into Multiplication: Division of Fractions

9. The final method gives the reason for the standard algorithm for dividing fractions. Like the previous method it uses division to be the conversion of a fraction and then looks to make the denominator 1 – a similar sort of process is used in rationalising denominators for both surds and complex numbers so it is useful to show this now and to allude to this as an important technique for later/higher mathematics.

Make Division into Multiplication: Division of Fractions

Here is a calculation to show that

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$$

1. Explain what is happening.

2. Do a similar calculation for

$$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$$

3. Do some more fraction divisions like this.

$$\frac{2}{3} \div \frac{3}{4} = \frac{\dfrac{2}{3}}{\dfrac{3}{4}}$$

$$= \frac{\dfrac{2}{3} \times \dfrac{4}{3}}{\dfrac{3}{4} \times \dfrac{4}{3}}$$

$$= \frac{2}{3} \times \frac{4}{3}$$

$$= \frac{8}{9}$$

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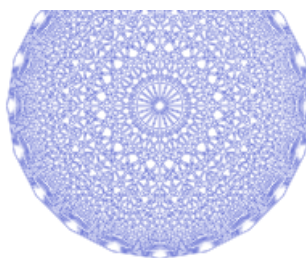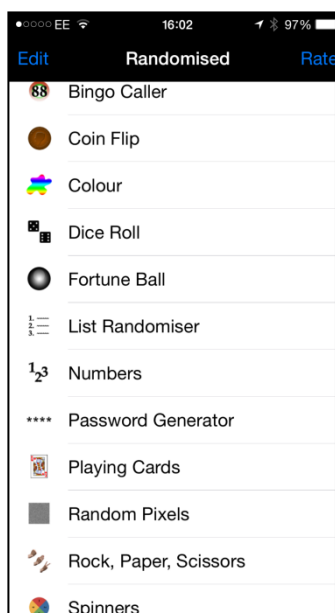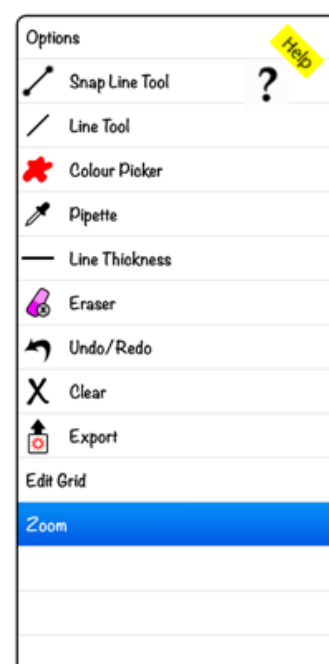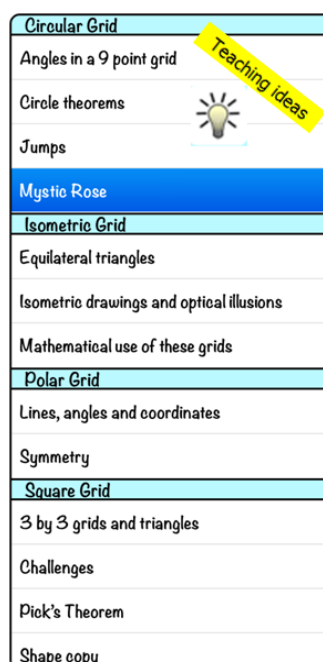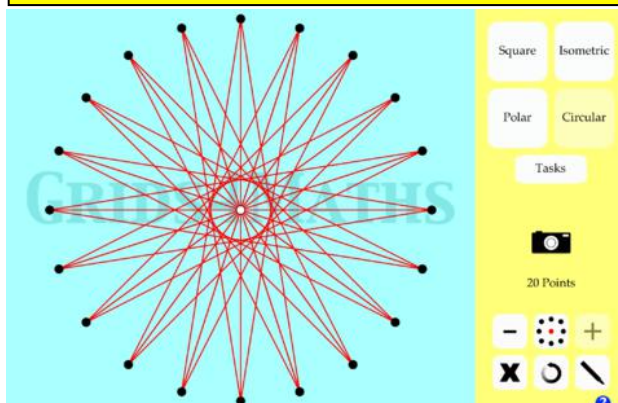- GeoDraw offers users a choice of 5 grids for use in mathematics and D&T lessons. Send/export images with/without grid using: Bluetooth, Email, Facebook, Twitter and into Pages or Keynote.

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Choose Time

21 years 10 months 28 days

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Age in Months: 262
Age in Weeks: 1143
Age in Days: 8003
Age in Hours: 192072
Age in Minutes: 11524320
Age in Seconds: 691459204

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